

EXCITED HADRONS AND QUARK–HADRON DUALITY* **

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We review how Quark–Hadron Duality (QHD) for (u, d) flavors at high energies and in the scaling regime suggests a radial and angular behaviour of mesonic and baryonic resonance masses of the Regge form $M_{nJ}^2 = \mu^2 n + \beta^2 J + M_0^2$. The radial mass dependence is asymptotically consistent with a common two-body dynamics for mesons and baryons in terms of the quark–anti-quark ($q\bar{q}$) and quark–diquark (qD) degrees of freedom, respectively. This formula is validated phenomenologically within an uncertainty determined by half the width of the resonances, $\Delta M_{nJ}^2 \sim \Gamma_{nJ} M_{nJ}$. With this error prescription, we find from the non-strange PDG hadrons different radial slopes $\mu_{q\bar{q}}^2 = 1.34(4) \text{ GeV}^2$ and $\mu_{qD}^2 = 0.75(3) \text{ GeV}^2$, but similar angular slopes $\beta_{q\bar{q}}^2 \sim \beta_{qD}^2 \sim 1.15 \text{ GeV}^2$.

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1. Introduction

Confinement requires hadronic physical states to be colour singlet, but what are the complete set of eigenstates of QCD spanning the Hilbert space \mathcal{H}_{QCD} ? In the hadronic sector with light (u, d) quarks, besides the normalizable bound states such as π^+ , π^- , π^0 or n , \bar{n} and p , \bar{p} (and, of course, stable atomic nuclei and anti-nuclei, such as ${}^2\text{H}$, ${}^3\text{H}$, ${}^3\text{He}$, ${}^4\text{He}$, *etc.*), all other states occur in the continuum as asymptotic states. Most of the states reported by the Particle Data Group (PDG) [1] are not bound states but unstable resonances such as σ , ρ , ω , a_1 or Δ , N^* , which in a pure (u, d) world would be produced from and would decay into pions and nucleons, subject to the selection rules imposed by conservation laws. So far, the states fitting into the quark model classification enter the PDG tables, so this is a practical definition of completeness, namely $\mathcal{H}_{\text{PDG}} = \mathcal{H}_{\bar{q}q} \oplus \mathcal{H}_{qqq} \oplus \mathcal{H}_{\bar{q}\bar{q}q} \oplus \dots$. However, in the case of baryons, more states have been theoretically predicted than experimentally found, hence these *missing resonances* defy this criterium. In a finite box, such as in lattice QCD, due to the quark and gluon field boundary conditions, all states are normalizable and their energies are discretized, hence resonances are associated *only* with those states whose energies are insensitive to the volume of the box, such that indeed $\mathcal{H}_{\text{PDG}} \subset \mathcal{H}_{\text{QCD}}$.

This article is based on Refs. [2, 3] where we point out that, at least asymptotically, quark–hadron duality (QHD) (for a review, see *e.g.* Ref. [4]) in inclusive processes and in the scaling regime, *i.e.*, for energies much larger than the resonance widths $\sqrt{s} \gg \Gamma$, sets a limit on the hadronic squared mass density for states with fixed quantum numbers J^{PC} .

2. Quark–hadron duality

The meaning of QHD can be best illustrated with a simple case. Let us consider the (conserved) vector current $B_\mu = \bar{q}\gamma_\mu q$ which vanishes in the vacuum, $\langle B_\mu \rangle = 0$, and compute the correlator represented by Fig. 1 (left)

$$\Pi_{\mu\nu}(q) = \int d^4x e^{iq \cdot x} i \langle 0 | T \{ B_\mu(x) B_\nu(0) \} | 0 \rangle = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) \Pi(q). \quad (1)$$

At the hadronic level, we assume a complete set of states, see Fig. 1 (middle), characterized by the Proca vector fields (such as ω , ω' , ω'' , \dots), as stable and elementary particles with masses M_n and vacuum decay amplitudes $\langle 0 | B^\mu(0) | \omega_n^\nu \rangle = f_n q^\nu \epsilon^\mu$, with $\epsilon \cdot q = 0$ implying gauge invariance $q_\mu \Pi^{\mu\nu} = 0$. For $s = q^2 \rightarrow \infty$, we replace the sum over n by an integral and get

$$\frac{1}{\pi} \text{Im} \Pi(s) = \sum_{n=0}^{\infty} f_n^2 \delta(s - M_n^2) \rightarrow \lim_{n \rightarrow \infty} \frac{f_n^2}{dM_n^2/dn}, \quad (2)$$

