THERMAL ENTROPY OF A QUARK–ANTIQUARK PAIR FROM A DYNAMICAL HOLOGRAPHIC EMD MODEL*

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(Received August 21, 2017)

Lattice QCD indicates a large amount of entropy associated with a quark–antiquark ($q\bar{q}$) pair near the deconfinement temperature. The entropy shows a sharp peak near the transition temperature and increases with the interquark distance. We use the gauge/gravity duality to reproduce these lattice results holographically. We consider a phenomenological bottom–up Einstein–Maxwell-dilaton (EMD) gravity model and analytically construct the gravity solutions, whose dual boundary theory satisfies the properties of confined/deconfined phases. We study the entropy of the $q\bar{q}$ pair and find that our holographic model qualitatively reproduces the corresponding lattice results. We further provide holographic results for the $q\bar{q}$ entropy with chemical potential.

DOI:10.5506/APhysPolBSupp.10.1145

1. Introduction

Lattice QCD data, summarized in Figs. 1 and 2, suggests a strong entanglement between the heavy bound states and the rest of QCD plasma. There are three main observations: *(i)* lattice data predicts a large amount of entropy associated with a heavy $q\bar{q}$ pair near the deconfinement transition temperature, *(ii)* away from the deconfinement temperature the entropy decreases with temperature, and *(iii)* the entropy grows as the separation between the quarks increases.

Since lattice results for the $q\bar{q}$ entropy also indicate the breakdown of the weak coupling approximation [1,2], it provides another avenue where the ideas of gauge/gravity duality can be further applied and tested. During the last two decades, the gauge/gravity duality has immersed as a standard tool

* Presented at "Excited QCD 2017", Sintra, Lisbon, Portugal, May 7–13, 2017.

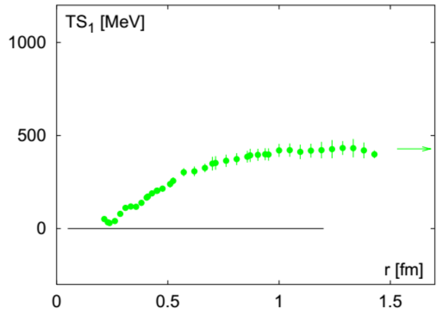

Fig. 1. Two flavour lattice QCD result for the entropy of a $q\bar{q}$ pair as a function of quark–antiquark separation at temperature $T \simeq 1.3\,T_c$. The result is taken from [3].

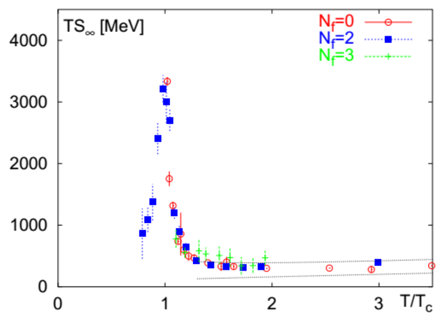

Fig. 2. Lattice QCD result for the entropy of a $q\bar{q}$ pair as a function of temperature T/T_c for large $q\bar{q}$ separation. The result is taken from [3].

to study strongly coupled gauge theories and indeed using this duality, a lot of new insights into the regime of strongly coupled QCD, which agrees qualitatively with lattice QCD, have been obtained both from "top–down" and phenomenological "bottom–up" models. Our aim in this proceeding would be to see whether this duality can provide new insights into the entropy of the $q\bar{q}$ pair as well.

