GENERAL UNQUENCHING PROPERTIES OF TWO-MESON SCATTERING AND PRODUCTION AMPLITUDES*

Eef van Beveren

Centro de Física da UC, Departamento de Física
Universidade de Coimbra, 3004-516 Coimbra, Portugal

George Rupp

Centro de Física e Engenharia de Materiais Avançados
Instituto Superior Técnico, Universidade de Lisboa, 1049-001 Lisboa, Portugal

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Besides the unitarity and symmetry requirements for a multi-resonance scattering amplitude, several other natural conditions can easily exclude unrealistic proposals. In particular, the behaviour of singularities under the variation of model parameters yields important information. We discuss how resonance poles should move in the complex-energy plane when coupling constants and masses are varied, how resonances above threshold can turn into bound states below threshold, and how the light-quark spectrum can be turned into the spectrum of heavy quarks, with one and the same analytic expression for the scattering amplitude. Moreover, it is shown that perturbative approximations usually do not satisfy these natural conditions.

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1. Introduction

It was a pleasure to participate in the Excited QCD 2017 workshop at Sintra (Portugal) and to attend short seminars on so many different approaches towards understanding the complicated relation between strong interactions and quantum chromodynamics (QCD). In particular, we were pleased to witness presentations on the study of hadron scattering, mass distributions, cross sections, and resonances. Nevertheless, although it did not seem to disturb most theoreticians, the lack of progress in measuring

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high-statistics multi-hadron data was very disappointing. Experiment does not yet provide the necessary conditions to confront model results with measured multi-hadron mass distributions. Several decades of accelerators and sensitive detectors have, unfortunately, still not resulted in sufficient data to allow for narrow binning and high statistics. From Fig. 1 in Ref. [1], one could even conclude that it will take quite a while before ATLAS statistics [2] competes with 25 years older data of the ARGUS Collaboration [3].

During the workshop, some criticism arose on the compilation of data by the Particle Data Group Collaboration (PDG) [4]. However, it would actually be more in place to direct such a criticism towards researchers who use section headings to fit the results of their models. Sure, the PDG reports on each significant enhancement observed in multi-hadron mass distributions as well as on its weighted central mass and width, and furthermore its quantum numbers based on available experimental data analyses. Subsequently, it introduces a new section in its Review of Particle Physics when suspected to be different from already reported enhancements. However, the PDG also supplies its readers with a list of published work related to each one of the enhancements. Serious researchers are thus free to go through the published mass distributions and draw their own conclusions on the nature of a certain enhancement or, even better, compare the full multi-hadron mass distribution with the results of their models.

2. Dimeson channels

Decades ago, pions or kaons were scattered from the proton's meson cloud (see *e.g.* Refs. [5, 6]) in order to produce pion–meson or kaon–meson cross sections, respectively. However, a more elegant production of dimesons stems from e^-e^+ scattering (see *e.g.* Refs. [7,8]), since via vector dominance, one is then pretty sure about the dimeson's quantum numbers. Consequently, the resulting $J^{PC} = 1^{--}$ mass distributions could be considered backbones of mesonic spectra and thus should have been given the highest priority in the past. Nevertheless, experiments for the vector charmonium spectrum only came up with binnings of 20 MeV [8] (see Fig. 1 in Ref. [9]) or 25 MeV [7] (see Fig. 3 in Ref. [10]) and extremely low statistics, whereas light-dimeson spectra are not in a better shape (see *e.g.* Ref. [11]).

Previously, we presented amplitudes for multi-resonance scattering (see Eq. (2) in Ref. [12]) and production (see Eq. (3) in Ref. [12]), which were based [13] on general scattering theory [14], applied to multi-channel dispersion in the presence of a tower of bound states and resonances, the so-called Resonance Spectrum Expansion (RSE) [15–17]. The RSE amplitude in Ref. [12] is restricted to single-channel scattering. However, a multi-channel generalisation is straightforward. For dimeson channels, the input spectrum

can be determined by the use of a bound-state model for quark–antiquark states. The amplitudes can be then converted into mass distributions and cross sections. Furthermore, one can extract the complex scattering singularities (poles in the total invariant mass $\sqrt{s}$) from the amplitudes and study their behaviour under variation of the model's parameters.

Quantum states of the RSE respect total angular momentum J, parity P, total flavour and isospin, moreover, when applicable, also charge conjugation C. However, they do not have well-defined orbital quantum numbers, relative angular momentum ℓ, and radial excitation n, but are rather mixtures of all possible orbital quantum numbers. In particular, vector S and D states mix. The latter phenomenon has interesting consequences, as dominantly D-wave resonances are found near the input spectrum and with small widths (a few MeV), whereas dominantly S-wave states have central resonance masses some 150 MeV or more below the masses of the input spectrum and with relatively large widths (tens of MeVs).

Coupled channels do not exhibit enhancements at the same place. A pole is determined by the full scattering matrix, but enhancements also depend on the kinematics of a specific channel. This can be nicely observed from Figs. 3 and 4 in Ref. [10], where mass distributions are depicted for $D^*\bar{D}^*$ [7] and $\Lambda_c^+\Lambda_c^-$ [18], respectively. Each of the two figures shows the $\Lambda_c^+\Lambda_c^-$ threshold enhancement and the $5S$, $4D$ charmonium resonances, but masses are different. Part of the discrepancy may stem from incompatibilities between the mass normalisations of the BaBar and Belle collaborations, but the larger part is due to differences in kinematics. Moreover, in some channels, no enhancement appears at all near the pole.