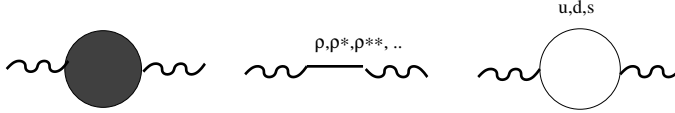


Fig. 1. Quark–hadron duality for VV correlators on the vacuum.

for the absorptive part. At the quark level, we have a loop with baryon charge of the quarks equal to $1/N_c$, see Fig. 1 (right), and massless quarks, with $\text{Im}\Pi(s) \rightarrow 1/(24\pi N_c)$ implying the asymptotic condition encoding QHD

$$\lim_{n \rightarrow \infty} f_n^2 / (dM_n^2/dn) = 1/(24\pi^2 N_c). \quad (3)$$

A similar discussion can be conducted for excited baryons when the forward scattering amplitude for the nucleon of Fig. 2 (left) is considered [5]

$$\begin{aligned} W_{\mu\nu}(p, q) &= \frac{1}{4\pi} \text{Im} \int d^4x e^{iq \cdot x} \sum_{\lambda} i \langle N_{\lambda}(p) | T \{ B_{\mu}(x) B_{\nu}(0) \} | N_{\lambda}(p) \rangle \\ &= \left(-g_{\mu\nu} + \frac{q_{\mu} q_{\nu}}{q^2} \right) W_1(s, q^2) + \left(p_{\mu} - q_{\mu} \frac{p \cdot q}{q^2} \right) \left(p_{\nu} - q_{\nu} \frac{p \cdot q}{q^2} \right) \frac{1}{M_N^2} W_2(s, q^2), \end{aligned} \quad (4)$$

with $s = (p+q)^2$. At the hadronic level, Fig. 2 (middle), we insert resonance states $|p+q, J\nu n\rangle$ with masses $M_{J\nu n}$ and transition form factors $G_{J\nu n}^{(i)}(q)$, to get (more details are in Refs. [3, 5])

$$W_i(s, q^2) = \sum_{J\nu n} \left[G_{J\nu n}^{(i)}(q) \right]^2 \delta(s - M_{J\nu n}^2). \quad (5)$$

In the Bjorken limit, $Q^2 \rightarrow \infty$ with $x = Q^2/2p \cdot q$ fixed, and $s = M_N^2 + Q^2(1/x - 1)$. At the quark level one obtains, Fig. 2 (right), both scaling $W_1(s, q^2) \rightarrow F_1(x)$, $W_2(s, q^2) \rightarrow F_2(x)$ and the Callan–Gross relation $F_2(x) = 2xF_1(x)$ due to the spin 1/2 nature of partons. The QHD requires

$$\sum_{J\nu} \lim_{n \rightarrow \infty} [G_{J\nu n}(q)]^2 / (dM_{J\nu n}^2/dn) = F_i(x), \quad (6)$$

which is satisfied if $G_{J\nu n}(q) \rightarrow F_{J\nu}(-Q^2/M_{J\nu n}^2)$ and $dM_{J\nu n}^2/dn \rightarrow \mu_{J\nu}^2$.

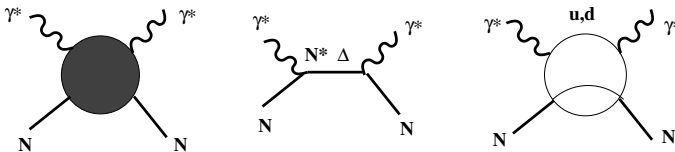


Fig. 2. Quark–hadron duality for VV correlators on the nucleon.