2. Einstein–Maxwell-dilaton gravity

We start with the Einstein–Maxwell-dilaton action in five dimensions

$$S_{\mathrm{EM}} = -\frac{1}{16\pi G_5}\int \mathrm{d}^5x\sqrt{-g}\left[R - \frac{f(\phi)}{4}F_{MN}F^{MN} - \frac{1}{2}\partial_M\phi\partial^M\phi - V(\phi)\right], \quad (1)$$

where G_5 is the Newton constant in five dimensions, $V(\phi)$ is the potential of the dilaton field, and $f(\phi)$ is a gauge kinetic function which represents the coupling between dilaton and gauge field A_M. To solve Einstein, Maxwell and dilaton equations, we consider the following Ansätze:

$$
\begin{aligned}
\mathrm{d}s^2 &= \frac{L^2 e^{2A(z)}}{z^2}\left(-g(z)\mathrm{d}t^2 + \frac{\mathrm{d}z^2}{g(z)} + \mathrm{d}y_1^2 + \mathrm{d}y_3^2 + \mathrm{d}y_3^2\right)\,, \\
A_M &= A_t(z)\,, \qquad \phi = \phi(z)\,,
\end{aligned} \tag{2}
$$

where we have assumed that the various fields depend only on the holographic coordinate z. Here, $z = 0$ corresponds to the asymptotic boundary and L is the AdS length scale. Importantly, these equations can be solved analytically in terms of a scale function $A(z)$

$$
\begin{aligned}
g(z) &= 1 - \frac{1}{\int_0^{z_\mathrm{h}} \mathrm{d}x\; x^3 e^{-3A(x)}}\left[\int\limits_0^z \mathrm{d}x\; x^3 e^{-3A(x)} + \frac{2c\mu^2}{\left(1 - e^{-cz_\mathrm{h}^2}\right)^2}\det\mathcal{G}\right]\,, \\
\phi'(z) &= \sqrt{6(A'^2 - A'' - 2A'/z)}\,, \\
A_t(z) &= \mu\frac{e^{-cz^2} - e^{-cz_\mathrm{h}^2}}{1 - e^{-cz_\mathrm{h}^2}}\,, \\
V(z) &= -3L^2z^2 g e^{-2A}\left[A'' + A'\left(3A' - \frac{6}{z} + \frac{3g'}{2g}\right) - \frac{1}{z}\left(-\frac{4}{z} + \frac{3g'}{2g}\right) + \frac{g''}{6g}\right]\,, \\
f(z) &= e^{-cz^2 - A(z)}\,,
\end{aligned} \tag{3}
$$

where

$$
\det\mathcal{G} = \begin{vmatrix} \int\limits_0^{z_\mathrm{h}} \mathrm{d}x\; x^3 e^{-3A(x)} & \int\limits_0^{z_\mathrm{h}} \mathrm{d}x\; x^3 e^{-3A(x) - cx^2} \\ \int\limits_{z_\mathrm{h}}^{z} \mathrm{d}x\; x^3 e^{-3A(x)} & \int\limits_{z_\mathrm{h}}^{z} \mathrm{d}x\; x^3 e^{-3A(x) - cx^2} \end{vmatrix}\,.
$$

Here, we have used the boundary condition that $g(z_\mathrm{h}) = 0$ at the horizon and $g(z)$ goes to 1 at the asymptotic boundary.

For $A(z)$, we consider the following simple form:

$$
A(z) = -\bar{a}\; z^2\,. \tag{4}
$$

We would like to emphasize that Eq. (3) is a solution of the Einstein–Maxwell-dilaton system for any $A(z)$, and Eq. (4) is just a particular form. This expression is chosen to reproduce some of the lattice QCD results holographically. The parameter $\bar{a}$ can be fixed by comparing with the lattice estimate for the confinement–deconfinement phase transition temperature at zero chemical potential. For example, our gravity background undergoes a thermal-AdS/black hole phase transition which on the dual boundary side corresponds to the confinement–deconfinement phase transition. Demanding the critical temperature at zero chemical potential to be around 270 MeV fixes the parameter $\bar{a} = 0.145$.

3. Black hole thermodynamics

The thermodynamic results of the gravity solution are shown in Figs. 3 and 4. There are two branches in the $(T, z_{\rm h})$ plane. The branch with negative (positive) slope is stable (unstable). The unstable branch, however, disappears for higher values of chemical potential. This defines a critical chemical potential $\mu_{\rm crit} = 0.673$ GeV. Moreover, the black hole branch does not exist below a certain minimal temperature $T_{\rm min}$. This suggests a phase transition from AdS black hole to thermal-AdS as the temperature decreases. The phase transition can be observed from the free energy behaviour shown in Fig. 4. We see that the free energy is positive for the unstable branch and becomes negative after some critical temperature $T_{\rm crit}$ along the stable branch, implying a first order Hawking/Page phase transition from AdS black hole to thermal-AdS as the temperature decreases.

