

A COLLISIONAL MODEL FOR SCALAR MESONS BELOW 1 GeV*

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(Received October 5, 2017)

A collisional model for hadron resonances appearing in hadron collisions is proposed. The given approach leads to a simple explanation of the scalar sector below 1 GeV with correct predictions for masses and dominant decay modes.

DOI:10.5506/APhysPolBSupp.10.1165

The scalar sector below and near 1 GeV is perhaps the most difficult for traditional approaches in the hadron spectroscopy. The usual quark model faces serious problems in explaining the existence and properties of light scalar mesons. Despite the recent progress in description of these states by dispersive methods [1, 2], the scalar sector still remains puzzling. It is highly desirable to have a simple and intuitively clear physical picture shedding light on the existence, observable masses, and main decay modes of light scalars. The purpose of this work is to propose a variant for such a picture.

The formation of hadron resonances is a complicated and largely mysterious quantum phenomenon. The resonance cannot be “visualized” as a kind of circular motion of quarks in analogy with a rough “visualization” of electron motion in atoms. The reason is that the hadron resonances do not represent bound states — a typical resonance lifetime is so small that the quarks usually have no time to make even one “circle” inside a space region of typical linear size around 1 fm. We will make an attempt to construct an alternative “visualization” based on a certain collisional picture describing formation of some resonances.

First, we recall the famous Gell-Mann–Oakes–Renner relation for the pion mass [3]

$$m_\pi^2 = -\frac{\langle \bar{q}q \rangle}{f_\pi^2} (m_u + m_d) = \Lambda^2 2m_q, \quad (1)$$

* Presented at “Excited QCD 2017”, Sintra, Lisbon, Portugal, May 7–13, 2017.

where we set $m_u = m_d \doteq m_q$ and $\Lambda \doteq -\frac{\langle \bar{q}q \rangle}{f_\pi^2}$. We will use the standard values for the quark condensate and masses of current quarks at the scale of the pion mass from the QCD sum rules [4] and chiral perturbation theory [5], $\langle \bar{q}q \rangle = -(250 \text{ MeV})^3$, $m_u + m_d = 11 \text{ MeV}$. Together with $f_\pi = 92.4 \text{ MeV}$ (the value of the weak pion decay constant in the normalization used in Eq. (1)), relation (1) yields $m_\pi = 140 \text{ MeV}$ and $\Lambda = 1830 \text{ MeV}$. This relation can be generalized to the strange pseudoscalar mesons, with the constant Λ being universal [5].

Consider a low-energy $\pi\pi$ scattering. Let us assume the existence of situation when a pion collides as a whole with one of quarks of another pion. This may happen when the first pion is faster, hence, has smaller de Broglie wavelength, *i.e.* when one pion wave packet penetrates another one¹. The collision lasts a very short time Δt which determines the lifetime of the formed coherent state. During the time Δt , the second quark (quark-spectator) “feels” the first one as a particle with unchanged color charge and spin (because pion is colorless and spinless) but with different proper mass: $m_q \rightarrow m_q + m_\pi$. Formally, doing this replacement for one of quarks in Eq. (1), we obtain the estimate on the mass of this coherent state (let us denote it σ beforehand)

$$m_\sigma^2 = \Lambda m_\pi + m_\pi^2 \quad (2)$$

that yields $m_\sigma \approx 525 \text{ MeV}$. This state must have the quantum numbers of scalar meson (a $\pi\pi$ state in S -wave), decay into two pions, and should be an extremely short-living particle. The formation of such a coherent state is favored by the Coulomb attraction between π^+ and π^- which leads to the dominance of isosinglet channel. All these observations strongly suggest that we must interpret this resonance as the $f_0(500)$ meson [1], widely known as σ meson.

We may propose a quantum-mechanical interpretation for relation (2). Consider a stationary state of some quantum system with energy E which is described by the Schrödinger equation, $H|\psi\rangle = E|\psi\rangle$. Consider now a sudden perturbation of this system, *e.g.* by a very fast particle. A sudden perturbation in Quantum Mechanics is a perturbation lasting so short time Δt that the wave function ψ is not changed during Δt (such a change always requires a finite time). The perturbed system is described by the equation $H'|\psi\rangle = E'|\psi\rangle$. But ψ is not an eigenfunction of the new Hamiltonian H' and the perturbed system becomes unstable. An important point for us is that the form of ψ determines the functional dependence of energy E from the parameters a_i of the Hamiltonian H , $E = E(a_i)$. Since ψ is unchanged during the time Δt , this dependence remains the same after the perturbation. It means that if H' is determined by a set of perturbed

¹ A picture of this process depends on a reference frame.

parameters a'_i , then $E' = E(a'_i)$. Returning to the $\pi\pi$ scattering, one of pions can be considered as a stationary system in which m_q plays the role of a parameter determining its energy m_π . A collision causes a sudden perturbation that changes the parameter, $m_q \rightarrow m_q + m_\pi$, for a short time Δt . This change must be substituted to Eq. (1) (for one of quarks) to obtain the energy (2) of unstable state σ .

