CRITICAL PHENOMENA IN THE NON-EXTENSIVE NAMBU–JONA-LASINIO MODEL*

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(Received April 26, 2017)

It is nowadays widely accepted that in many branches of physics, experimental data indicate the necessity of a departure from the standard extensive Boltzmann–Gibbs (BG) statistics, which is then replaced by a non-extensive statistics. Here, non-extensive calculation in a dense nuclear matter (NM) is proposed.

DOI:10.5506/APhysPolBSupp.10.705

We can distinguish two physical systems, in which knowledge of the nuclear Equation of State for the density several times bigger than the density of atomic nuclei is significant: the interior of neutron stars [1] and high-energy scattering of heavy ions [2]. In the recently discovered heavy neutron stars (with the mass of $2M_{\odot}$ and the radius of 10 km), the gravity compress an NM to the density of 10^{15} g/cm^3. Thus, the neutron matter has a density which exceeds the density achieved inside the heavy atomic nuclei twice and it is surrounded by the crust of a lower density. In a good approximation, this is the relativistic gas of neutrons with an admixture of protons immersed in the meson field of mutual attraction at greater distances and repulsion at small distances, less than 1 fm (10^{-13} m). Higher density and higher temperatures of an NM occur in the scattering of heavy ions, where quarks and gluons are the basic degrees of freedom of NM description. Their mutual interactions have a long-range nature leading to correlations, which become important with the increase of density and persist above the temperature of the phase transition from the hadron matter to the quark–gluon matter. To obtain the full information about the Equation of State (EoS) of the NM, heavy-ion experiments are conducted for different energies and using different nuclear targets. This way we can change the basic parameters of the

* Presented at the "Critical Point and Onset of Deconfinement" Conference, Wrocław, Poland, May 30–June 4, 2016.

state equation — temperature T and density ρ; *e.g.* in RHIC experiment [2] for Au+Au beams of energy $s_{NN} = (7.7, 11.5, 19.6, 27, 39)$ GeV were used, which allows us to get distributions for the increasing temperatures inside the created "fireball" of the quark–gluon matter — QCD plasma. Due to the long-ranged gluon interactions, the quark–gluon plasma created in the event after the collision is in the excited state and then it seeks the thermal and chemical balance between quarks and gluons increasing entropy. The presentation [2] shows the schematic scattering of heavy ions with energies in the center of mass (10–40) GeV. The quark–gluon plasma begins formation after the initial alignment of the energy distribution, when $\tau = 1$–4 fm$/c$. Then, after a period about $\tau = 5$ fm$/c$, it hadronizes into independent hadrons and its temperature drops in the process of kinematic freezing.

Such a strongly correlated system can be well-described by using the non-extensive Tsallis approach [3]. In this approach, the entropy of the system S_q is not the sum of entropies of individual subsystems, and in addition to the standard parameter T (temperature) occurring in the conventional BG statistics, there is a new parameter q called the non-extensive parameter. For $q = 1$, BG and Tsallis distributions are identical, so the quantity $|q-1|$ is the measure of departure from the equilibrium (usually to the state of being some type of stationary state). Relations $S_q(q > 1) > S_{\mathrm{BG}}(q = 1) > S_q(q < 1)$; in the literature determine appropriate systems, as being "superadditive", "additive" and "subadditive". This happens wherever the investigated system is "small" (which means that it is of the size comparable to the range of operating forces), when non-statistical fluctuations, correlations and all types of "memory" effects occur in it. Thermodynamically this means that the so-called "heat bath" is finite and inhomogeneous. When considering dense NM some kind of mean field theory is used, either in terms of nuclear degrees of freedom (like the Walecka model (WM) [4]), or in terms of quark and anti-quark degrees of freedom (like in the Nambu–Jona-Lasino (NJL) model [5–7]).

Both descriptions have been reformulated using a non-extensive approach (for the q-WM in [8–11] and for the q-NJL model in [12]). All these models try to find the non-extensive traces in the nuclear Equation of State (EoS) using the non-extensive single particle distributions. However, it is very difficult to compare their results in a conclusive way because they use different forms of single, non-extensive particle distributions. In [14], analysing a Fermi Gas model we compare and discuss these distributions following the method presented in [13], both for the momentum dependent fermion distributions $n_q(p)$ and for anti-fermion distributions $\bar{n}_q(\bar{p})$. Based on the above results, the non-extensive variant of the NJL model was formulated for the dense quark matter, with the liquid and gas phase [15]. Here, among others we analysed, in the area of phase transition, the critical temperature as a function of the departure from the BG statistics, given by the parameter

$|q-1|$. Thus, the dependence of pressure, entropy, specific heat and baryon susceptibility (related to the observed fluctuations of the baryon numbers) from the non-extensive parameter q, density and temperature T, was calculated. Let us note that in the critical area of the phase transition from hadrons to the quark–gluon plasma, the volume effects or non-extensive effects have the same source, namely a release of volume energy related to the disappearance of the hadron structure. The behaviour of these parameters in the critical area (that is for density and temperature near the phase transition) allowed the examination of possible changes of the phase transition for the non-extensive systems. We have shown how nature of the phase transition and the location of the critical point depend on the non-extensivity in nuclear medium. The main result of this part of the paper is the presentation of differences between the NJL calculations with different arrangements (entropies) of the quark system (Fig. 3 in [15]). With a better ordering, there is smaller entropy of the quark system ($q < 1$), and the NM is characterized by a lower pressure in the critical area. The expected phase transition is smoother compared to the system of higher entropy ($q > 1$) in which the change of such critical parameters, like specific heat or baryon susceptibility, has a more singular course.

