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PRODUCTION OF $f_0(500)$, $f_0(980)$ AND $a_0(980)$ IN THE $\chi_{c1} \to \eta\pi^+\pi^-$ AND $\eta_c \to \eta\pi^+\pi^-$ DECAY*

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Using the chiral unitary approach in coupled channels and SU(3) symmetry, we describe the production of $f_0(500)$, $f_0(980)$ and $a_0(980)$ in the $\chi_{c1} \to \eta\pi^+\pi^-$ reaction, recently performed by the BESIII Collaboration. A very strong peak for the $a_0(980)$ can be seen in the $\eta\pi$ invariant mass, while clear signals for the $f_0(500)$ and $f_0(980)$ appear in the one of $\pi^+\pi^-$. Next, we make predictions for the analogous decay $\eta_c \to \eta\pi^+\pi^-$, which could also be measured experimentally. We discuss the differences of these reactions which are interesting to test the picture where these scalar mesons are dynamically generated from the interaction of pairs of pseudoscalars.

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1. Introduction

The experiment on the $\chi_{c1} \to \eta\pi^+\pi^-$ decay performed with high statistics by the BESIII Collaboration [1], and previously by the CLEO Collaboration [2], presents an interesting opportunity to test the picture where the scalar mesons $f_0(500)$, $f_0(980)$ and $a_0(980)$ are dynamically generated from the final-state interaction of meson pairs $\pi^+\pi^-$ and $\eta\pi^\pm$. Indeed, it is found that the most dominant two-body structure comes from $a_0(980)^\pm\pi^\mp$, with $a_0(980)^\pm \to \eta\pi^\pm$.

In this short paper, we will briefly discuss the work of Refs. [3, 4], where the chiral unitary approach and SU(3) symmetry were used to describe the production of these three scalars in the BESIII experiment and to make

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predictions for the analogous reaction with η_c instead of χ_{c1}. We will make a short discussion on SU(3) scalars and compare the treatment of the amplitude and mass distribution used to describe each decay.

2. Common formalism

We start by considering that the charmonium states $c\bar{c}$ behave as an SU(3) scalar, and use the following ϕ matrix to get the weight of every trio of pseudoscalar mesons created in the χ_{c1} or η_c decay:

$$\phi \equiv \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & K^0 \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta' \end{pmatrix} . \tag{1}$$

If we think of ϕ as a $q\bar{q}$ matrix, as discussed in Ref. [3], it is natural to build an SU(3) scalar by taking

$$\begin{aligned} \mathrm{SU(3)[scalar]} \equiv \mathrm{Tr}(\phi\phi\phi) &= 2\sqrt{3}\eta\pi^+\pi^- + \sqrt{3}\eta\pi^0\pi^0 + \frac{\sqrt{3}}{9}\eta\eta\eta \\ &\quad +3\pi^+K^0K^- + 3\pi^-K^+\bar{K}^0 \,, \end{aligned} \tag{2}$$

where we have already neglected the η' which plays only a marginal role in the building of the $f_0(500)$, $f_0(980)$, $a_0(980)$ resonances, because of its large mass and small couplings. We have also neglected the terms that cannot make a transition to the final state $\eta\pi^+\pi^-$.

In fact, there are four SU(3) scalars: $\mathrm{Tr}(\phi\phi\phi)$, $\mathrm{Tr}(\phi)\,\mathrm{Tr}(\phi\phi)$, $[\mathrm{Tr}(\phi)]^3$ and $\mathrm{Det}(\phi)$. But by the Cayley–Hamilton relation,

$$2\ \mathrm{Tr}(\phi\phi\phi) - 6\,\mathrm{Det}(\phi) - 3\ \mathrm{Tr}(\phi)\,\mathrm{Tr}(\phi\phi) + [\mathrm{Tr}(\phi)]^3 = 0\,, \tag{3}$$

only three of them are independent. In Ref. [4], we discussed other possibilities and concluded that the best choice is indeed $\mathrm{Tr}(\phi\phi\phi)$.

