

# TEMPERATURE EFFECTS ON SUPERFLUID PHASE TRANSITION IN BOSE–HUBBARD MODEL WITH THREE-BODY INTERACTION\*

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We study the combined effects of two- and three-body local interactions as well as the finite temperature on the phase diagram of the Bose–Hubbard model. To handle the system with strong local interactions, we use the resolvent expansion technique, based on the contour integral representation of a partition function, and to find the phase diagram, we derive Landau-type expansion for free energy in terms of the superfluid order parameter.

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## 1. Introduction

The behaviour of atoms contained in an optical lattice is governed mainly by two-body interactions. However, there are indications that also three-body interactions should be taken into account [1]. In this work, we present the finite temperature phase diagram of the Bose–Hubbard model and study its dependence on the three-body interactions strength.

## 2. The model

To describe an ultracold gas of bosons in an optical lattice, the Bose–Hubbard model [2] is used

$$\hat{H} = -J \sum_{\{ij\}} \left( \hat{a}_i^\dagger \hat{a}_j + \hat{a}_j^\dagger \hat{a}_i \right) + \sum_i \hat{n}_i \left[ \frac{U}{2} (\hat{n}_i - 1) + \frac{W}{6} (\hat{n}_i - 1)(\hat{n}_i - 2) - \mu \right],$$

where  $U$  and  $W$  measure strength of two- and three-body local, repulsive interactions, respectively, and  $J$  is the tunnelling strength. In the strong coupling regime,  $J$  is much smaller than  $U$ .

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### 3. Phase diagram

The mean-field approximation leads to the following Hamiltonian:

$$\hat{H}' = \sum_i \left[ Jz\Phi \left( \Phi - \hat{a}_i - \hat{a}_i^\dagger \right) + \hat{n}_i \left( \frac{U}{2}(\hat{n}_i - 1) + \frac{W}{6}(\hat{n}_i - 1)(\hat{n}_i - 2) - \mu \right) \right],$$

where  $z$  is the coordination number of a lattice and  $\Phi$  is the superfluid order parameter. The corresponding partition function has been calculated using the resolvent method [3]. To obtain the phase diagram, we used the Landau theory (for details, see Ref. [4]). The phase diagram for various temperatures and values of three-body interaction strength is shown in Fig. 1. For  $T = 0$ , the subsequent Mott lobes are widened by three-body interactions. Finite temperature smears the lobes — the effect is stronger for higher temperatures.

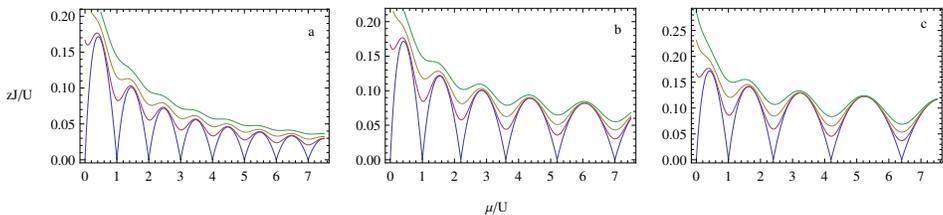


Fig. 1. Phase diagram of the Bose–Hubbard model for several values of the three-body interaction strength  $W/U = 0, 0.2$  and  $0.4$  (panel (a), (b) and (c), respectively) in the  $(\mu/U, zJ/U)$  plane for several values of the temperature ( $kT/U = 0, 0.1, 0.15$  and  $0.20$ , curves from the bottom to the top, respectively).

### 4. Conclusions

We have investigated the effect of the three-body interactions on the Bose–Hubbard model using both the mean-field approach and the resolvent method. We have also found the phase diagram and depicted its dependence on various parameters of interest.

### REFERENCES

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