Vol. 10 (2017) *Acta Physica Polonica B Proceedings Supplement* No 4

HADRONIC VACUUM POLARIZATION IN $e^+e^- \to \mu^+\mu^-$ PROCESS BELOW 3 GeV*

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(Received October 5, 2017)

The interference effect between leptonic radiative corrections and hadronic polarization functions is calculated via optical theorem for μ-pair productions. It is achieved by using the data for dominant channels of the production cross section $\sigma_{\rm h}(e^+e^- \to \text{hadrons})$. The result is compared with the KLOE experiment for $\mu^-\mu^+$ production at ϕ-meson energy for which we take into account specific experimental conditions. Moreover, running fine structure coupling is compared with the KLOE2 experiment for radiative return $\mu^-\mu^+$ production at ω/ρ-meson energy.

DOI:10.5506/APhysPolBSupp.10.1159

1. Introduction

Comparisons between theory and experiment have been used for decades to test Standard Theory. For an accurate measurement, the studies require consistent account of leptonic as well as hadronic virtual corrections. The hadronic contribution to photon vacuum polarization function plays a particularly important role, since it is the main source of uncertainties in theoretical calculation of muon anomalous magnetic moment a_μ. The last precise measurement of a_μ, together with the last decades data for electrohadron production, leads to an evidence of tension between the Standard Theory and experiments [1, 2]. Similar confrontation of the theoretical technique with the experimental accuracy is offered by a long-time known [3, 4] interference effect between leptonic and hadronic vacuum polarization functions in close vicinity of narrow resonances: ω and ϕ as well as heavier quarkonia the $J/\Psi, \Psi$ and Υs. There, the hadronic vacuum polarization is enhanced several orders of magnitude above remaining hadronic background. In practice, the effect is explored in the so-called B-factories like BaBar, Belle or BESS, or more earlier in Frascati with detectors and accelerator tuned in ϕ-meson mode. Most recent precise measurements of muon production $e^+e^- \to \mu^+\mu^-$

* Presented at "Excited QCD 2017", Sintra, Lisbon, Portugal, May 7–13, 2017.

by SND, CMD-2,3 and the KLOE(2) detectors allow to test the standard theoretical assumptions in the quite similar manner as for a_μ. With incomes of many new precise data for hadroproduction cross section $\sigma_{\rm h}$, we reanalyzed the calculation for [5] within the new data used. Having extracted hadron polarization function, we provide independent comparison of theory and recent KLOE2 [6] measurement of the fine structure coupling constant.

2. $\sigma_{\mu\mu}$ for KLOE 2004

In the first part of the paper, I present the comparison between calculated cross section $\sigma(e^+e^- \to \mu^+\mu^-)$ and the high precision measurement obtained by the KLOE detector [5]. The method of extraction of hadronic polarization function $\Pi_{\rm h}$ will be briefly mentioned. As we shall see, the recent knowledge of $\sigma_{\rm h}$ allows theoretical determination of $\sigma_{\mu\mu}$ with theoretical error comparable to KLOE $\sigma_{\rm stat}$ for $\sigma_{\mu\mu}$. Thus, the error of theory due to the uncertainty in $\sigma_{\rm h}$ becomes 10 times smaller then the experimental error of 2004 KLOE measurement. The result presented here represents a precise prediction for a future more accurate experiments.

The details of theory of $e^+e^- \to \mu^+\mu^-$ for KLOE detector can be found in [7]. The integral cross-section formula is proportional to the square of the fine structure constant $\alpha(s)$, which reads

$$\alpha(s) = \frac{\alpha}{1-\Pi(s)}\,, \tag{1}$$

with $\alpha = \alpha(0) = 1/137.0359991390$ and where the polarization function $\Pi(s) = \Pi_{\rm l}(s) + \Pi_{\rm h}(s)$ is completed from the leptonic 'l' and the hadronic 'h' part.

