MOTION OF CONFINED PARTICLES*

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In honour of Helmut Satz on the occasion of his 80th birthday and the recent awarding of an Honorary Doctor title at the University of Wrocław

We carry out numerical evaluations of the motion of classical particles in Minkowski space $\mathbb{M}^4$ which are confined to the inside of a bag. In particular, we analyze the structure of the paths evolving from the breaking of the dilatation symmetry, the conformal symmetry and the combination of both together. The confining forces arise directly from the corresponding nonconserved currents. We demonstrate in our evaluations that these particles under certain initial conditions move toward the interior of the bag.

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1. Introduction

The problem of the confinement of quarks and gluons has been discussed at many different levels. Here, we investigate the motion of a particle which is confined inside a region from the inward pressure B of the surrounding vacuum with a fixed energy density B. These properties are basic to the Bag Model [1]. In the following, we limit our discussion to the consequences of this structure on the motion of a single particle (parton) confined to the interior of the bag. Next, we discuss the dilatation current $D^\mu(x)$ and the conformal currents $K^{\mu\alpha}(x)$ for $\mathbb{M}^4$. Due to the confining conditions in the Bag Model, the presence of a finite trace for the energy-momentum tensor leads to the breaking of the dilatation and conformal symmetries. In the following sections, we show how this fact changes the motion.

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2. Fundamental currents

In this section, we describe the dilatation current $D^\mu(x)$ and the four conformal currents $K^{\mu\alpha}(x)$ in relation to the energy-momentum tensor $T^{\mu\nu}(x)$ where $\alpha, \mu, \nu = 0, 1, 2, 3$ in terms of the space-time vector $x^\mu \in \mathbb{M}^4$ with a positive-time metric $g_{\mu\nu}$. The dilatation current is defined as [2]

$$D^\mu(x) = x_\nu T^{\mu\nu}(x) . \tag{2.1}$$

The four conformal currents are written as

$$K^{\mu\alpha}(x) = \left(2x^\alpha x_\nu - g^\alpha_\nu x^2\right) T^{\mu\nu}(x) . \tag{2.2}$$

The basic equations for these currents with finite trace T^μ_μ are given in terms of the divergence in $\mathbb{M}^4$ as

$$\partial_\mu D^\mu(x) \;=\; T^\mu_\mu , \tag{2.3}$$
$$\partial_\mu K^{\mu\alpha}(x) \;=\; 2x^\alpha T^\mu_\mu . \tag{2.4}$$

3. Broken symmetries

The presence of a finite trace $T^\mu_\mu \neq 0$ in Eqs. (2.3) and (2.4) provides for broken dilatation and conformal symmetries. This situation arises from the contrapositive statement of Noether's theorem, which yields from the nonconserved currents the broken symmetries. Thereby, we conclude that

$$\partial_\mu D^\mu(x) \neq 0 \;\Rightarrow\; \text{scale symmetry is violated} , \tag{3.1}$$
$$\partial_\mu K^{\mu\alpha}(x) \neq 0 \;\Rightarrow\; \text{the conformal symmetry is broken} . \tag{3.2}$$

Physically it is known that for a violation of translational symmetry forces arose that change the motion. Similarly for a broken rotational symmetry, a torque was present. We have previously discussed the implications of Eqs. (2.3) and (2.4) for the 1+1 dimensional space-time $\mathbb{M}^2$ [3, 4].

4. Confining forces inside the bag

We recall that the inward bag pressure $-B$ and the energy density B give rise to a diagonal energy-momentum tensor $(+B, -B, -B, -B)$, which yields

$$T^\mu_\mu = g_{\mu\nu} T^{\mu\nu} = 4B . \tag{4.1}$$

Next, we describe the attractive force along the world line of the moving particle by the *dyxle force* $\mathcal{D}(x)$:

$$\mathcal{D}(x) = D^\mu x_\mu = B x^\mu x_\mu = B\tau^2 , \tag{4.2}$$

whereby τ is the proper time. Similarly, the *fourspan* $\mathcal{K}^{\mu\alpha}(x)$ is the combination of the four conformal currents $K^{\mu\alpha}(x)$ so that [5]

$$\mathcal{K}^{00}(x) = B\left(2\left(\left(x^0\right)^2 - \tau^2\right)\right)\,; \tag{4.3}$$

$$\mathcal{K}^{ii}(x) = B\left(2\left(x^i\right)^2 + \tau^2\right) \quad \text{with} \quad i = 1,2,3\,; \tag{4.4}$$

$$\mathcal{K}^{\iota\kappa} = 2Bx^\iota x^\kappa \quad \text{with} \quad \iota,\kappa = 0,1,2,3 \quad \text{and} \quad \iota \neq \kappa\,. \tag{4.5}$$

