

# HEAVY AND HEAVY–LIGHT MESONS AND THE LORENTZ STRUCTURE OF THE QUARK–ANTIQUARK INTERACTION\*

S. LEITÃO<sup>a</sup>, A. STADLER<sup>b,a</sup>, M.T. PEÑA<sup>c,a</sup>, E.P. BIERNAT<sup>a</sup>

<sup>a</sup>Centro de Física Teórica de Partículas (CFTP), Instituto Superior Técnico,  
Universidade de Lisboa, 1049-001 Lisboa, Portugal

<sup>b</sup>Departamento de Física, Universidade de Évora, 7000-671 Évora, Portugal

<sup>c</sup>Departamento de Física, Instituto Superior Técnico (IST)  
Universidade de Lisboa, 1049-001 Lisboa, Portugal

(Received September 4, 2017)

We solve a Minkowski-space integral equation, derived in the Covariant Spectator Theory, for quark–antiquark bound states describing heavy and heavy–light mesons. The equation’s kernel contains a one-gluon exchange interaction and a covariant generalization of a linear confining potential with a mixed scalar, pseudoscalar, and vector Lorentz structure, characterized by a continuous mixing parameter. We investigate to what extent the Lorentz structure of the confining kernel can be determined by fitting the mixing parameter to the meson spectrum.

DOI:10.5506/APhysPolBSupp.10.939

## 1. Introduction

Already several decades ago it has become clear that many mesons can be well described as bound states of a quark and an antiquark, interacting through a linear and a one-gluon exchange (OGE) potential [1, 2]. However, modern unified descriptions of all mesons have to go beyond these early works in several aspects, among which we emphasize a proper treatment of chiral symmetry and relativity. Among the frameworks that meet these requirements figure prominently Lattice QCD (*e.g.*, [3–6]) and the Dyson–Schwinger/Bethe–Salpeter (DS/BS) approach [7–12].

We use the Covariant Spectator Theory (CST), which is related to the BS equation, but goes beyond ladder approximation by effectively taking contributions from crossed-ladder diagrams into account [13, 14]. Chiral

---

\* Presented by A. Stadler at “Excited QCD 2017”, Sintra, Lisbon, Portugal, May 7–13, 2017.

symmetry can also be implemented exactly in CST [15, 16], but in this work, we calculate heavy and heavy–light mesons, where this aspect is of minor importance. Instead, we focus on the role of relativity and, in particular, on the Lorentz structure of the confining interaction, which is at present still an open problem.

### 2. The CST bound-state equation

The one-channel spectator equation (1CSE) for the bound-state vertex function  $\Gamma$  is shown graphically in Fig. 1. It has the same structure as the BS equation, except that particle 1 (by convention the heavier particle, unless they have equal mass) is on its positive-energy mass shell. It is an approximation to the full, four-channel equation, which should work very well in systems with at least one heavy particle [17].

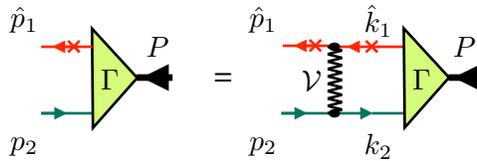


Fig. 1. The one-channel spectator equation (1CSE) for the bound-state vertex function  $\Gamma$  of a quark (particle 1) and an antiquark (particle 2), interacting through a kernel  $\mathcal{V}$ . An “ $\times$ ” on a line means that the particle is on its positive-energy mass shell, and the corresponding four-momentum carries a “ $\hat{\phantom{x}}$ ”.

We work in the meson’s rest frame, where the total momentum is  $P = \hat{p}_1 - p_2 = (\mu, \mathbf{0})$ ,  $\hat{p}_1 = (E_{1p}, \mathbf{p})$  is the on-shell momentum of particle 1, with  $E_{1p} = \sqrt{m_1^2 + \mathbf{p}^2}$ , and the external relative momentum is  $p = (\hat{p}_1 + p_2)/2 = (E_{1p} - \mu/2, \mathbf{p})$  (analogous expressions hold for the internal relative momentum  $k$ ). The 1CSE can be written as [17]

$$\Gamma(\hat{p}_1, p_2) = - \int \frac{d^3k}{(2\pi)^3} \frac{m_1}{E_{1k}} \mathcal{V}(p, \hat{k}_1 - P/2) \frac{m_1 + \hat{k}_1}{2m_1} \Gamma(\hat{k}_1, k_2) \frac{m_2 + \not{k}_2}{m_2^2 - k_2^2 - i\epsilon}, \tag{1}$$