Purely QED contributions, represented by $\Pi_{\rm l}$, are well-known and listed in [7], while $\Pi_{\rm h}$ is not directly calculable from the equation of motions, but through the knowledge of many other experiments $e^+e^- \to$ hadrons*, where the star means that the final-state photons should be included as well, while as opposed to it, the initial ones should be subtracted. In fact, the evaluation of $\Pi_{\rm h}$ rely on numerical evaluation of the following singular integral [8,9]:

$$\Pi_{\rm h}(s) = \frac{s}{4\pi^2\alpha}\int\limits_{m_\pi^2}^{\infty} {\rm d}\omega \frac{\sigma_{\rm h}(\omega)\left[\frac{\alpha}{\alpha(\omega)}\right]^2}{\omega - s + i\epsilon}\,. \tag{2}$$

Recall that the use of experimental data straightforwardly would lead to a large numerical noise and lost of required accuracy. I found that the usual way (for the method of clusters, see [10,11]) produces cusps and spikes. They cause the error stemming from such an interpolation, averaging and integration procedure is hard to estimate due to the presence of principal value integration (recall for clarity, such a numerical problem is avoided in the case of a_μ evaluation as the integral is regular one).

Therefore, instead of direct use of experimental data, the fit (including errors) is made for each combinations of measurements. I used the fine selection method of the data, which is based on the following simple criterion:

$$\sigma^2_{\rm syst} + \sigma^2_{\rm stat} < \epsilon^2_{\rm h}\,, \tag{3}$$

where on the left-hand side, there is a sum of statistical and systematical error of the data points and on the right-hand side, a suited choice of error function evaluated at data points is employed. The data satisfying inequality (3) are used to establish the fits, wherein the experimental error are replaced by the inflated error function (IEF) $\epsilon_{\rm h}$, while all data points not satisfying rule (3), are not used for making a fits.

Let stress here that with the choice of a given IEF $\epsilon_{\rm h}$, condition (3) does not automatically ensure the existence of a good global fit satisfying $\chi^2 < 1$. It actually happens to/in cases when one combines various experiments with the error underestimated by the experimental group (assume here they are the source of data for comparison (3)), which usually happens in the case where the systematical error is not completely known (not named explicitly, the older threshold data extracted by ISR method are a typical example). Impossibility of minimizing χ^2 such that $\chi^2 < 1$ (note that $\chi^2 \simeq 1$ is valid only for non-inflated error) indicates the badness, or rather, the incompatibility of the data. In this case, we are either forced to further inflate the IEF (by changing the prescription for $\epsilon_{\rm h}$) or we discard the problematic data set from the fine selection.

There is an obvious price to pay due to a certain lost of information, which is however under a good control (through the error). Moreover, one must perform aforementioned fits explicitly, which is not always a cheap and painless procedure. To this point, well-established interpolating fits to the existing experimental data for $\sigma_{\rm h}$ at each exclusive channel that have been found during several last years. Using these, the large number of generated quasi-data points makes systematic error from the principal value integration in (2) immaterial and the error of theory for $\sigma_{\mu\mu}$ is almost solely due to the propagation of "inflated" error $\epsilon_{\rm h}$. More explicitly, the systematic error due to the integration procedure has been minimized with the relative precision smaller then 0.005/40 for the muon production cross section (the absolute value is approximately hundred times smaller then the experimental error [5]).

The functional form of the error function $\epsilon_{\rm h}$ can be taken arbitrary, however the choice, which does not reduce n.d.f. too much and is simultaneously simple enough, is preferred. For this purpose, we have used the following IEF

$$\epsilon_{\rm h}(s) = c_l\sqrt{\sigma_{\rm h}(s)}\,, \qquad \epsilon_{\rm h}(s) = c_s\sigma_{\rm h}(s)\,, \tag{4}$$

where the left equation in (4) is used for $\sigma_{\rm h}(s)$ larger the 1 nb, while the right one is used for small $\sigma_{\rm h}(s) < 1$ nb. It is enough to take constant values for parameters $c_{l,s}$, noting that values $c_l = 0.8$ nb$^{1/2}$, $c_s = 1/3$ in Eq. (4) were most recently used in almost all of hadronic channels.

Recent measurements are taken completely into account and the complete list of selected experimental data and their fits will be presented in an updated version of [7]. Without specifications of channels, choice (4) fully accepts the last experiments, *e.g.* CMD-3, BESSIII, KLOE, also most CMD-2 measurements as well as large-s BaBar data are fully taken, it cuts partially some data from SND, CMD-2, it also cuts on resonance data from BaBar, while in practice, we can freely discard the data from and old experiments completely (CMD, DM, NA7, OLYA, TOF).