Next, we use the chiral unitary approach to describe how the scalar mesons are dynamically generated from the interaction of pairs of pseudoscalars in coupled channels. We follow the framework of Ref. [5], using an effective chiral Lagrangian where mesons are the degrees of freedom

$$\mathcal{L}_2 = \frac{1}{12\,f_\pi^2}\,\mathrm{Tr}\left[(\partial_\mu\phi\,\phi - \phi\,\partial_\mu\phi)^2 + M\phi^4\right]\,, \tag{4}$$

where ϕ is the SU(3) matrix, f_π is pion decay constant and

$$M = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2m_K^2 - m_\pi^2 \end{pmatrix} . \tag{5}$$

From this Lagrangian, we extract the kernel of each channel, which in charge basis are: (1) $\pi^+\pi^-$, (2) $\pi^0\pi^0$, (3) K^+K^-, (4) $K^0\bar{K}^0$, (5) $\eta\eta$,(6) $\pi^0\eta$ and can be found in Refs. [6,7]. These kernels are used to build the V-matrix which is then inserted into the Bethe–Salpeter equation, summing the contribution of every meson–meson loop

$$T = (1 - VG)^{-1}\, V \,, \tag{6}$$

where G is the meson–meson loop function, which we regularize with a cutoff using $q_{\rm max} \sim 600$ MeV. After the integration in q^0 and $\cos\theta$, we have

$$G = \int\limits_0^{q_{\rm max}} \frac{q^2 {\rm d}q}{(2\pi)^2} \frac{\omega_1 + \omega_2}{\omega_1\omega_2 \left[(P^0)^2 - (\omega_1 + \omega_2) + i\epsilon\right]} \tag{7}$$

with $\omega_i = \sqrt{q^2 + m_i^2}$, $P^0 = \sqrt{s}$. Each kernel is projected in S-wave and a normalization factor is included when identical particles are present, which later needs to be restored. Finally, the T-matrix will give us the scattering and transition amplitudes between each channel, and isospin symmetry is used to obtain the amplitude of channels with different charges [3].

3. Theoretical description of $\chi_{c1} \to \eta\pi^+\pi^-$

Following the assumption that $c\bar{c}$ behaves as an SU(3) scalar, we look at the quantum numbers of the initial and final states, combining them in two cases: η leaves in P-wave, while $\pi^+\pi^-$ go through the final-state interaction with $I = 0$ to form the $f_0(500)$ and $f_0(980)$ in S-wave; and π^- (or π^+) leaves in P-wave, while $\eta\pi^+$ (or $\eta\pi^-$) go through the final-state interaction with $I = 1$ to form the $a_0^\pm(980)$ in S-wave.

To illustrate our method, we will describe the case where η leaves in P-wave and $\pi^+\pi^-$ interact. In this case, we will consider the diagrams of Fig. 1. Then from the SU(3) scalar in Eq. (2), we select the terms in which

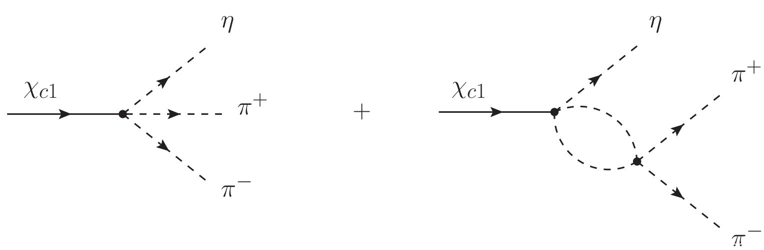

Fig. 1. Diagrams considered in the description of $f_0(500)$ and $f_0(980)$ production in $\chi_{c1} \to \eta\pi^+\pi^-$ reaction: tree-level (left) and rescattering of $\pi^+\pi^-$ pair (right).

we can isolate one η and let the other pairs rescatter, since our coupled channels approach allows them to make a transition to $\pi^+\pi^-$ final state

$$\eta\left(2\sqrt{3}\pi^+\pi^- + \sqrt{3}\pi^0\pi^0 + \frac{\sqrt{3}}{9}\eta\eta\right) . \tag{8}$$

Then we will have the sum of tree-level and rescattering

$$\begin{aligned} t_\eta &= V_P\left(\vec{\epsilon}_{\chi_{c1}}\cdot\vec{p}_\eta\right) \\ &\quad\times\left(h_{\pi^+\pi^-} + \sum_i h_i S_i G_i[M_{\rm inv}(\pi^+\pi^-)]t_{i,\pi^+\pi^-}[M_{\rm inv}(\pi^+\pi^-)]\right) , \end{aligned} \tag{9}$$

where h_i are the weights of Eq. (8), S_i are symmetry and combination factors for the identical particles and the factor V_P provides a global normalization, which is adjusted to the data in the $a_0(980)$ peak.