3. Finite widths effects

A remarkable feature of hadronic resonances noticed by Suranyi as early as 1967 [5] is that they are narrow, since $\gamma \equiv \Gamma/M \sim 0.13$, a natural value in the light of the large- N_c expansion, where $\gamma = \mathcal{O}(N_c^{-1})$ [6]. An upgrade of the Suranyi ratio yields in average the value $\Gamma/M = 0.12(8)$ both for mesons and baryons [7]. This has interesting implications for QHD at *finite* energies. For correlator (1), the finite width of the resonances can be implemented in Eq. (2) by an energy-dependent Breit–Wigner distribution

$$\frac{1}{\pi} \text{Im} \Pi(s) = \frac{1}{\pi} \sum_{n=0}^N \frac{f_n^2 \Gamma_n \sqrt{s}}{(s - M_n^2)^2 + \Gamma_n^2 s}, \quad (7)$$

where we put a cut-off N in the sum and assume no quark thresholds. We make the following Ansatz compatible with Eq. (3) and $\Gamma_n/M_n \rightarrow \gamma$

$$f_n^2 = 1/(24\pi^2 N_c), \quad M_n^2 = n + 1, \quad M_n \Gamma_n = \gamma n, \quad n = 0, 1, 2, \dots, N. \quad (8)$$

The dependence of $\text{Im} \Pi(s)$ on both Suranyi's ratio γ and the high-energy cut-off N is illustrated in Fig. 3. As we see, the number of visible peaks strongly depends on this ratio, but regardless of its numerical value we find the *finite* limit at large s which not only corresponds to the narrow resonance limit, but also coincides with the finite s result averaged over resonances. On the other hand, if we cut off the sum over states, we comply with the expected partonic behaviour *only* below the cut-off, *i.e.* $s \lesssim M_N^2 \sim NM_0^2$. Similar patterns hold for the correlator in Eq. (4) for finite Q^2 and x [5].

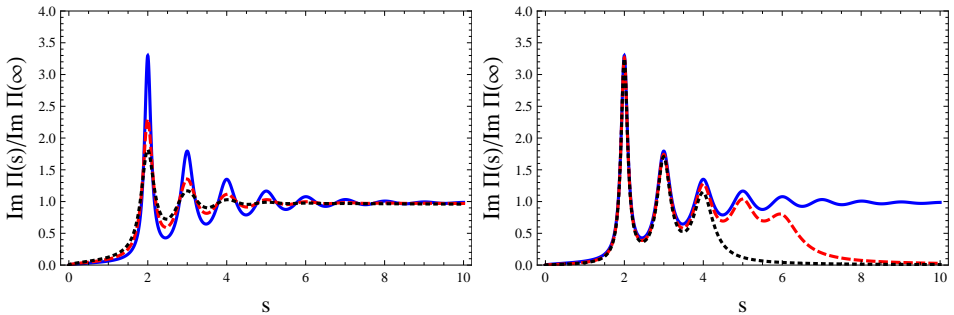


Fig. 3. Asymptotically normalized absorptive part for the baryon polarization operator as a function of the CM squared energy when $M_n^2 = n + 1$ and $M_n \Gamma_n = \gamma n$ for $n = 0, \dots, N$. Left panel: $\gamma = 0.05$ (solid), 0.10 (dashed), 0.15 (dotted) for $N = 100$. Right panel: $N = 10$ (solid), 5 (dashed), 3 (dotted) for $\gamma = 0.1$.

4. Two-body dynamics with linear potential and Regge fits

For highly excited states, relativity can be accommodated by the Salpeter equation [8], where the (classical) mass operator in the CM frame reads

$$M = \sqrt{p^2 + m_1^2} + \sqrt{p^2 + m_2^2} + V_{12}(r). \quad (9)$$

The spinor structure is included in the potential $V_{12}(r)$, which accounts for the static interaction in the heavy-quark limit, $m_Q, m_{\bar{Q}} \gg p$, implying $V_{\bar{Q}Q}(r) \rightarrow \sigma_{\bar{Q}Q}r$ for $\bar{Q}Q$ states (from charmonium data $\sigma_{\bar{Q}Q} \sim 4.5 \text{ fm}^{-2}$ [9]). For light quarks $m_q, m_{\bar{q}} \rightarrow 0$ and $M \rightarrow 2p + \sigma_{\bar{q}q}r$ with $p^2 = p_r^2 + L^2/r^2$. At large M , we take $p \rightarrow p_r$, and the semiclassical quantization rule $\oint p_r dr = 2\pi(n + \alpha)$, with α denoting a constant, leads to [8]