where  $k_2 = \hat{k}_1 - P$ , and we use fixed quark masses  $m_i$  (which will be replaced by dynamical mass functions in future work). Our interaction kernel  $\mathcal{V}$  has the form

$$\mathcal{V}(p, k) = [(1 - y)(\mathbf{1}_1 \otimes \mathbf{1}_2 + \gamma_1^5 \otimes \gamma_2^5) - y\gamma_1^\mu \otimes \gamma_{\mu 2}] V_L(p, k) - \gamma_1^\mu \otimes \gamma_{\mu 2} [V_{\text{OGE}}(p, k) + V_C(p, k)], \tag{2}$$

where  $V_L(p, k)$  is a covariant generalization of a linear confining potential,  $V_{\text{OGE}}(p, k)$  is the short-range one-gluon-exchange interaction in the Feynman

gauge, and  $V_C(p, k)$  a covariant form of a constant potential. We use a Pauli-Villars regularization with a cut-off parameter  $\Lambda = 2m_1$  for all mesons. The detailed form of the kernel is given in [17].

The OGE and constant kernels are Lorentz-vector interactions. The Lorentz structure of the linear confining kernel in (2) is a mixture of equally-weighted scalar and pseudoscalar coupling (S+P) on the one hand, which satisfies the axial-vector Ward-Takahashi identity [18], and vector coupling (V) on the other hand. By adjusting the mixing parameter  $y$ , we can continuously vary the relative weight of these structures, with  $y = 0$  yielding a pure S+P coupling, and  $y = 1$  a pure V coupling. For any value of  $y$ , the non-relativistic limit of this kernel is always, when transformed into coordinate space, the Cornell-type potential  $V(r) = \sigma r - \alpha_s/r - C$ .

For numerical calculations, Eq. (1) is converted into an eigenvalue equation for the so-called “relativistic wave functions”, defined as spinor matrix elements of the vertex functions multiplied by the quark propagators. In the nonrelativistic limit, they turn then into Schrödinger wave functions. By solving this eigenvalue problem, we obtain, very conveniently, the masses and wave functions of a whole tower of excited states.

### 3. Numerical results and discussion

Working with the 1CSE, in [19], we performed least-square fits of the model parameters  $\sigma$ ,  $\alpha_s$ , and  $C$ , to the masses of selected data sets containing heavy and heavy-light mesons with  $J^P = 0^\pm$  and  $1^\pm$ , while choosing fixed values for the quark masses and keeping  $y = 0$ . In this work, we allow  $y$  and the quark masses to vary, which represents a considerable increase in required computing time. The data set S1 contains 9 meson states with  $0^-$ , the set S2 (S2') contains 25 (24) states with  $0^\pm$  and  $1^-$ , and the set S3 contains 39 states with  $0^\pm$  and  $1^\pm$  (the states are listed in [17]).

Table I shows the parameters of the various models determined in our fits, which include the ones found in [19], M0S1 and M0S2, for comparison. The results of four of our models are compared to the data in Fig. 2. In fact, all models of Table I reproduce the data very well, with r.m.s. differences to the largest data set S3 ranging from 30 to 40 MeV. This is a remarkable agreement, considering that we are performing *global* fits, using the same model parameters for the whole range of mesons, with masses from below 2 GeV to over 10 GeV.

Fitting the quark masses does not lead to a significant improvement over the results of [19]. The light and charm quark masses can be varied by a few hundred MeV (the bottom quark mass to a somewhat lesser degree) without noticeably deteriorating the quality of the fit.

TABLE I

Kernel parameters of the different models considered in this work. Quark masses and  $C$  are in GeV,  $\sigma$  in  $\text{GeV}^2$ ,  $\alpha_s$  and  $y$  are dimensionless. The values in boldface were held fixed during the fits of the respective models.

