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EXTENDED SOFT-WALL MODEL FOR THE QCD PHASE DIAGRAM*

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The soft-wall model, emerging as bottom-up holographic scenario anchored in the AdS/CFT correspondence, displays the disappearance of normalisable modes referring to vector mesons at a temperature $T_{\rm dis}$ depending on the chemical potential μ, $T_{\rm dis}(\mu)$. We explore options for making $T_{\rm dis}(\mu)$ consistent with the freeze-out curve $T_{\rm fo}(\mu)$ from relativistic heavy-ion collisions and the cross-over curve $T_{\rm c}(\mu)$ from QCD at small values of μ.

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1. Introduction

In still lacking a convincing top–down approach from the string theory to a proper gravity dual of QCD, one must resort to bottom-up models which are designed to mimic certain wanted features of QCD. Among such approaches is the soft-wall model [1] as a particular realisation of the AdS/CFT correspondence w.r.t. the hadron spectrum, especially vector mesons. While being a phenomenological set-up, the original soft-wall model [1] can be modified to accommodate the Regge-type spectrum of radial excitations of selected hadron species at vanishing temperature T and chemical potential μ. Extending the model further to $T > 0$, one finds that, at temperatures $T \geq T_{\rm dis}$, hadrons as normalisable states disappear [2]. It is tempting to consider such a scenario as an emulation of deconfinement. As shown in [3], one can tune the model further to achieve $T_{\rm dis} = T_{\rm c}$, where $T_{\rm c} \approx 150$ MeV is the cross-over temperature known from lattice QCD evaluations for $2+1$ flavours with physical quark masses. There are options to let disappear all hadron states at $T_{\rm dis}$ (instantaneous disappearance) or only the ground

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state, and excited states already disappeared in a narrow corridor below $T_{\rm c}$ (sequential disappearance). For steering these details, the Hawking–Page transition is a central issue.

Reference [3] focused on purely thermal effects. Here, we investigate the options for $T_{\rm dis}(\mu)$. We provide a special modification of the soft-wall model such as to make $T_{\rm dis}(\mu)$ consistent with $T_{\rm fo}(\mu)$ and $T_{\rm c}(\mu)$, where "fo" labels the chemical freeze-out and "c" is for the cross over. The dependence of $T_{\rm fo}$ on μ is determined nowadays from hadron multiplicities observed in relativistic heavy-ion collisions at varying beam energy; system size and centrality dependencies help to consolidate the freeze-out curve $T_{\rm fo}(\mu)$. The map of hadron multiplicities on the freeze-out data is provided traditionally by thermo-statistical models of the hadron resonance gas [4–6], may be supplemented by effects of inelastic, post-hadronization reactions [7]. The results are in agreement with data analyses using mean and variance of net baryon number and net-electric charge distributions based on lattice QCD input [8]. On the other hand, lattice QCD provides *ab initio* calculations of $T_{\rm c}(\mu)$, albeit restricted to a region of $\mu/T < 3$ due to the sign problem.

All these attempts have the goal to pin down the QCD phase diagram and to seek a critical point that marks the onset of a curve of first-order phase transitions when going to larger values of μ, realised experimentally by lowing the beam energy. Experimentally, dedicated efforts are devoted to the search for a critical point, most notably the beam energy scan at RHIC [9–11] and the program at NA61/SHINE [12–14]. Besides many models envisaging statements on the phase diagram of strong interaction matter [15–20], also holographic approaches are to be mentioned. These aim essentially at mimicking the thermodynamics [21, 22] rather than individual hadron properties, but can address issues of deconfinement as well [23, 24]. Here, we report on the modified soft-wall model with regard of non-zero temperature and non-zero chemical potential.