It should be remarked that the product Λm_π in Eq. (2) is not renorminvariant and Λ should be taken at the scale of m_π . If we replace pion in Eq. (2) by some other particle, we should take Λ at the scale of that particle.

Relation (2) suggests that the chiral limit $m_\pi \rightarrow 0$ entails $m_\sigma \rightarrow 0$. The available phase space and decay amplitude, however, are not changed drastically, *i.e.*, in the chiral limit, σ remains a very broad resonance. The obtained scalar state is thus different from the sigma meson of linear sigma model or Nambu–Jona-Lasinio model where it is not massless in the chiral limit. One should take into account that the large- N_c limit is inherent for such models as the first approximation, while for analysis of very broad resonances this limit is hardly appropriate [2].

The recent lattice simulation of Ref. [6] reported an effect of evolving σ meson into a stable bound state lying below the $\pi\pi$ threshold as m_π is increased. This observation follows directly from the mass relation (2): Imposing $m_\sigma \geq 2m_\pi$, we obtain $m_\pi \leq \Lambda/3$. This restriction is nontrivial as Λ depends on the mass scale. If we normalize Λ to the numerical results of Ref. [6], $m_\sigma = 758$ MeV when $m_\pi = 391$ MeV, we get $\Lambda = 1078$ MeV that gives the estimate $m_\pi \leq 359$ MeV. This restriction agrees with the lattice results of Ref. [6]: σ represents a bound state at $m_\pi = 391$ MeV and a broad resonance at $m_\pi = 236$ MeV.

A calculation of total decay width in our approach is an open problem. The resonance width is a measure of the mass uncertainty. Within our framework, this uncertainty can appear from the uncertainty of Λ , *i.e.* of the quark condensate $\langle \bar{q}q \rangle$. Perhaps a definite value of $\langle \bar{q}q \rangle$ exists inside the quark bound states only. In the space region of hadron collisions, however, a “stabilization” of $\langle \bar{q}q \rangle$ around this value may require a finite time and this causes a large mass uncertainty typical for resonances. For calculation of strong decay width, one then needs a microscopic dynamical picture of QCD vacuum which is unknown.

Let us define now a general notion of collisional resonance. Consider a scattering of hadrons A and B where the latter has a smaller de Broglie wavelength. Then, by assumption, B can excite one of quarks inside A in the collisional way. The formed coherent state will be denoted as A_B (means “ B inside A ”). The σ meson represents the $\pi\pi$ collisional resonance in this notation. Adding now the K and η mesons, we can construct other scalar collisional states which can be formed, *e.g.*, in the $K\pi$ scattering.

Consider the state π_K . Making the replacement $m_q \rightarrow m_q + m_K$ for one of quarks in Eq. (1), we obtain its mass $m_{\pi_K} \approx 970$ MeV. As in the case of σ , the expected isospin of π_K is zero. Its natural decay mode would be $\pi_K \rightarrow K\pi$ but such a decay is forbidden by the isospin conservation if π_K represents a genuine resonance. π_K should be then relatively narrow and, as its mass lies slightly below the KK threshold, its dominant decay mode is expected to be $\pi_K \rightarrow \pi\pi$. The scalar resonance $f_0(980)$ meets all these expectations [1].

Let us include now the η meson. We predict the following characteristics of π_η resonance: $m_{\pi_\eta} \approx 1010$ MeV, $I_{\pi_\eta} = 1$ (it inherits I_π), the dominant decay mode $\pi_\eta \rightarrow \eta\pi$ and it should be broader than π_K because this mode is not forbidden. The scalar resonance $a_0(980)$ satisfies these predictions [1].