Finally, we can write the differential mass distribution for $\pi^+\pi^-$

$$\frac{{\rm d}\Gamma}{{\rm d}M_{\rm inv}(\pi^+\pi^-)} = \frac{1}{(2\pi)^3}\frac{1}{4M^2_{\chi_{c1}}}\frac{1}{3}p^2_\eta p_\eta \tilde{p}_\pi\left|t_\eta\right|^2 , \tag{10}$$

where p_η is the η momentum in the χ_{c1} rest frame and $\tilde{p}_\pi$ is the pion momentum in the $\pi^+\pi^-$ rest frame.

In Fig. 2, we show the results using the method of Ref. [3] and the experimental data of Ref. [1]. We also compare the results using Tr$(\phi\phi\phi)$ as the SU(3) scalar to the case where only Tr(ϕ) Tr$(\phi\phi)$ was used, and see that the latter is completely off from the experiment.

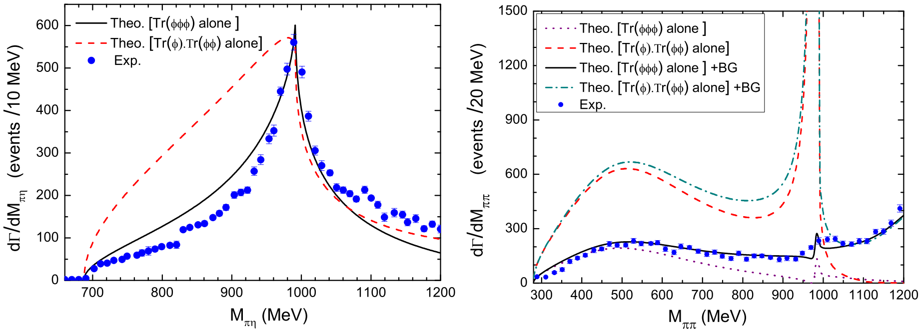

Fig. 2. Results for the $\pi\eta$ (left) and $\pi^+\pi^-$ (right) mass distribution in the $\chi_{c1}\to\eta\pi^+\pi^-$ reaction, using Tr$(\phi\phi\phi)$ or Tr(ϕ) Tr$(\phi\phi)$. A linear background is adjusted to the data in the $\pi^+\pi^-$ mass distribution. Data from Ref. [1].

4. Predictions for $\eta_c \to \eta\pi^+\pi^-$

In the analogous reaction, $\eta_c \to \eta\pi^+\pi^-$, the dominant structure will be the one where every final-state meson goes out in S-wave. Therefore, one must consider the interference between each term in the amplitude, then

$$t = t_{\text{tree}} + t_\eta + t_{\pi^+} + t_{\pi^-} \,, \qquad t_{\text{tree}} = V_P \, h_{\eta\pi^+\pi^-} \,. \tag{11}$$

Each of the later three terms is a function of an invariant mass, analogous to Eq. (9). We select $M_{\text{inv}}(\pi^+\pi^-)$ and $M_{\text{inv}}(\pi^+\eta)$ as variables and the third one is determined by the relation: $M_{13}^2 = M_{\eta_c}^2 + 2m_\pi^2 + m_\eta^2 - M_{12}^2 - M_{23}^2$. It is also necessary to consider the double differential mass distribution [8]

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}M_{\text{inv}}(\pi^+\pi^-)\mathrm{d}M_{\text{inv}}(\pi^+\eta)} = \frac{1}{(2\pi)^3}\frac{1}{8M_{\eta_c}^3} M_{\text{inv}}(\pi^+\pi^-)M_{\text{inv}}(\pi^+\eta)|t|^2 \,, \tag{12}$$

where we need to integrate in one of the invariant masses to get the distribution of the other one. This way the background of $\pi^+\eta$ appears naturally in the $\pi^+\pi^-$ mass distribution and *vice versa.*