2. Modified soft-wall model

The model pursued here is based on the action

$$S_V = -\frac{1}{4k_V}\int \mathrm{d}z\,\mathrm{d}^4x\,\sqrt{g}e^{-\Phi(z)}F^2 \tag{1}$$

with k_V chosen to render S_V dimensionless. The dilaton field Φ acts as a conformal symmetry breaker. The quantity g denotes the determinant of the metric tensor. Equation (1) is utilised to describe the dynamics of an U(1) vector field with the components V_M, where $F_{MN} = \partial_M V_N - \partial_N V_M$ (indices $M, N = 0, \ldots, 4$) is the field strength tensor, dual to the boundary vector current, *e.g.* $J_\mu \sim \bar{q}\gamma_\mu q$. A special five-dimensional Riemann space with coordinates $x_{0,1,2,3}$ and holographic coordinate z is described by the infinitesimal distance squared

$$\mathrm{d}s^2 = e^{A(z)} \left(f(z)\mathrm{d}t^2 - \mathrm{d}\vec{x}^{\,2} - \frac{1}{f(z)}\mathrm{d}z^2 \right) , \tag{2}$$

where $A(z)$ is a warp function and $f(z)$ is the blackness function, both to be specified below. The equation of motion follows from (1), with the metric determinant to be read off (2) and $\psi = \varphi \exp\{-(A-\Phi)/2\}$, as

$$\left(\partial_\xi^2 - \left(U_T - m_n^2\right)\right)\psi = 0 \,, \tag{3}$$

where ξ is the tortoise coordinate determined by $\mathrm{d}\xi = \mathrm{d}z/f(z)$ and U_T is the Schrödinger equivalent potential

$$U_T = \left(\tfrac{1}{2}\left(\tfrac{1}{2}\partial_z^2 A - \partial_z^2 \Phi\right) + \tfrac{1}{4}\left(\tfrac{1}{2}\partial_z A - \partial_z \Phi\right)^2 \right) f^2 + \tfrac{1}{4}\left(\tfrac{1}{2}\partial_z A - \partial_z \Phi\right)\partial_z f^2 \,. \tag{4}$$

To arrive at (3), the Ansatz $V_\mu = \epsilon_\mu \varphi(z) \exp\{ip_\nu x^\nu\}$ and the gauges $V_z = 0$ and $\partial_\mu V^\mu = 0$ (Greek indices run in the range $0,\ldots,3$) are employed. The normalisable solutions of (3) determine squared vector meson masses $m_n^2 = p_M p^M$, where $n = 0$ denotes the ground state (gs) and $n \geq 1$ counts the radial excitations, labelled with 1^{st}, 2^{nd}, *etc.*

In the spirit of [1], the soft-wall model sets a "soft wall" by the dilaton profile $\Phi(z) = (cz)^p$ with a scale c; we employ the warp factor $A(z) = \ln(L^2/z^2 + \tilde{\mu}^2)$ with the AdS radius $L = 1/c$. Our Ansatz for the blackness function is with $\vartheta(z_{\mathrm{H}}) = \pi z_{\mathrm{H}} T(z_{\mathrm{H}}) - 1$ (see Appendix A)

$$f(z) = 1 - \frac{z^4}{z_{\mathrm{H}}^4}\left(1 + \frac{2\vartheta(z_{\mathrm{H}})}{\exp\left\{\frac{2}{e}\vartheta(z_{\mathrm{H}}) + 4\hat{\mu}^2\right\}}\left[\left(\frac{z}{z_{\mathrm{H}}}\right)^{2\exp\left\{\frac{2}{e}\vartheta(z_{\mathrm{H}})+4\hat{\mu}^2\right\}} - 1\right]\right) \tag{5}$$

providing from $\partial_z f(z)\,|_{z=z_{\mathrm{H}}} = -4\pi T(z_{\mathrm{H}})$ the Hawking temperature

$$T(z_{\mathrm{H}}) = \tilde{T}(z_{\mathrm{H}})\left(1 - \hat{\mu}^2\right) \tag{6}$$

