FLUCTUATIONS OF CHARGES AT THE PHASE BOUNDARY*

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(Received February 16, 2017)

Properties of fluctuations of conserved charges in thermal equilibrium are discussed. Particular emphasis is put on possible origin of deviations from the Skellam distribution in higher order cumulants of net-baryon-number fluctuations around the chiral critical line and those of net-electric charge fluctuations at the chemical freeze-out. Importance of understanding the reference distribution is stressed.

DOI:10.5506/APhysPolBSupp.10.675

1. Introduction

The search for Quantum Chromodynamics (QCD) phase transitions has been one of central objectives in heavy-ion collision experiments. The successful description of various particle yields produced in the collisions with the grand canonical ensemble of a hadron gas [1] may allow for an interpretation of event-by-event fluctuations of particle numbers as the equilibrium fluctuations in a subsystem, which can be calculated in first-principle lattice QCD (LQCD) and effective models [2].

Recent LQCD calculations indicate that the transition at finite temperature and small baryon density is of smooth crossover at physical quark masses [3]. Moreover, the crossover region seems consistent with the O(4) scaling behavior [4] in the case of broken $\mathrm{U}(1)_{\mathrm{A}}$. Although the phase structure at larger density is not yet known from LQCD due to the sign problem, model calculations suggest rich phase structure [5]. In particular, discovery of the QCD critical point (CP) is the primary object of the beam energy

* Presented at the "Critical Point and Onset of Deconfinement" Conference, Wrocław, Poland, May 30–June 4, 2016.

scan program in heavy-ion collision experiments [6]. It is expected that enhancement of the fluctuations of conserved charges signals the existence of the critical point [7]. While the fluctuations will be the strongest at CP, those induced by the criticality also take place at the crossover region. Since the crossover transition can be regarded as a remnant of the second order chiral phase transition in the chiral limit, behavior of the fluctuations near chiral crossover also inherits this in the chiral limit. In this case, divergence is smeared into sign changes [8, 9]. As the system goes from the transition point, the remnants from the criticality become weaker and, finally, may result in a small deviation from non-critical reference value which is normally taken to be that of non-interacting gas [10]. Therefore, one also needs to understand the non-critical references under experimental circumstances.

In this proceedings, I outline the property of the fluctuations of the conserved charges near the equilibrium chiral transition. I start with divergent cumulants of net baryon number in the chiral limit based on $\mathrm{O}(N)$ scaling theory. Then I discuss the smeared signal of the chiral transition at finite volume or finite quark mass on the basis of explicit model calculations [9]. Finally, I briefly discuss the electric charge fluctuations in a hadron gas which does not have any criticality to demonstrate a non-trivial deviation from the reference distribution [11].

2. O(4) criticality in net-baryon-number fluctuations

Cumulants of conserved charges $\langle(\delta N)^n\rangle_{\mathrm{c}}$ are convenient to characterize the property of fluctuations and theoretically can be obtained from derivatives of thermodynamic pressure $p(T, \{\mu_i\})$ with respect to chemical potentials μ_i $(i = B, Q, S)$

$$\chi_n^{(i)} \equiv \frac{\partial^n \left[p(T, \{\mu_i\})/T^4\right]}{\partial(\mu_i/T)^n} = \frac{\langle(\delta N)^n\rangle_{\mathrm{c}}}{VT^3} \,. \tag{1}$$

Provided that the pressure consists of a regular part p_0 and a singular part p_{sing} close to the chiral phase transition, the $\mathrm{O}(N)$ scaling theory gives $p_{\mathrm{sing}} \sim |T/T_{\mathrm{c}} - 1|^{2-\alpha}$, where α is the critical exponent of the specific heat and $\alpha \simeq -0.21(-0.08)$ in 3d O(4) (O(2)) universality class [8]. This feature leads to the divergent sixth order cumulant as the first divergent one at $\mu = 0$, since $\chi_{2n}^{(B)} \sim |T/T_{\mathrm{c}} - 1|^{2-\alpha-n}$. At $\mu_q = \mu_B/3 \neq 0$, the leading singularity is given by $\chi_n^{(B)} \sim (\mu_q/T)^n|T/T_{\mathrm{c}} - 1 + \kappa\mu_q/T|^{2-\alpha-n}$, where κ corresponds to the curvature of the $\mathrm{O}(N)$ critical line. Therefore, the third order net-baryon cumulant is the first divergent cumulant at finite baryon density. However, this divergent signal is smeared when the system has a finite size or there is explicit chiral symmetry breaking. Then the signal is weakened by the prefactor $(\mu_q/T)^n$ in the small μ_q region.