| Model             | $\sigma$ | $\alpha_s$ | $C$    | $y$           | $m_b$        | $m_c$        | $m_s$         | $m_q$         |
|-------------------|----------|------------|--------|---------------|--------------|--------------|---------------|---------------|
| M0 <sub>S1</sub>  | 0.2493   | 0.3643     | 0.3491 | <b>0.0000</b> | <b>4.892</b> | <b>1.600</b> | <b>0.4478</b> | <b>0.3455</b> |
| M1 <sub>S1</sub>  | 0.2235   | 0.3941     | 0.0591 | 0.0000        | 4.768        | 1.398        | 0.2547        | 0.1230        |
| M0 <sub>S2</sub>  | 0.2247   | 0.3614     | 0.3377 | <b>0.0000</b> | <b>4.892</b> | <b>1.600</b> | <b>0.4478</b> | <b>0.3455</b> |
| M1 <sub>S2</sub>  | 0.1893   | 0.4126     | 0.1085 | 0.2537        | 4.825        | 1.470        | 0.2349        | 0.1000        |
| M1 <sub>S2'</sub> | 0.2017   | 0.4013     | 0.1311 | 0.2677        | 4.822        | 1.464        | 0.2365        | 0.1000        |
| M1 <sub>S3</sub>  | 0.2022   | 0.4129     | 0.2145 | 0.2002        | 4.875        | 1.553        | 0.3679        | 0.2493        |
| M0 <sub>S3</sub>  | 0.2058   | 0.4172     | 0.2821 | <b>0.0000</b> | 4.917        | 1.624        | 0.4616        | 0.3514        |

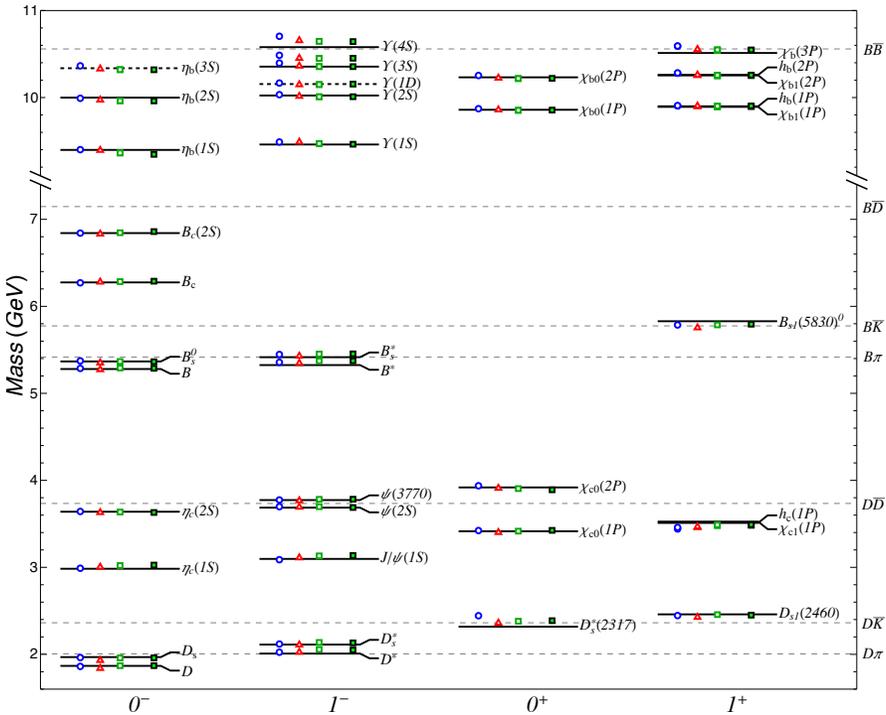


Fig. 2. Masses of heavy–light and heavy mesons with  $J^P = 0^\pm$  and  $1^\pm$ . The symbols represent the 1CSE results calculated with the models M1<sub>S1</sub> (circle), M1<sub>S2'</sub> (triangle), M1<sub>S3</sub> (open square), and M0<sub>S3</sub> (filled square) of Table I. Solid horizontal lines are the measured meson masses [20]. Dashed horizontal lines across the figure indicate open flavor thresholds.

The Lorentz mixing parameter still prefers  $y = 0$  when we fit to pseudoscalar states only (set S1), but a minimum between  $y = 0.2$  and  $0.3$  is found when larger data sets are used. To see if this contribution of V coupling is significant, we perform a series of calculations where the value of  $y$  is fixed at different values and all other parameters are refitted. Figure 3 shows the r.m.s. difference to various data sets as functions of  $y$ . Clearly, the minima for data set S2 and S3 are so shallow that values of  $y$  between 0 and roughly 0.25 (for S3) are equally compatible with the meson spectrum.

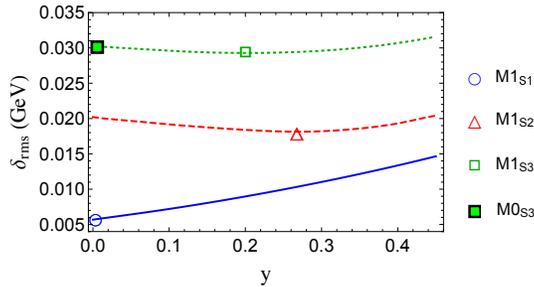


Fig. 3. R.m.s. difference between calculated and experimental meson masses as a function of the mixing parameter  $y$ , using data set S1 (solid line), S2' (dashed line), and S3 (dotted line). The symbols indicate the values for the models shown in Fig. 2.