with $\tilde{T}(z_{\mathrm{H}}) = \tilde{T}_{\min}(1 + [1/x - 2 + x]/\Theta)$, where $x = z_{\mathrm{H}}/\tilde{z}_{\min}$ and $\Theta = \pi \tilde{T}_{\min}\tilde{z}_{\min}$. In the special case of $\tilde{T}(z_{\mathrm{H}}) = 1/(\pi z_{\mathrm{H}})$, (5) belongs to the metric of a Reissner–Nordström black hole embedded in an asymptotic Anti-de Sitter space. It is customary to identify $\mu = \sqrt{2}\hat{\mu}\gamma z_{\mathrm{H}}^{-1}$ as baryo-chemical potential and T as the temperature of the boundary theory. The parameter γ arises as a ratio of two coupling strengths when deriving the AdS Reissner–Nordström black brane (*cf.* [25] and Appendix A). Equation (5) keeps the required properties of a black hole: it has a simple zero at horizon $z = z_{\mathrm{H}}$,

$f(z=0,z_{\rm H})=1$ and $(\partial_z^i f)_{z\to 0}=0$ for $i=1,2,3$. The above parameters c, p and $\tilde{\mu}$ can be tuned at $T=0$ to reproduce a Regge-type spectrum $m_n^2=\alpha+\beta n$ in agreement with the known vector meson states forming a trajectory of radial excitations parametrised by α and β [3]. Note that, for $\mu=0$, Eq. (5) facilitates numerical results agreeing on the sub-percent level with those of [3].

3. Non-zero chemical potential

Depending on μ, $\tilde{T}_{\rm min}$ and $\tilde{z}_{\rm min}$, $T(z_{\rm H})$ can display a minimum of $T_{\rm min}$ at $z_{\rm min}$ which translates into $T_{\rm min}(\mu)$. If so, then (5) must be replaced by the trivial, non-black-hole function $f=1$ for all $T<T_{\rm min}$, *i.e.* due to the Hawking–Page transition, the thermal gas solution is the stable configuration. What remains is a selection of parameters $\tilde{T}_{\rm min}$, $\tilde{z}_{\rm min}$ and γ to achieve $T_{\rm dis}^{\rm gs}(\mu)\cong T_{\rm fo}(\mu)\cong T_{\rm c}(\mu)$. We take the leading order shape

$$T_{\rm fo}(\mu)\cong T_{\rm c}(\mu)\cong T_0\left(1-\kappa\left(\frac{\mu}{T_0}\right)^2+\ldots\right) \tag{7}$$

with $\kappa=0.005\ldots0.01$ from [8] (*cf.* also [7]) and put for simplicity $T_0=T_{\rm c}(\mu=0)=155$ MeV without an error band.

The dependence of $T_{\rm dis}^{\rm gs}$ follows from numerical solutions of (3) with potential (4), where the μ dependence comes from (5) and (6). We employ here the parameters $p=1.99$, $\tilde{\mu}=0.5$ and $c=443$ MeV which provide one possible setting of a Regge trajectory with $\alpha=0.71$ GeV^2 and $\beta=0.75$ GeV^2 at $T=\mu=0$, as shown in [3]. The particular choice $\tilde{T}_{\rm min}=155$ MeV and $c\tilde{z}_{\rm min}=2$ is for a scenario, where for $\mu=0$, the thermal gas solution is valid for all temperatures $T<\tilde{T}_{\rm min}$. That is, for $T<\tilde{T}_{\rm min}$, the vector meson spectrum is as at $T=0$ with the implication that the thermo-statistical model analysis applies in that region with standard vacuum masses of hadrons. At $T>\tilde{T}_{\rm min}$, however, the black-hole solution must be accomplished. Equation (3) does not allow for normalisable solutions at $T>\tilde{T}_{\rm min}$, *i.e.* just at $T=\tilde{T}_{\rm min}$, the hadron states (here shown only for vector mesons) disappear. In such a special setting, one therefore identifies both the (chiral) cross over point and the chemical freeze-out temperature at $\mu=0$ with (de)confinement. We adjust the remaining parameter γ such to put the disappearance temperature of the ground state, $T_{\rm dis}^{\rm gs}(\mu)$ (upper dashed curve), on the freeze-out/cross-over curve (solid/blue curve) in parametrisation (7). Using the above quoted values of curvature measure κ in the spirit of upper and lower bounds, we find the results exhibited in Fig. 1. Up to a certain critical value of the chemical potential, the disappearance curve of the lowest vector meson states is on the top of the freeze-out/cross-over curve for a given value of κ. The related physical interpretation is that once a cooling piece of deconfinement matter reaches $T_{\rm dis}^{{\rm gs},1^{\rm st},\dots}(\mu)$, it hadronizes by occupying statistically the available hadron states.