Figure 1 displays an example of the smearing of the divergent fluctuations of the net baryon number near $O(N)$ criticality. The results are obtained from LQCD in the strong coupling limit with the auxiliary field Monte-Carlo method, at vanishing quark masses[1]. Both third and fourth order cumulants are divergent according to the $O(N)$ scaling, but the finite volume effect smears the divergence to oscillation across the critical temperature. Such oscillation can be attributed to the peak structure of the second-order cumulant, as higher order cumulant is obtained by differentiating it with the chemical potential. Introducing non-vanishing quark mass also induces the similar effects even in the thermodynamic limit, as seen in model calculations [8]. Consequently, the criticality might emerge as a small deviation from the reference value, *i.e.*, hadron resonance gas.

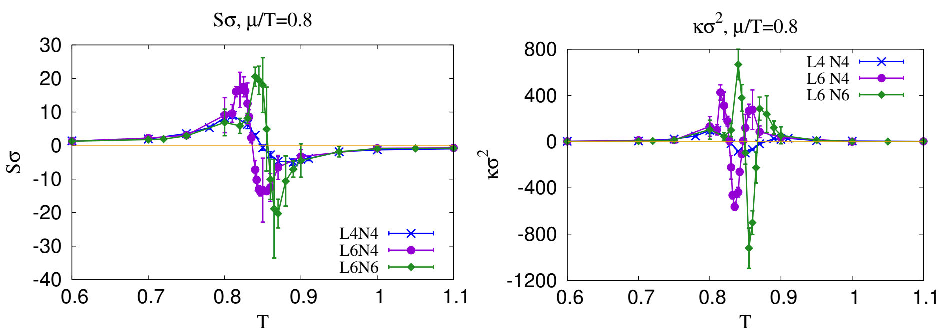

Fig. 1. The third (left) and fourth (right) order cumulants of the net baryon number normalized by the second order one ($S\sigma = \chi_3/\chi_2$ and $\kappa\sigma^2 = \chi_4/\chi_2$), obtained from LQCD in the strong coupling and chiral limit. Figures are adopted from Ref. [9].

In the hadron resonance gas, the cumulants are expressed by the sum over hadron species j [10]

$$\chi_n^{(i)} = \sum_j \left(\frac{m_j}{\pi T}\right)^2 \sum_{k=1}^{\infty} k^{n-2} K_2(km_j/T) \times \begin{cases} \cosh(k\mu_i/T)\,, & n = \text{even}\,, \\ \sinh(k\mu_i/T)\,, & n = \text{odd}\,. \end{cases} \tag{2}$$

Taking only the leading term, $k = 1$ corresponds to the Boltzmann approximation. In this case, the cumulants have only two independent values, $\chi_{2m}^{(i)} = \chi_2^{(i)}$ and $\chi_{2m+1}^{(i)} = \chi_1^{(i)}$, and the underlying probability distribution is the Skellam distribution [12]. Then the smeared critical fluctuation appears as a deviation from the Skellam distribution in the higher order cumulants and in the tail part of the probability distribution [13].

[1] In this framework, the relevant symmetry is O(2) because of the use of the staggered fermion.

3. Influence of quantum statistics in net-electric charge fluctuations

When non-critical effects cause deviation from the Skellam distribution, they have to be carefully examined before considering the critical effects. Equilibrium net-baryon fluctuations of the hadron gas have only negligible deviation from the Skellam distribution according to the sufficiently heavy baryon masses. However, it is not the case for the net-electric charge fluctuations in which pions play a crucial role. This can be seen from Eq. (1), where the series can be interpreted as a multicomponent Boltzmann gas with mass km_j, charge k and degeneracy k^{n-4}. Higher order fluctuations of the quantum gases behave as multi-charged one, which results in deviation from the Skellam distribution [12]. An immediate consequence of this property is a non-trivial dependence of the fluctuations on the momentum cut. Figure 2 (left) displays the $\chi_1^{(Q)}$ in a single-component free Bose gas for various particle masses with low transverse momentum cut below p_{tmin} normalized by the value without cut. The response to the cut is stronger for lighter particles, reflecting the influence of the Bose statistics. In Fig. 2 (right), a cumulant ratio $\chi_4^{(Q)}/\chi_2^{(Q)}$ of the free π gas is shown for various temperature with respect to the low p_{t} cut. By increasing p_{tmin}, the cumulant ratio approaches the value of the Skellam distribution. Thus, the deviation from the Skellam distribution arises from the quantum statistics and its p_{tmin} dependence has a non-trivial structure. It has been shown that the difference of $\chi_1^{(Q)}/\chi_2^{(Q)}$ between STAR [14] and PHENIX [15] data can be partly attributed to p_{t} cut effect [11], through analyses with the hadron resonance gas model.