We, therefore, cannot make a definite conclusion about whether the confining interaction is of pure S+P nature, or if it contains also V coupling, by looking at the meson masses only. However, other observables may be more sensitive to  $y$ . As an example, Fig. 4 shows sizeable changes in the contributions of spin singlet and triplet  $P$  waves to the norm integral of the  $c\bar{c}$  ground-state wave function when  $y$  is varied in the series of fits de-

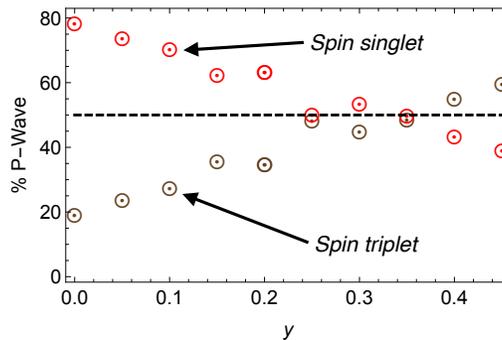


Fig. 4. Probabilities of spin triplet and singlet  $P$  waves in the charmonium ground-state as a function of the mixing parameter  $y$ .

scribed above. Although wave functions are not observables themselves, there are observables that depend strongly on the detailed structure of the wave functions, such as decay constants. By studying the dependence of decay constants on  $y$ , we should be able to obtain more precise constraints on the Lorentz structure of the confining interaction. This work is planned for the near future.

We thank Fundação para a Ciência e a Tecnologia (FCT) for support under contracts SFRH/BD/92637/2013, SFRH/BPD/100578/2014, and UID/FIS/0777/2013.

## REFERENCES

- [1] E. Eichten *et al.*, *Phys. Rev. D* **17**, 3090 (1978) [*Erratum ibid.* **21**, 313 (1980)].
- [2] S. Godfrey, N. Isgur, *Phys. Rev. D* **32**, 189 (1985).
- [3] C. McNeile, C. Michael [UKQCD Collaboration], *Phys. Rev. D* **74**, 014508 (2006).
- [4] T. Burch *et al.* [Bern–Graz–Regensburg Collaboration], *Phys. Rev. D* **73**, 094505 (2006).
- [5] J.J. Dudek, R.G. Edwards, N. Mathur, D.G. Richards, *Phys. Rev. D* **77**, 034501 (2008).
- [6] R.A. Briceno, J.J. Dudek, R.D. Young, [arXiv:1706.06223](https://arxiv.org/abs/1706.06223) [hep-lat].
- [7] C.J. Burden *et al.*, *Phys. Rev. C* **55**, 2649 (1997).
- [8] P. Maris, C.D. Roberts, *Phys. Rev. C* **56**, 3369 (1997).
- [9] C.S. Fischer, R. Alkofer, *Phys. Rev. D* **67**, 094020 (2003).
- [10] G. Eichmann *et al.*, *Phys. Rev. C* **79**, 012202 (2009).
- [11] T. Hilger, C. Popovici, M. Gómez-Rocha, A. Krassnigg, *Phys. Rev. D* **91**, 034013 (2015).
- [12] G. Eichmann *et al.*, *Prog. Part. Nucl. Phys.* **91**, 1 (2016).
- [13] F. Gross, *Phys. Rev.* **186**, 1448 (1969).
- [14] A. Stadler, F. Gross, *Few-Body Syst.* **49**, 91 (2011).
- [15] F. Gross, J. Milana, *Phys. Rev. D* **43**, 2401 (1991).
- [16] E.P. Biernat, F. Gross, M.T. Peña, A. Stadler, *Phys. Rev. D* **89**, 016005 (2014).
- [17] S. Leitão, A. Stadler, M.T. Peña, E.P. Biernat, *Phys. Rev. D* **96**, 074007 (2017).
- [18] E.P. Biernat *et al.*, *Phys. Rev. D* **90**, 096008 (2014).
- [19] S. Leitão, A. Stadler, M. Peña, E.P. Biernat, *Phys. Lett. B* **764**, 38 (2017).
- [20] K.A. Olive *et al.* [Particle Data Group], *Chin. Phys. C* **38**, 090001 (2014).