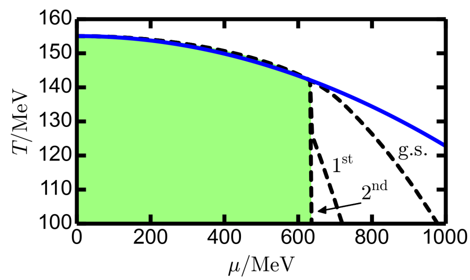

Fig. 1. (Colour on-line) QCD phase diagram with two options of the freeze-out/cross-over curve (7) (solid/blue curves, left panel: $\kappa = 0.005$, right panel: $\kappa = 0.01$); note that (7) without higher-order terms holds true only in the small-μ region. In the shaded/green areas, the thermal gas solution applies. Its upper boundary is given by $T_{\rm min}(\mu)$. The disappearance temperatures $T_{\rm dis}$ as a function of μ (dashed curves) of the first three vector meson states according to Eq. (3) with potential (4) (parameters: $p = 1.99$, $\tilde{\mu} = 0.5$, $c = 443$ MeV (*cf.* set 2.0 of [3]), $\tilde{T}_{\rm min} = 155$ MeV, $c\tilde{z}_{\rm min} = 2$) are adjusted by $\gamma = 7.85$ (left) and $\gamma = 5.55$ (right). Up to $\mu = 620$ MeV (left) or $\mu = 440$ MeV (right), all states disappear instantaneously at $T = T_{\rm min}$. For larger values of μ, where only the black-hole solution is valid (white regions), the third and all higher states do not exist at all (indicated by the vertical dashed lines); the ground state and the first excited state disappear sequentially.

The above sketched scenario can be relaxed by minor parameter variations to have $T_{\rm dis}^{\rm gs} > T_{\rm dis}^{1^{\rm st}} > T_{\rm dis}^{2^{\rm nd}} \dots$, *i.e.* a sequential appearance of vector meson states upon cooling. Figure 2 exhibits a possibility where the first two states appear sequentially in a narrow corridor centred at $T_{\rm c}(\mu)$ for small μ. If such a behaviour can be established for other hadron species too, it is still consistent with the application of the thermo-statistical models.

It is premature to extrapolate the described scenario to too large values of μ, and thus to critical point issues, since *(i)* Eq. (7) relies on the leading-order term and *(ii)* lacking knowledge on $T_{\rm c}(\mu)$, *i.e.* whether $T_{\rm fo}(\mu) \cong T_{\rm c}(\mu)$ at larger values of μ, and *(iii)* unsettled options in constructing other blackness functions beyond (5) and (6).

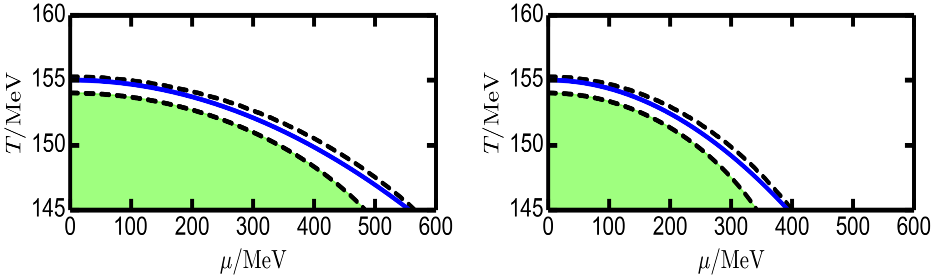

Fig. 2. As in Fig. 1 but for $\tilde{T}_{\min} = 154$ MeV, $c\tilde{z}_{\min} = 2.5$, $\gamma = 8.79$ (left) and $\gamma = 6.22$ (right). For all values of μ, the ground state disappears at a temperature higher than the radial excitations. While the thermal gas solution is valid, all excited states disappear instantaneously.