5. The confinement of the particles inside the bag

For the formation of the hadrons in the Bag Model [1,6], we take M_h as the hadronic mass and V_h as the hadronic volume. Thus, the hadronic energy density ϵ_h is given by

$$\epsilon_h = M_h/V_h\,. \tag{5.1}$$

In the case of the Bag Model, it has been pointed out [1,6] that

$$\epsilon_h = 4B\,. \tag{5.2}$$

It has been stated above that the hadronic energy density inside the bag is B. In addition, it has been shown that there are three parts of the parton kinetic energy density adding up to $3B$. Thus, putting it together for the Bag Model, from Eqs. (4.1) and (5.2), we find that

$$T^\mu_\mu = 4B = \epsilon_h\,. \tag{5.3}$$

6. Numerical evaluation of the motion in $\mathbb{M}^4$

We calculated the trajectories of the discussed forces in a semi-classical fashion. The trajectory of the dyxle force is shown in Fig. 1. The data show that the parton moves from the outer light cone in the interior of the bag. The trajectory is first more directed to the time axis as an effect of the Minkowski metric. As a comparison, in $\mathbb{M}^2$, we can use Fig. 1 (left) in [4], for which our present results show a strong similarity in the plane between the x_1 and x_2 axes. In both the cases of $\mathbb{M}^2$ and $\mathbb{M}^4$, the dyxle force converges rapidly toward the origin of the confined system.

The effect of the fourspan is shown in Fig. 2. The two-dimensional plot shows that the parton moves between positive and negative values of the spatial axes x_1 and x_2 (left). From the calculated data, one can see that the parton moves aside and is forced on a kind of rounded orbit in four-dimensional space-time. The dependence of the fourspan on the time also causes a loop in the time direction (right). The comparison with Fig. 1 (right) in [4] brings out clearly the additional structure of $\mathbb{M}^4$ of the fourspan.

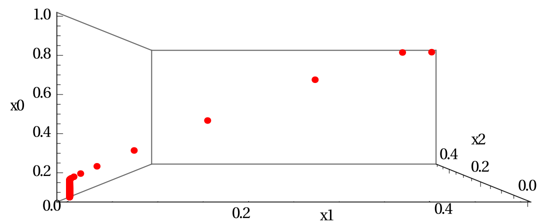

Fig. 1. Three-dimensional representation of the trajectory due to only the dyxle force with time axis x_0, the two spatial axes x_1 and x_2, with $B = 0.1$ GeV^4. The initial values are $x_0 = 1.0$; $x_1 = x_2 = x_3 = 0.5$, in fm.

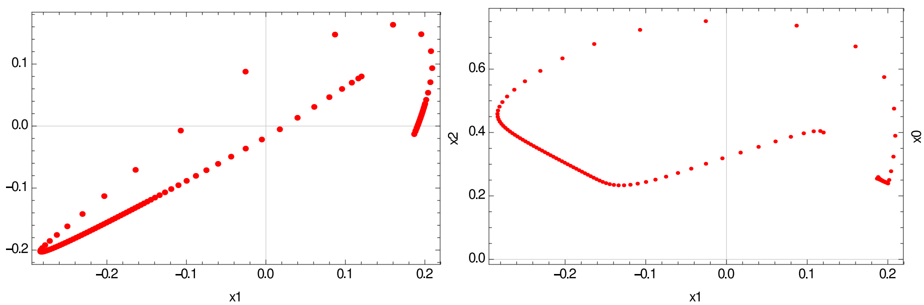

Fig. 2. Two-dimensional representations of the trajectory due to only the fourspan with the spatial axes x_1 and x_2 (left) and with the space-time axes x_1 and x_0 (right), with $B = 0.01$ GeV^4. The initial values are $x_0 = 0.4$, $x_1 = 0.12$, $x_2 = 0.08$, $x_3 = 0.06$, in fm.

The physically relevant situation is the combination of dyxle force and fourspan since the trace anomaly breaks both symmetries simultaneously, see Fig. 3. The trajectory makes only one movement to the negative spatial

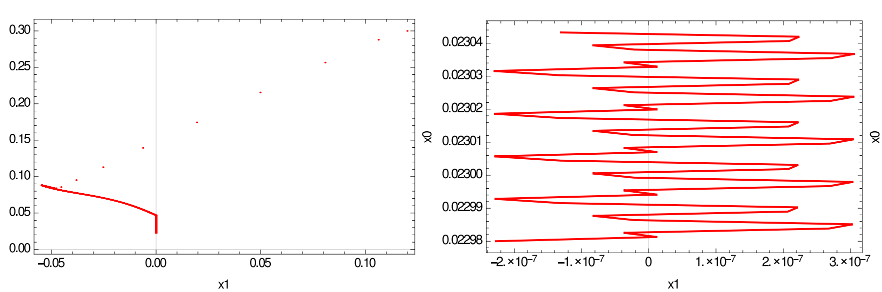

Fig. 3. Two-dimensional representation of the trajectory due to the dyxle and the fourspan together with the time axis x_0 and the spatial axis x_1, with $B = 0.1$ GeV^4. Full range of date (left) and the last 50 values (right). The initial values are $x_0 = 0.3$, $x_1 = 0.12$, $x_2 = 0.08$, $x_3 = 0.1$, in fm.

axis and then the overwhelming dyxle force pulls the parton toward the time axis and further toward the origin (left). The fourspan component of these combined forces possesses a permanent fluctuation around the time axis (right).

7. Summary and conclusions

In this work, we have presented our results on the motion of the internal constituents (partons) inside the confining structure of the bag. Whereby we have seen how the presence of the bag pressure has led to symmetry breakings which brought the presence of physical forces which altered the parton motion. Under certain conditions, we have found that the resulting motion is toward the properly chosen center of the bag.

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