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SYMMETRY BREAKING EFFECT ON THE INHOMOGENEOUS CHIRAL PHASE IN THE EXTERNAL MAGNETIC FIELD*

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We investigate the effect of the current quark mass on the inhomogeneous chiral phase in the QCD phase diagram, to discuss the properties of the phase transition using the generalized Ginzburg–Landau (GL) expansion. The external magnetic field spreads this phase over the low chemical potential region even if the current quark mass is finite, which implies that the existence of this phase can be explored by the lattice QCD simulation.

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1. Introduction

Exploring the finite density region of the QCD phase diagram is one of the challenging issues in nuclear physics. Recently, the possible existence of the inhomogeneous chiral phase has been actively discussed by the analysis of some effective models. In this phase, the scalar and pseudoscalar quark condensates spatially modulate and the complex order parameter, $\phi(\boldsymbol{r})$, representing this phase takes the form of

$$\phi(\boldsymbol{r}) \equiv \left\langle \bar{\psi}\psi \right\rangle + i \left\langle \bar{\psi} i\gamma^5 \tau_3 \psi \right\rangle = \Delta(\boldsymbol{r}) e^{i\theta(\boldsymbol{r})} \,. \quad (1)$$

Using some inhomogeneous configurations, most analyses have shown that the inhomogeneous chiral phase appears as an intermediate phase during the standard chiral phase transition.

In QCD, various magnetic aspects have attracted much interest. One of the interesting subjects is the symmetry behavior in the magnetic field (B). It has been suggested that the chiral symmetry breaking is enhanced due to B in the effective model, *magnetic catalysis* (MC). However, the recent lattice simulations have shown *inverse magnetic catalysis* (IMC) at finite temperature. This phenomenon has not been well-understood yet.

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In the magnetic field, the dual chiral density wave (DCDW) phase ($\Delta(\boldsymbol{r}) = \Delta, \theta(\boldsymbol{r}) = qz$) is remarkably extended in the low chemical potential (μ) region [1]. The energy spectrum of quarks exhibits asymmetry, which gives rise to the distinctive phenomenon [2]. Note that complex $\phi(\boldsymbol{r})$ is necessary for the energy spectrum to be asymmetric about zero. A peculiar role of the spectral asymmetry can be also seen around the transition point: it induces a new term in the thermodynamic potential and, consequently, a new Lifshitz point should appear on the $\mu = 0$ line in the chiral limit [2]. If this is the case, one may expect a direct observation of DCDW by a lattice QCD simulations, although the extrapolation to the finite μ region is restricted due to the sign problem.

To discuss this issue in a realistic situation, we should consider the effect of the current quark mass. For DCDW, although no exact solution is known, a variational method may work well in the absence of B [4]. As a consequence, the function form of DCDW is largely deformed near the transition point and, accordingly, the DCDW region of the phase diagram is reduced. We shall follow the similar approach here and find the proper solution of $\theta(\boldsymbol{r})$ instead of qz near the transition point in the presence of B.

2. Thermodynamic potential with finite current quark mass

The thermodynamic potential near the transition point is given by the generalized GL expansion based on the NJL model [3]. The NJL model Lagrangian takes the following form within the mean field approximation,

$$\mathcal{L}_{\mathrm{MF}} = \bar{\psi}\left[i\not{D} - m_c - m\left(\cos\theta(z) + i\gamma^5\tau^3\sin\theta(z)\right)\right]\psi - \frac{m^2}{4G} \tag{2}$$

with the covariant derivative, $D_\mu = \partial_\mu + i\mathcal{Q}A_\mu$, where $\mathcal{Q}$ is the electric charge matrix in the flavor space, and the SU(2) symmetric quark mass, $m_c \equiv m_u = m_d \simeq 5$ MeV. Here, we assume the mean field of the quark condensates, $-2G\phi(\boldsymbol{r}) = me^{i\theta(z)}$, where m plays a role of the dynamical quark mass, and the direction of modulation is taken to be parallel to $\boldsymbol{B}$.