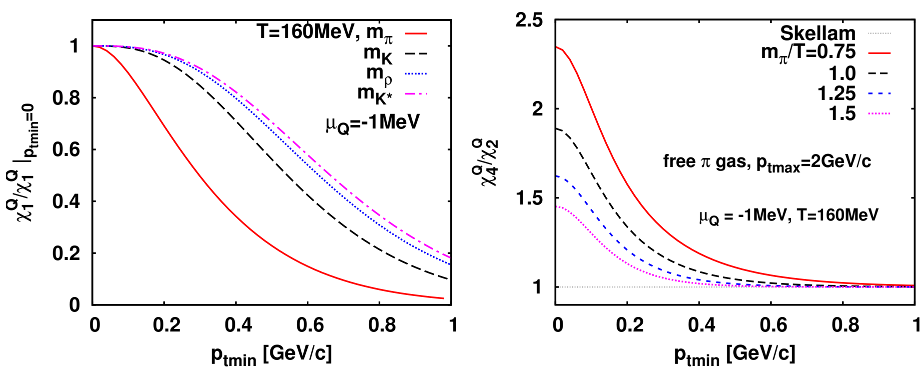

Fig. 2. Cumulants of net-electric charge of a free Bose gas with low p_{t} cuts. Figures are from [11].

This work was supported by the Grants-in-Aid for Scientific Research on Innovative Area from MEXT, Japan (No. 24105008), Grants-in-Aid for Scientific Research from JSPS, Japan (No. 16K05349) and the National Science Centre, Poland (NCN) under grants: Maestro DEC-2013/10/A/ST2/00106.

REFERENCES

[1] P. Braun-Munzinger, K. Redlich, J. Stachel, in: R.C. Hwa, X.N. Wang (Eds.), *Quark–Gluon Plasma 3*, World Scientific, 2004.

[2] M. Asakawa, M. Kitazawa, *Prog. Part. Nucl. Phys.* **90**, 299 (2016).

[3] R. Bellwied *et al.*, *Phys. Lett. B* **751**, 559 (2015); A. Bazavov *et al.*, *Phys. Rev. D* **93**, 014512 (2016).

[4] S. Ejiri *et al.*, *Phys. Rev. D* **80**, 094505 (2009).

[5] K. Fukushima, T. Hatsuda, *Rep. Prog. Phys.* **74**, 014001 (2011); K. Fukushima, C. Sasaki, *Prog. Part. Nucl. Phys.* **72**, 99 (2013).

[6] X. Luo, *Nucl. Phys. A* **956**, 75 (2016).

[7] M. Stephanov, K. Rajagopal, E. Shuryak, *Phys. Rev. D* **60**, 114028 (1999); Y. Hatta, M.A. Stephanov, *Phys. Rev. Lett.* **85**, 2076 (2000); M.A. Stephanov, *Phys. Rev. Lett.* **102**, 032301 (2009); M. Asakawa, S. Ejiri, M. Kitazawa, *Phys. Rev. Lett.* **103**, 262301 (2009).

[8] C. Sasaki, B. Friman, K. Redlich, *Phys. Rev. D* **75**, 074013 (2007); V. Skokov, B. Friman, K. Redlich, *Phys. Rev. C* **83**, 054904 (2011); B. Friman. F. Karsch, K. Redlich, V. Skokov, *Eur. Phys. J. C* **71**, 1694 (2011).

[9] T. Ichihara, K. Morita, A. Ohnishi, *Prog. Theor. Exp. Phys.* **2015**, 113D01 (2015).

[10] F. Karsch, K. Redlich, *Phys. Lett. B* **695**, 136 (2011).

[11] F. Karsch, K. Morita, K. Redlich, *Phys. Rev. C* **93**, 034907 (2016).

[12] P. Braun-Munzinger *et al.*, *Phys. Rev. C* **84**, 064911 (2011); *Nucl. Phys. A* **880**, 48 (2012).

[13] K. Morita, V. Skokov, B. Friman, K. Redlich, *Eur. Phys. J. C* **74**, 2706 (2014); K. Morita, B. Friman, K. Redlich, V. Skokov, *Phys. Rev. C* **88**, 034903 (2013); K. Morita, B. Friman, K. Redlich, *Phys. Lett. B* **741**, 178 (2015); K. Morita, K. Redlich, *Acta Phys. Pol. B Proc. Suppl.* **7**, 69 (2014).

[14] L. Adamczyk *et al.* [STAR Collboration], *Phys. Rev. Lett.* **113**, 092301 (2014).

[15] A. Adare *et al.* [PHENIX Collaboration], *Phys. Rev. C* **93**, 011901 (2016).