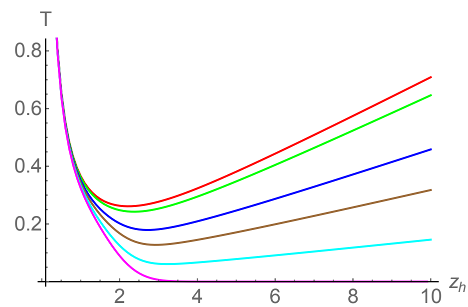

Fig. 3. (Colour on-line) The Hawking temperature (T) as a function of $z_{\rm h}$ for various values of the chemical potential μ. Here, red, green, blue, brown, cyan and magenta (from top to bottom) curves correspond to $\mu = 0$, 0.2, 0.4, 0.5, 0.6 and 0.673 [GeV], respectively.

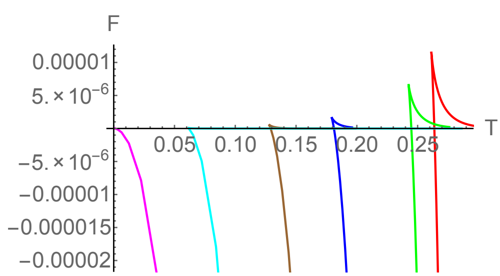

Fig. 4. (Colour on-line) Free energy F as a function of T for various values of the chemical potential μ. Here, red, green, blue, brown and cyan (from right to left) curves correspond to $\mu = 0$, 0.2, 0.4, 0.5, 0.6 and 0.673 [GeV], respectively.

4. Free energy of quark–antiquark pair

Our main aim of this section is to show that the thermal-AdS (AdS black hole) phase corresponds to confinement (deconfinement) in the boundary theory. Again, by confinement we simply mean a phase for which the expectation value of the Wilson loop satisfies the area law, while the expectation value of the Polyakov loop is zero. In order to show these properties, we first need to calculate the free energy of the $q\bar{q}$ pair. Via gauge/gravity duality, the free energy $\mathcal{F}$ of the $q\bar{q}$ pair can be calculated from the on-shell fundamental string world sheet action. Generally, there are two world sheet configurations that minimize the string action: an ∪-shaped connected configuration which extends from the boundary ($z = 0$) into the bulk and a disconnected configuration which consists of two lines separated by distance ℓ, extending from the boundary to the end of the spacetime.

The free energy $\mathcal{F}$ of a $q\bar{q}$ pair in the thermal-AdS background is shown in Fig. 5. It turns out that the connected ∪-shape configuration is the only relevant string solution here and this solution does not penetrate deep into the bulk. Some kind of an "imaginary wall" in the bulk space appears near $z \simeq 1.185$ GeV^{-1} which cannot be penetrated by the string world sheet. Around this wall, ℓ increases rapidly. Importantly, the free energy of the $q\bar{q}$ pair is found to have the Cornell expression, $\mathcal{F}_{\text{con}} = -\frac{\kappa}{\ell} + \sigma_s \ell$. This suggests that the $q\bar{q}$ pair is connected by the string and forms a confined state in the dual boundary theory. Similarly, the Polyakov loop expectation value, calculated from the single quark free energy, also vanishes. This suggests that thermal-AdS phase corresponds to confined phase in the dual boundary theory.

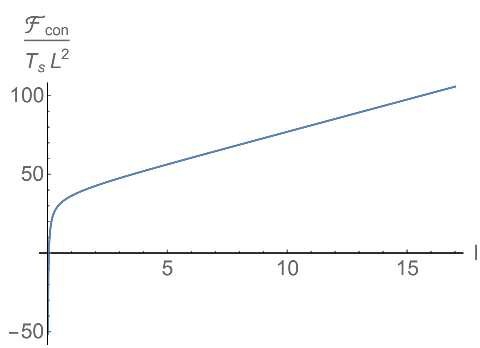

Fig. 5. $\mathcal{F}_{\text{con}}$ as a function of ℓ in the thermal-AdS background (in GeV).