Since our approach is valid only for energies up to 1.2 GeV, we need to introduce a cut in each amplitude to perform the integration. To do that, we evaluate $Gt(M_{\text{inv}})$ combinations up to $M_{\text{inv}} = M_{\text{cut}}$. From there on, we multiply Gt by a smooth factor to make it gradually decrease at large M_{inv}

$$Gt(M_{\text{inv}}) = Gt(M_{\text{cut}})e^{-\alpha(M_{\text{inv}}-M_{\text{cut}})} \quad \text{for} \quad M_{\text{inv}} > M_{\text{cut}} \,. \tag{13}$$

In Fig. 3, we show the predictions for the production of $f_0(500)$, $f_0(980)$ and $a_0(980)$ in the $\eta_c \to \eta\pi^+\pi^-$ decay. To compare qualitatively with the

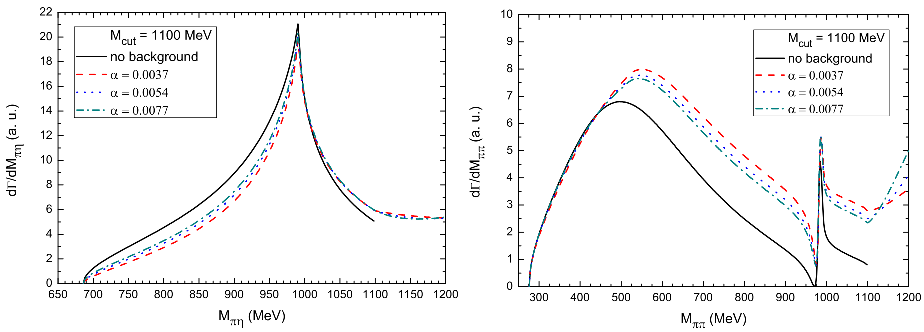

Fig. 3. Predictions from Ref. [4] for the mass distribution of $\pi\eta$ (left) and $\pi^+\pi^-$ (right) in $\eta_c \to \eta\pi^+\pi^-$, using $M_{\text{cut}} = 1100$ MeV and $\alpha = 0.0037$, 0.0054, 0.0077 MeV^{-1}, which reduce Gt by a factor of 3, 5 and 10, respectively, at $M_{\text{cut}} + 300$ MeV. The "no background" solid curve is obtained by keeping only the tree-level and the main rescattering amplitude.

results of the previous section, we show with the solid curves, denoted by "no background", the results obtained by keeping only the tree-level and the main rescattering amplitude $t_{\pi^-}[M_{\rm inv}(\pi^+\eta)]$ in the case of $a_0(980)$, and $t_\eta[M_{\rm inv}(\pi^+\pi^-)]$ in the case of the $f_0(500)$ and $f_0(980)$. We can see that the background introduced goes in the direction where there was a small discrepancy between the results of Ref. [3] and the data of Ref. [1] in the $\chi_{c1} \to \eta\pi^+\pi^-$ reaction.

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REFERENCES

[1] M. Ablikim *et al.* [BESIII Collaboration], *Phys. Rev. D* **95**, 032002 (2017).
[2] G.S. Adams *et al.* [CLEO Collaboration], *Phys. Rev. D* **84**, 112009 (2011).
[3] W.H. Liang, J.J. Xie, E. Oset, *Eur. Phys. J. C* **76**, 700 (2016).
[4] V.R. Debastiani, W.H. Liang, J.J. Xie, E. Oset, *Phys. Lett. B* **766**, 59 (2017).
[5] J.A. Oller, E. Oset, *Nucl. Phys. A* **620**, 438 (1997) [*Erratum ibid.*, **652**, 407 (1999)].
[6] W.H. Liang, E. Oset, *Phys. Lett. B* **737**, 70 (2014).
[7] J.J. Xie, L.R. Dai, E. Oset, *Phys. Lett. B* **742**, 363 (2015).
[8] C. Patrignani *et al.* [Particle Data Group], *Chin. Phys. C* **40**, 100001 (2016).