$$M_n^2 = \mu^2 n + M_0^2, \quad (10)$$

with $\mu_{\bar{q}q}^2 = 4\pi\sigma_{\bar{q}q}$. This agrees with QHD of Eq. (3) if $f_n^2 \rightarrow \mu^2/(24\pi^2 N_c)$. From the QHD condition for baryons, Eq. (6), we get a mass formula similar to Eq. (10) and we infer a quark-diquark (qD) long-distance dynamics with $V_{qD}(r) \rightarrow \sigma_{qD}r$ and $\mu_{qD}^2 = 4\pi\sigma_{qD}$ of the form proposed in Ref. [10] for the non-strange sector; their $\sigma_{qD} = 1.57 \text{ fm}^{-2}$ (called β there) produces $\mu_{qD}^2 = 0.76 \text{ GeV}^2$. The QHD scaling requirements at the hadronic level, Eqs. (3) and (6), are consistent with an equidistant mass squared spectrum for the intermediate hadronic states. More generally, Regge trajectories can be parameterized as $M_{n,J}^2 = a + \mu^2 n + \beta^2 J$. Our fits for the radial and angular slopes in the case of non-strange mesons [2] and baryons [3] assume an uncertainty given by the finite width of the resonances

$$\chi^2 = \sum_{\alpha} \left(\frac{M_{\alpha}^2 - (M_{\alpha}^{\text{exp}})^2}{\Gamma_{\alpha}^{\text{exp}} M_{\alpha}^{\text{exp}}} \right)^2, \quad (11)$$

and are summarized in Fig. 4. We find that $\mu_{\bar{q}q}^2 = 1.34(4) \text{ GeV}^2 \sim 2\mu_{qD}^2$ but $\beta_{\bar{q}q}^2 \sim \beta_{qD}^2 \sim 1.15 \text{ GeV}^2$. The value $\mu_{qD}^2 = 0.75(3) \text{ GeV}^2$ agrees remarkably well with the relativistic qD model findings [10]. Thus, our QHD-based analysis supports the idea that possibly $\mathcal{H}_{\text{PDG}} = \mathcal{H}_{\bar{q}q} \oplus \mathcal{H}_{qD} \oplus \mathcal{H}_{\bar{q}\bar{D}} \oplus \dots$

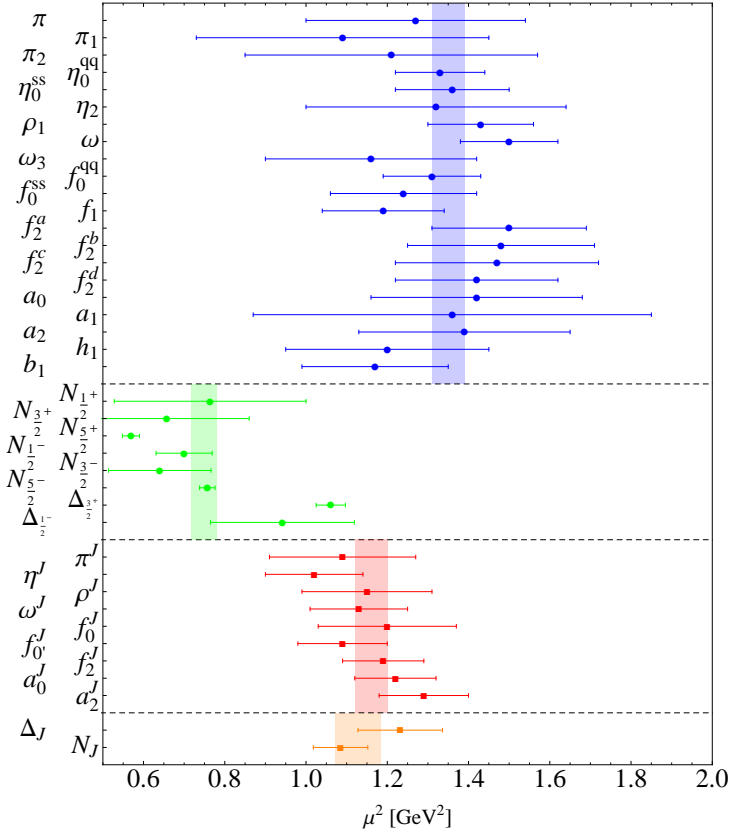


Fig. 4. Radial and angular slopes for non-strange mesons or baryons containing (u, d) quarks and anti-quarks or diquarks, respectively.

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