On the other hand, the situation is completely different with the AdS black hole background. The results are presented in Fig. 6, where the free energy difference between the connected and disconnected string solutions is shown. It turns out that with black hole background, there appears an ℓ_{max}

above which connected solution does not exist and only the disconnected solution remains. A phase transition from a connected to a disconnected solution occurs as we increase the $q\bar{q}$ separation length ℓ. Importantly, for higher separations, it is the disconnected solution which has the lower free energy and this free energy is independent of ℓ. It implies that the string tension is zero and there is no linear law confinement for the boundary theory dual to the AdS black hole phase. Moreover, one can easily show that the Polyakov loop expectation value is now finite. It implies that the AdS black hole phase corresponds to a deconfined phase in the dual boundary theory.

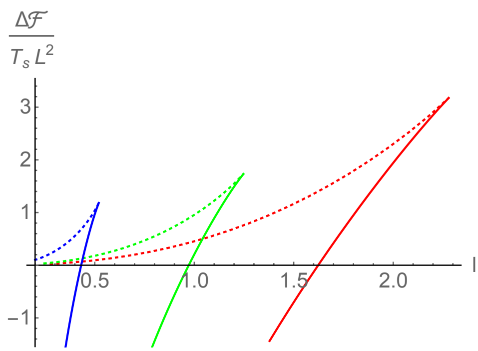

Fig. 6. (Colour on-line) $\Delta\mathcal{F} = \mathcal{F}_{\mathrm{con}} - \mathcal{F}_{\mathrm{discon}}$ as a function of ℓ in the AdS black hole background for various values of z_{h}. Here, $\mu = 0$ and red, green and blue (from right to left) curves correspond to $z_{\mathrm{h}} = 1.5$, 1.0 and 0.5 [GeV], respectively.

5. Entropy of quark–antiquark pair

We now move on to examine the entropy S of the $q\bar{q}$ pair in both phases. The results are shown in Figs. 7 and 8. We find that our holographic model qualitatively captures lattice QCD results for the $q\bar{q}$ entropy. In particular, we find a large entropy associated with $q\bar{q}$ pair near the deconfinement transition temperature. One can also notice a sharp peak in the entropy near the transition temperature, mimicking another important lattice result. Similar results in the deconfined phase were found in [7]. Moreover, the higher temperature asymptotics of the $q\bar{q}$ entropy in our holographic model is identical to those of [7]. In particular, for $T \gtrsim 2T_{\mathrm{c}}$, we find a tendency that TS increases with T as noticed by lattice QCD well. Our study further predicts analogous asymptotic behaviour of TS in the presence of a chemical potential too.

Another lattice prediction which our holographic model reproduces is the increase in the entropy of the $q\bar{q}$ pair as a function of distance between them, see Fig. 8. We see that for each temperature, S increases with ℓ. Moreover,

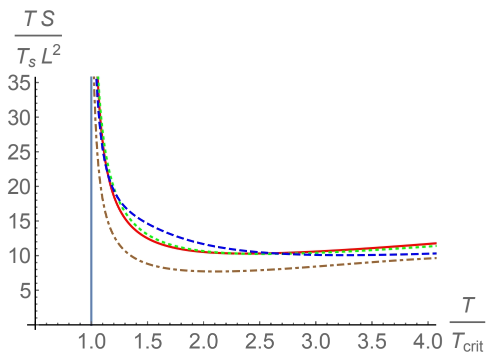

Fig. 7. (Colour on-line) Entropy of the $q\bar{q}$ pair as a function of temperature in the deconfined phase for various values of chemical potential μ. Here, solid/red, doted/green, dashed/blue and dot-dashed/brown curves correspond to $\mu = 0$, 0.2, 0.4 and 0.6 [GeV], respectively.