Taking the external magnetic field along the z axis, the thermodynamic potential can be written up to the fourth order about the order parameters, and its derivative and the first order in m_c as

$$\begin{aligned}\Omega(\mu,T,B) \;=\; & \Omega_0 + \int\frac{\mathrm{d}^3\boldsymbol{x}}{V}\Big\{\alpha_1 m\cos\theta + \frac{1}{2}\left(\alpha_2 + \frac{1}{2G}\right)m^2 + \tilde{\alpha}_2 m\,(\sin\theta)' \\ & + \frac{\alpha_3}{4}\left[4m^3\cos\theta - m\,(\cos\theta)''\right] + \tilde{\alpha}_3 m^2\theta' + \frac{\alpha_4}{4}\left(m^4 - m^2\theta\theta''\right) \\ & + 3\tilde{\alpha}_{4a}m^3\,(\sin\theta)' + \tilde{\alpha}_{4b}m\,(\sin\theta)'''\Big\}\end{aligned} \tag{3}$$

with a shorthand notation, $\theta' \equiv \partial\theta/\partial z$, where the GL coefficients depend on μ, T and B, and Ω_0 is the constant term independent of the order parameters. Note that the effect of m_c appears in α_1, α_3, $\tilde{\alpha}_2$, $\tilde{\alpha}_{4a}$ and $\tilde{\alpha}_{4b}$, which are proportional to m_c. The coefficients α_i ($i = 1$–4) include a UV divergence and the Pauli–Villars regularization is used in the present calculation.

It may be worth mentioning that the $\tilde{\alpha}_3$ term is originated from the spectral asymmetry of the quark energy eigenvalues and proportional to B. The presence of such term has been shown in the chiral limit and a close relation to chiral anomaly has been demonstrated [2]. Note that the $\tilde{\alpha}_3$ term remarkably extends the DCDW phase in the presence of B.

From the stationary condition: $\delta\Omega/\delta\theta(z) = 0$, we find the equation in the sine-Gordon form,

$$\theta'' + \mathrm{sign}\left(\alpha_1 + m^2\alpha_3\right) m_\pi^{*2} \sin\theta = 0\,, \tag{4}$$

with $m_\pi^{*2} \equiv 2\frac{|\alpha_1 + m^2\alpha_3|}{m\alpha_4}$, and the relevant solution to Eq. (4) is obtained as

$$\theta(z) = 2\,\mathrm{am}\left(\frac{m_\pi^*}{k} z, k\right)\,, \tag{5}$$

where "am" is the amplitude function with modulus $k \in [0, 1]$. Then, the wave number (Q) of condensates is defined by the relations, $Q = \frac{\pi m_\pi^*}{kK(k)}$, where $K(k)$ is the complete elliptic integral of the first kind. There are two order parameters, m and k (or Q), where m characterizes the magnitude of SSB, and k measures a degree of the inhomogeneity. When $k = 1$, $\theta(z)$ behaves like the single kink and Q vanishes. Then, we can see that the thermodynamic potential is reduced to the one in the homogeneous phase. On the other hand, when k and m_c simultaneously go to zero and $2m_\pi^*/k \to q$, the original DCDW phase is recovered, $\theta = qz$. In the following, we call the phase where $0 < k < 1, m \neq 0$ the *massive DCDW phase*.

3. Results and discussions

In the present calculation, we use the parameter set in Ref. [5]: $\Lambda = 851$ MeV and $G\Lambda^2 = 2.87$. In Fig. 1, we show the resulting phase diagram. There are drawn the phase boundary between the massive DCDW phase and the homogeneous phase, and the crossover line given by the pseudocritical temperature defined as the peak of the chiral susceptibility: $-\partial m/\partial T$. In our original paper [6], the change of the phase boundary has also been analyzed when m_c or B changes. We have found that the massive DCDW phase is extended to the low μ region with the decrease of m_c. Thus, the result in Ref. [2] is recovered in the chiral limit. Moreover, B raises the critical temperature, which is consistent with MC. Consequently, we can see that B enlarges the massive DCDW phase over the low μ and high T region even if m_c is finite.