Consider a hypothetical K_π collisional resonance. The formation of the coherent state K_π is much harder than π_K because π has larger de Broglie wavelength, *i.e.* the pion wave packet is larger than the kaon one. This is also true for the measured mean sizes, $\langle r_\pi \rangle > \langle r_K \rangle$ [1]. But one might assume the existence of non-zero probability for K_π due to some quantum effects. The Coulomb attraction would favor then the $K_{\pi^-}^+$ or $K_{\pi^+}^-$ channel, the $K_{\pi^0}^\pm$ and $K_{\pi^\pm}^0$ are much less plausible. In any case, the mass of K_π would be given by

$$m_{K_\pi}^2 = \Lambda(m_\pi + m_q + m_s) = \Lambda m_\pi + m_K^2 \quad (3)$$

resulting in $m_{K_\pi} \approx 710$ MeV. Here, $m_s \approx 130$ MeV is the mass of the strange quark below 1 GeV. It is tempting to interpret K_π as the unconfirmed scalar resonance $K_0^*(800)$ called also k meson [1]. The Particle Data reports the following mass for this elusive resonance: 682 ± 29 MeV [1]. Comparison of Eq. (3) with Eq. (2) shows that, as expected, K_π would be a partner of σ in which one of u or d quarks is substituted by the s quark. The observed isospin $I_k = \frac{1}{2}$, however, contradicts the favorable isospin zero predicted by the assumed mechanism of K_π formation. We see thus that the existence of k meson, if confirmed, is not in conflict with our general principle for collisional resonances and one should look for a correct formation mechanism.

We propose the following explanation. Let us take a closer look at the production of k meson. The main source of information on k are decays of J/ψ meson into kaons and pions. The decays of vector charmonia are always accompanied by an abundant photon background. In all this “mixture”, one can have situations when photons produce π^0 meson inside a kaon. The formed coherent state would then inherit the kaon isospin, *i.e.* would give rise to the scalar partners of the pseudoscalar K^+ , K^- , K^0 , and $\bar{K}^0$ mesons, with the mass being described by relation (3).

Our approach predicts other collisional scalar resonances as well: K_η with mass about 1120 MeV, η_η with mass about 1150 MeV, almost unfeasible η_π having mass near 760 MeV, and a formation mechanism similar to that of k meson, and resonances with η' like the $\pi_{\eta'}$ state of mass about 1330 MeV. It is likely very hard to detect these resonances in the $\pi\pi$, $K\pi$ and KK scattering (in experiments of direct $\eta\pi$ and ηK scattering, this would be easier) but they may contribute to the strong background emerging in these reactions.

Within the framework of our collisional interpretation, the scalar resonances below and slightly above 1 GeV represent thus two-meson states. In terms of the quark degrees of freedom, they are tetraquarks, as is also suggested by many other models and observations [2].

It should be noted that the collisional interpretation described above can be considered not only for the light scalar mesons but also for some other hadrons. For instance, imagine that a ρ meson was formed “inside” a pion and excited one of pion quarks in the collisional way. The coherent state π_ρ is then formed. According to our prescription (*i.e.*, the replacement $m_q \rightarrow m_q + m_\rho$ for one of quarks in Eq. (1)), the mass of this state is given by the relation

$$m_{\pi_\rho}^2 = \Lambda m_\rho + m_\pi^2, \quad (4)$$

where Λ should be taken at the scale m_ρ . For making estimates in the first approximation, we will consider Λ as a universal constant and set as before $\Lambda = 1830$ MeV. We obtain $m_\rho \approx 1190$ MeV. As in the σ case, the formation of the coherent state π_ρ should be favored by the Coulomb attraction, *i.e.* the favored channel is $\pi_{\rho^-}^+$ or $\pi_{\rho^+}^-$ that entails zero isospin. The PC-parities are $(PC) = (-+)(--)= (+-)$. The obtained coherent state seems to be nothing but the resonance $h_1(1170)$ [1]. A natural consequence of π_ρ structure of h_1 is the absolute dominance of the decay mode $h_1 \rightarrow \rho\pi$ [1]. The isotriplet partner of $h_1(1170)$ — the $b_1(1230)$ meson — represents the collisional resonance π_ω that determines its isospin 1 (it inherits the pion isospin) and dominant decay $b_1 \rightarrow \omega\pi$ [1]. b_1 is expected to be heavier than h_1 because $m_\omega > m_\rho$ and is narrower than h_1 because $\Gamma_\omega < \Gamma_\rho$. These expectations agree with the experimental data [1], at least qualitatively.

In summary, we have proposed a novel interpretation for the scalar sector below 1 GeV which allows to predict the masses and dominant decay modes of the light scalar resonances. It is based on a simple collisional interpretation of the resonance formation. Our approach can be extended and applied to some other hadrons. For instance, the $h_1(1170)$ meson can be understood as the π_ρ collisional resonance. It seems that many highly excited N and Δ baryons can be described as collisional excitations of the kind M_B , where M is a meson (typically the π meson and resonances which

are abundantly produced in reactions with π , like ρ (ω) and f_J mesons) and B is a baryon (typically the proton and $\Delta(1232)$). A further development of our observations could be an interesting subject for a future work.

The present work was supported by the Saint Petersburg State University travel grant 11.41.421.2017 and by the RFBR grant 16-02-00348-a.

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