4. Summary

The famous soft-wall model [1] represents a particular realisation of ideas anchored in the AdS/CFT correspondence. It can be modified to accommodate a Regge-type spectrum of radial excitations of vector mesons. Considering vector mesons as prototypical representatives of hadrons, one can further modify such a gravity field duality model to study the fate of certain hadron species immersed in a hot and dense ambient medium. Parameters can be tuned to let disappear vector mesons as normalisable modes above a temperature to be identified tentatively with "deconfinement temperature" or, more specifically, with the chiral cross-over temperature T_{c} [3], thus extending the approach in [2]. Following, *e.g.* [8] (see also [26]) in identifying the chemical potential dependence of $T_{\mathrm{c}}(\mu)$ with the freeze-out systematics found from heavy-ion experiments and condensed in $T_{\mathrm{fo}}(\mu)$ at small μ, we have demonstrated that the suitably adopted soft-wall model allows for a consistent scenario: Once a temperature T_{dis} is reached upon cooling of a piece of "deconfined matter", hadrons appear, either suddenly at once or sequentially in a narrow corridor of temperatures, and are ready for statistical distribution.

Appendix A

The goal is to extend the black-hole function in AdS, $f_{\mathrm{BH}}(z) = 1 - (z/z_{\mathrm{H}})^4$, yielding $T_{\mathrm{BH}}(z_{\mathrm{H}}) = 1/(\pi z_{\mathrm{H}})$, and the Reissner–Nordström black-hole function, $f_{\mathrm{RN}}(z) = 1 - (1 + \frac{1}{2}\hat{\mu}^2)(z/z_{\mathrm{H}})^4 + \frac{1}{2}\hat{\mu}^2(z/z_{\mathrm{H}})^6$ in AdS, yielding $T_{\mathrm{RN}}(z_{\mathrm{H}}) = (\pi z_{\mathrm{H}})^{-1}(1 - \hat{\mu}^2)$ [25]. Clearly, $f_{\mathrm{RN}}(z; z_{\mathrm{H}}, \hat{\mu} = 0) = f_{\mathrm{BH}}(z; z_{\mathrm{H}})$. As in [3], we start from the general statement that for all positive i with $i > 4(\pi z_{\mathrm{H}} T(z_{\mathrm{H}}) - 1) =: 4r$, the function f defined by

$$f(z) = 1 - \frac{z^4}{z_{\mathrm{H}}^4}\left(1 + \frac{4r}{i}\left(\frac{z^i}{z_{\mathrm{H}}^i} - 1\right)\right) \tag{A.1}$$

is a suitable blackness function, *i.e.* $f(z = 0, z_{\mathrm{H}}) = 1$, $(\partial_z^i f)_{z\to 0} = 0$ for $i = 1, 2, 3$ and the simple zero at the horizon, $f(z = z_{\mathrm{H}}; z_{\mathrm{H}}) = 0$. To recover the Reissner–Nordström case, we observe that $r = -\hat{\mu}^2$ and $i = 2$ are required. To construct a proper blackness function, we can apply any function $h : \mathbb{R} \to \mathbb{R}$ with h positive, $h(x) > x$ for all $x \in \mathbb{R}$ and $h(-4\hat{\mu}^2) = 2$ and set $i = h(4r)$. One possibility is $h(x) = 2e^{ax+4\hat{\mu}^2}$ for all $a \geq 1/2e$ which yields (5) for $a = 1/2e$.

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