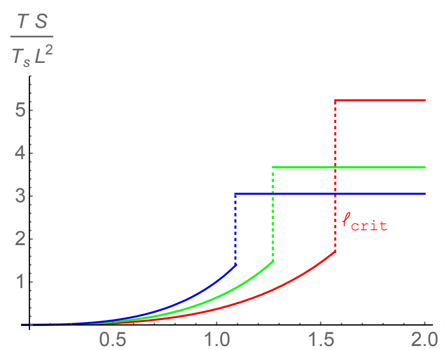

Fig. 8. (Colour on-line) Entropy of the $q\bar{q}$ pair as a function of distance in the deconfined phase for various temperatures. Here $\mu = 0$ and red, green and blue (from bottom to top) curves correspond to $T/T_{\text{crit}} = 1.1$, 1.2 and 1.3 [GeV], respectively.

for large ℓ, S saturates to a constant value and becomes independent of it[1]. This is due to the fact that the disconnected string configuration has lower free energy at large separations, while it is independent of ℓ. Therefore, the corresponding entropy is also independent of ℓ. We see that these results qualitatively match with the results predicted by lattice QCD (shown in Fig. 1). However, as opposed to the latter, the entropy here does not smoothly go to saturation. There is a discontinuity in the entropy at ℓ_{crit} (denoted by dotted lines in Fig. 8). This discontinuity in the entropy arises precisely due to a first order transition between the different string configurations at ℓ_{crit}.

[1] In order to make it more readable, the magnitude of S has been suppressed by a factor of 2 in $\ell > \ell_{\text{crit}}$ region of Fig. 8.

6. Conclusions

We considered an Einstein–Maxwell-dilaton gravity model to study the entropy of a $q\bar{q}$ pair using the gauge/gravity correspondence. We first expressed the gravity solution in terms of a scale function $A(z)$ and then considered a particular profile for it, which led to a thermal-AdS/black hole phase transition on the gravity side. We showed that this phase transition corresponds to the confinement/deconfinement phase transition in the dual boundary theory. We then studied the free energy and entropy of such a $q\bar{q}$ pair and showed that our holographic model qualitatively reproduces the known lattice QCD results. In particular, our holographic model correctly reproduced: *(i)* a growth in the entropy of a $q\bar{q}$ pair with interquark distance and *(ii)* a sharp rise in the entropy near the deconfinement transition. We, moreover, provided a holographic estimate for the $q\bar{q}$ entropy with chemical potential, which hopefully can be compared with lattice QCD in the near future.

One major drawback of the current model is that it predicts zero $q\bar{q}$ entropy in the confined phase. This is strictly due to the fact that the latter phase is dual to thermal-AdS, which is independent of temperature. This drawback can be rectified by computing $1/N$ corrections or by constructing a black hole solution for the confined phase as well. Here, our ongoing investigation suggests that by choosing a different form for $A(z)$, it is possible to construct a black hole solution on the gravity side whose dual boundary theory closely resembles a confined phase. Using such a black hole solution, one can study temperature-dependent properties of the confined phase as well. This work is in preparation and will be reported elsewhere.

REFERENCES

[1] O. Kaczmarek, F. Zantow, *Phys. Rev. D* **71**, 114510 (2005).

[2] K. Hashimoto, D.E. Kharzeev, *Phys. Rev. D* **90**, 125012 (2014).

[3] O. Kaczmarek, F. Zantow, *PoS* **LAT2005**, 192 (2006).

[4] J.M. Maldacena, *Int. J. Theor. Phys.* **38**, 1113 (1999) [*Adv. Theor. Math. Phys.* **2**, 231 (1998)].

[5] S.S. Gubser, I.R. Klebanov, A.M. Polyakov, *Phys. Lett. B* **428**, 105 (1998).

[6] E. Witten, *Adv. Theor. Math. Phys.* **2**, 253 (1998).

[7] I. Iatrakis, D.E. Kharzeev, *Phys. Rev. D* **93**, 086009 (2016).