Summary of NJL calculations [15] of the quark distributions in the area of phase transition from the hadron matter to the quark matter can be found in Fig. 1. It presents the distribution of critical points for the quark and gluon plasma as a function of temperature and chemical potential for different values of parameter q. Changes of the critical temperature of the BG statistics were examined for all possible implementations of the non-extensive Tsallis statistics [14] giving different thermodynamic dependence between density or chemical potential and the basic thermodynamic functions, such as entropy, pressure or energy. It turns out that for $q > 1$, the entropy increases and the phase transition occurs for higher chemical potential like in excited fireball in HI collision. In contrary, for the systems with $q < 1$, for which entropy decreases (that is the ordering increases like probably in a dense NM in neutron stars), the critical temperature of the transition from the hadron phase to the quark phase increases (for $q = 0.9$ from 68 MeV to 71 MeV Fig. 1). It is the temperature where constituent quark mass 300 MeV (part of the nucleon mass) changes to current quark mass 5 MeV. In this case, the smaller chemical potential (of 6 MeV) confirms long-range correlations responsible for the better arrangement of quarks inside NM. Such a change of a quark chemical potential gives the increase of positive correlations and approximately compensates for a lack of nucleon degrees of freedom in the NJL model by energy 3×6 MeV= 18 MeV (in a reasonable comparison with the nucleon binding energy in NM — 16 MeV). In other words, the lower entropy is obtained by the attractive correlations, therefore, we need higher critical temperature at which the “constituent” quark mass decreases to the

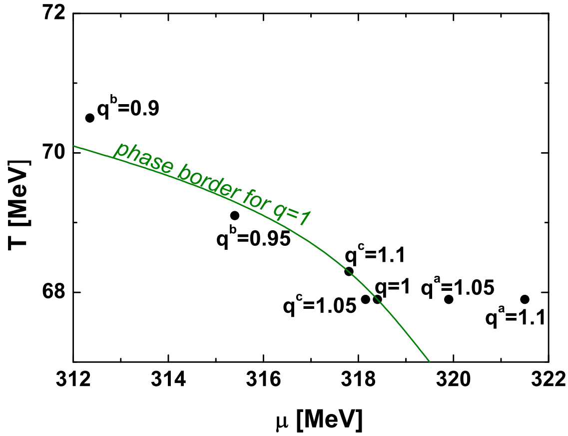

Fig. 1. Phase diagram in the q-NJL model in the T–μ plane. The critical end points (CEP) for different values of q corresponding to different realizations of the q-NJL model and compared with the BG case of $q = 1$. The choice q^a denotes $q > 1$, choice q^b denotes $q < 1$, choice q^c denotes hybrid realization (see the text).

value of "current" mass, associated with the "dissociation" of nucleons in the phase transition of hadron matter do QGP. The parametrization of the effects of non-extensive statistics recently proposed in [16], which involves the description of two areas of the phase space, below and above the Fermi surface. Such a hybrid selection of q-statistics, in which $(q-1)$ changes the sign in one-particle distributions [14] allows to avoid all limitations of the phase space, seems to be interesting (marked as q^c in Fig. 1). However, there appears a very significant problem of the physical interpretation associated with the jump of the distribution functions on the Fermi surface and, in consequence, jumps in other thermodynamic functions. The papers [14, 15] showed that it is important how parameter q changes its value on the Fermi surface. Two cases can be considered. In the first one, the change of q parameter is an approximation describing the gradual changes of the mean field, in which the particle is located. The transition to the positive values of the parameter $(q-1)$ above the Fermi level, where particles are no longer bound, is a gradual transition, in which we expect values $q \sim 1$ for the Fermi momentum of a nucleon. In this case, there is no actual discontinuity of thermodynamic functions because, in reality, the transition should be described by the continuous transition of the parameter q. In the second case, when we deal with the discontinuous change of the parameter q, we describe rather a more complex system *e.g.* a crystal network, interacting with the gas of electrons or systems of quantum dots, which we can control changing the external potential. In this case, the real jumps of the non-extensive parameter are possible (discontinuities), which, in effect, may discontinuously change the number of particles or one-particle energies. However, it seems that the above-mentioned discontinuities are not confirmed in the nuclear physics and that is why this method is not applicable in our case (and that is why it was not used in [17] in the quasi-particle description).

Supported in part by the National Science Centre, Poland (NCN) under contract DEC-2013/09/B/ST2/02897. I would like to thank G. Wilk for comments.

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