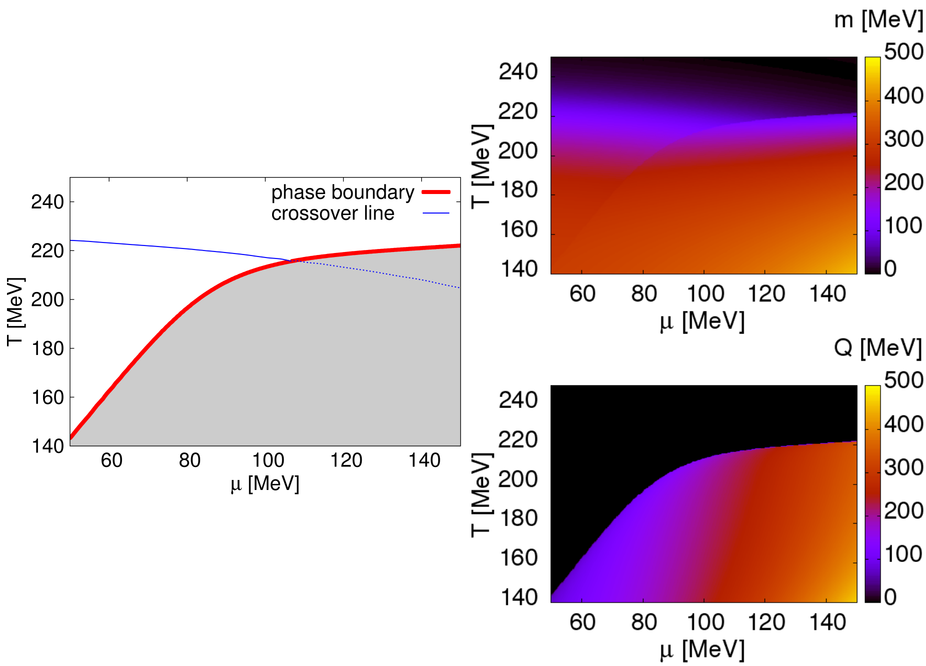

Fig. 1. (Color online) Phase diagram at $m_c = 5$ MeV, $\sqrt{eB} = 1$ GeV (left panel). The thick black/red line describes the phase boundary between the massive DCDW phase (shaded area) and the homogeneous phase. The solid/blue line describes the crossover line. The conventional crossover line without the massive DCDW phase corresponds to the dotted blue line. The right upper (lower) panel shows the value of m (Q) at the same range of μ–T as the left panel.

To discuss the effect of IMC in the present model, it is assumed that the effect is simulated by giving a B dependence to the coupling constant of the NJL model (G). According to Ref. [7], G is fitted to reproduce the result of the lattice QCD simulation [8]. In the following, we consider the case at $\sqrt{eB} = 1$ GeV. The coupling constant is put as $G\Lambda^2 = 1.85$. In Fig. 2, the change of the phase boundary by IMC is shown. The region of the massive DCDW phase shrinks and the critical temperature decreases due to the effect. However, the massive DCDW phase survives in the $\mu/T < 1$ region if m_c is sufficiently small.

In Ref. [9], the possibility of the observation of the DCDW phase has been discussed in the case with the singular line at $\mu = 0$ in the chiral limit. However, the phase boundary is moved to $\mu \neq 0$ region due to m_c.

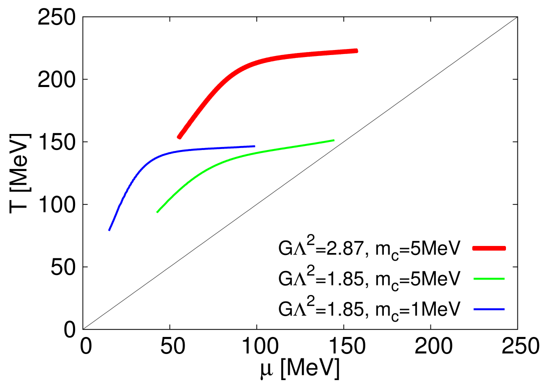

Fig. 2. (Color online) Phase boundary obtained including IMC. The thick gray/red line corresponds to the phase boundary in the Fig. 1. On the other hand, the gray/green and black/blue lines describe one at $m_c = 5, 1$ MeV with IMC.

In the Taylor expansion method and the analytic continuation method from imaginary μ, the singularity at $\mu \neq 0$ cannot be described and the massive DCDW phase cannot be grasped. In the reweighting method, the importance sampling is carried out for some parameter choice, where there is no sign problem. Therefore, we need to find a special region with the massive DCDW phase and no sign problem there. In the canonical approach, it may be found that the quark number density has a discontinuity derived from some first-order phase transition, if there is the massive DCDW phase in $\mu \neq 0$ region. However, the phase transition cannot be identified as one from the homogeneous phase to the massive DCDW phase. Therefore, we need to find some specific order parameters on the phase transition.

4. Summary and concluding remarks

We have discussed the inhomogeneous chiral phase at $B \neq 0$ and $m_c \neq 0$. It is found that B extends the massive DCDW phase over the low μ region similar to the DCDW phase in the chiral limit though m_c tends to reduce this phase region. Within our analysis based on the NJL model, B seems to raise the critical temperature for MC. So we tune the coupling constant of the NJL model to estimate the qualitative influence of IMC. Consequently, the critical temperature decreases. However, the massive DCDW phase can develop in the region: $\mu/T < 1$ if m_c is sufficiently small. Therefore we suggest that the inhomogeneous chiral phase can be explored by the lattice QCD simulations by choosing some proper method, for example the reweighting method or the canonical approach.

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