In Ref. [19], we compared to experiment [5,6] our predicted cross sections for S-wave isodoublet dispersion of $K\pi$ (see Fig. 2), $K\eta$ (see Fig. 6) and $K\eta'$ (see Fig. 7). For $K\pi$, we showed results for three different values of the overall coupling constant λ. For very small values of λ, one observes the scalar $n\bar{s}$ input spectrum. When λ takes about half its model value, one notices some more structure for low invariant masses. At the model's standard value of λ, this structure is dominant and well in agreement with the experimental data [5,6]. The behaviour of the poles under variation of λ for the two lowest-lying $K\pi$ resonances was also studied in Ref. [19] (see Fig. 3). The scattering pole for $K_0^*(1430)$ can directly be connected to the $n\bar{s}$ input spectrum. However, the scattering pole for $K_0^*(800)$ does not stem from the input spectrum, but is dynamically generated [20].

Under variation of λ, poles can also move below the lowest threshold, thus representing bound states. The passage through threshold is different for S-waves and higher waves. This issue was studied in Ref. [21] (see Figs. 4.1 and 4.2). The resonance pole for P- or higher-wave dispersion moves smoothly towards threshold under variation of λ (see Fig. 6 in Ref. [22]). Below the

threshold, it behaves as expected for a bound-state pole. In contrast, a complex S-wave resonance pole can have a real part smaller than the threshold value of $\sqrt{s}$, and only end up on the real axis well below the threshold, thus representing a virtual bound state. Thereafter, under variation of λ, the pole moves back towards the threshold along the real $\sqrt{s}$ axis, and only after touching the threshold, it turns into a true bound state (see Fig. 5 in Ref. [22], or Fig. 1 in Ref. [23]). Moreover, one can also continuously vary quark masses and the corresponding threshold values. Resonance poles then move smoothly from one established resonance to another (see Fig. 1 in Ref. [24]).

Any model that claims to describe resonances of multi-hadron scattering or production should exhibit the above properties for the corresponding resonance poles. Such poles for perturbative scattering amplitudes usually do not satisfy this behaviour at the threshold, as was studied in Ref. [25] (see Fig. 5).

3. Resonances

In the harmonic-oscillator approximation of the RSE (HORSE), one can predict mass distributions for dimeson channels. It was observed that the harmonic-oscillator frequency can be taken the same (0.19 GeV) for all flavours.

The results for $K\pi$ [20], $K\eta$, and $K\eta'$ [19] were already discussed above. In Table 2 in Ref. [19], we showed the five lowest-lying resonance poles that we found in the scattering matrix for isodoublet S-wave channels. We thus expect to find 10 plus 5 corresponding poles in the isosinglet and isotriplet S-wave channels, respectively, many more than observed in experiment [4].

For vector states, we also found many resonances that have not yet been confirmed in experiment. Our assignments for D_s^* resonances are collected in Fig. 1 and Table 3 in Ref. [26]. The 20-MeV binning of the data [27] does not allow for firm conclusions. Higher dominantly S- and dominantly D-wave charmonium resonances from HORSE have been reported by us at various occasions, as *e.g.* in Fig. 5 in Ref. [10], where one may observe that 25 MeV bins and low statistics [7] do not allow for any definite conclusions.

Nevertheless, R_b data of the BaBar Collaboration [28] can be compared to the HORSE predictions for bottomonium (see Fig. 1 in Ref. [29]). However, in Ref. [4], the non-resonant $B\bar{B}$ threshold enhancement is classified as the $\Upsilon(4S)$ resonance, whereas HORSE predicts the central mass of that resonance to be some 150 MeV heavier. Threshold enhancements can easily be observed for $B\bar{B}$, $B^*\bar{B}$, and $B^*\bar{B}^*$ (see Fig. 3 in Ref. [29]). Moreover, non-resonant threshold enhancements for production amplitudes were predicted in Ref. [16]. In Fig. 6 in Ref. [29], one observes how the $\Upsilon(4S)$

resonance interferes with the $B_s\bar{B}_s$ threshold enhancement. In Figs. 6 and 7 in Ref. [30], we extracted the $\Upsilon\left(2\ ^3D_1\right)$ state from data published by the BaBar Collaboration [31].

Finally, the discovery of a very light hadronic particle, the $E(38)$, with a mass of about 38 MeV, was not completely unexpected. A flavour-independent HORSE parameter, the average $q\bar{q}$/meson–meson interaction distance, was observed to be related to such a small mass. But it was not before 25 years later that we became aware of an interference effect that might be associated with the existence of a 38-MeV quantum [32]. Further evidence [33] resulted from leptonic bottomonium decays published by the BaBar Collaboration [31]. High-statistics data (see Fig. 8 in Ref. [34]) published by the COMPASS Collaboration [35] exhibits a very clear diphoton signal, but the collaboration changed *a posteriori* its electronically published article, claiming that the enhancement is an artifact of their experimental setup. However, that claim is not substantiated by their own Monte Carlo simulation (see Fig. 8 in Ref. [36]), which, moreover, refers to data with much lower statistics, published in Ref. [37].

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