In order to evaluate $\mathit{\Pi}_{\rm h}$, the main exclusive channels: $\pi\pi$, K^+K^-, $K_{\rm L}K_{\rm S}$ and $\pi\pi\pi$ as well as $\eta\gamma$ and $\pi\gamma$ have been considered. Final states with higher multiplicity were neglected, noting that their total contribution seems to be smaller then the one from $\eta\gamma$ channel for ϕ-meson region or from $\pi\gamma$ channel for ω/ρ-meson region. The effect of well-established vector charmonia and bottomonia has been included through $\sigma_{\rm h}$ by using their BW forms with PDG experimentally determined values. The result is shown in Fig. 1, where the old analysis is compared with the new one. The new one includes in

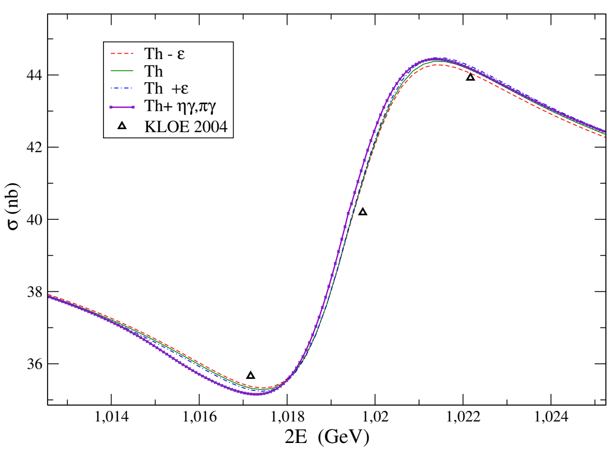

Fig. 1. Muon pair cross section. The comparison between theory and experiment as described in text.

addition the data from KLOE, BESSIII and BaBar for KK channels, which have no practical effect on the final curve. However, more importantly, it newly includes $\eta\gamma$ and $\pi\gamma$ channels of σ_{h}, which have been neglected in the previous analyses of ϕ-meson study. The bands between $\pm\epsilon$ reflects the propagation of inflated error ϵ_{h} into the muon pair cross section and were obtained with $c_l = 1$ nb$^{1/2}$, $c_s = 1$ in previous analyses. The error band is even more tight and not shown for the new analyses. The KLOE data points are represented by triangles, noting that the statistical deviations roughly correspond with the size of the triangle.

To conclude the first part, the observed ϕ-meson interference effect on $\mu\mu$ spectrum is roughly reproduced by the Standard Theory dispersion relation. Should be reminded that the detector measurement provided three points with very small statistical error ($\sigma_{\mathrm{stat}} = 0.1$ nb), unhappily the total error was governed by systematical error due to the luminosity and detection uncertainties ($\delta_{\mathrm{syst}} = 1.2\%$). Small $1.7\sigma_{\mathrm{tot}}$ difference from SM prediction does not represent large tension between the theory and experiment. Lowering the systematical error would be not only an experimental challenge for precise experimental facilities like KLOE2, CMD3, but also for the Standard Theory.

3. α_{QED} at KLOE2

As a bonus, we also get similar interference effect in the ω energy region. This, somehow smaller, 3 nb sized zig-zag structure has been only very recently measured by radiative return method by the KLOE2 Collaboration [6]. In this case, the experiment is in a complete agreement with the theory, noting that the relative errors are much larger in the ρ/ω region.

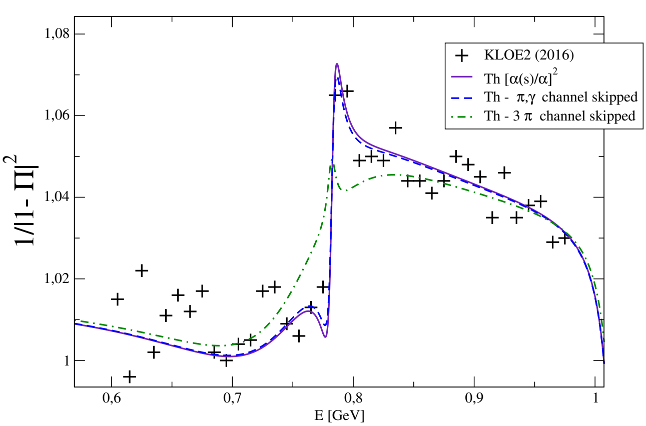

Fig. 2. Square of fine structure constant. To see the portion of 3π and π,γ contributions, we also show the results with these channels subtracted from σ_{h}.

The comparison with the KLOE2 experiment and the way the ρ/ω peak is pronounced in the QED running coupling is shown in Fig. 2. More precise comparison is a remaining challenge for both the Standard Theory and new generation experiments.

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