FROM THE GRIBOV AMBIGUITY TO CONFINING EFFECTIVE MODELS* **

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We briefly review a confining quark model that aims at the extension of the refined Gribov–Zwanziger framework to the matter sector. Interactions are encoded in a non-local quark propagator that displays dynamical chiral symmetry breaking in the infrared, while reproducing the perturbative expectations in the deep ultraviolet limit.

1. Introduction

The confinement mechanism in Quantum Chromodynamics (QCD) has been an elusive issue for decades. Even though the capability of computing hadronic observables from (lattice) QCD has improved considerably — with remarkable success [1] — there is still no proof of what drives the fundamental degrees of freedom of this theory to be always confined in colourless bound states. Nevertheless, the absence of quarks from the physical spectrum is a strong constraint, and any low-energy effective theory written in terms of them could be greatly affected by this feature.

To address this question, a well-defined infrared QCD framework has to be applied. In what follows, we adopt the Gribov–Zwanziger (GZ) quantization of Yang–Mills theories [2] — briefly summarized in the next section — to avoid non-perturbative zero-modes that hinder the applicability of the standard gauge path integral in the infrared regime. An extension of this setup to describe confined quarks [3, 4] is presented in Sec. 3, followed by thermodynamic results [5] and some final remarks.

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2. Gribov copies and non-perturbative quantization

The starting point for describing a QCD system is to quantize the Yang–Mills action. The standard procedure of defining a functional integral in terms of gauge fields is the Faddeev–Popov method. In the Landau gauge and Euclidean space, we have

$$\int \mathcal{D}A\mathcal{D}\bar{c}\mathcal{D}c\mathcal{D}b\, e^{-S_{\mathrm{YM}}-S_{\mathrm{gf}}}\,, \tag{1}$$

where the standard Yang–Mills action and the gauge-fixing part read

$$S_{\mathrm{YM}} = \int \mathrm{d}^4 x \tfrac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}\,, \qquad S_{\mathrm{gf}} = \int \mathrm{d}^4 x \left[b^a \partial_\mu A_\mu - \bar{c}^a \mathcal{M}^{ab} c^b \right]\,, \tag{2}$$

with $\mathcal{M}^{ab}$ being the Faddeev–Popov operator in Landau gauge

$$\mathcal{M}^{ab} = -\partial_\mu \left(\delta^{ab}\partial_\mu + g f^{abc} A^c_\mu \right) = -\partial_\mu D^{ab}_\mu\,. \tag{3}$$

As pointed out by Gribov a long time ago [6][1], this path integral becomes ill-defined whenever the Faddeev–Popov operator develops a zero mode, since the derivation of Eq. (1) involves a change of variables whose Jacobian is exactly the determinant of this operator. These modes exist, are called Gribov copies and several explicit examples have already been worked out. It was also shown that these copies cannot be reached by small fluctuations around the trivial field $A = 0$, so that deep within the perturbative regime perturbation theory works, as it is well-known from its very successful predictions for high-energy processes. In our case, however, infrared phenomena are our main interest and this quantization procedure has to be revised.

The basic idea is to restrict the path integral measure to a region Ω in which the Faddeev–Popov operator is positive definite, so that no (infinitesimal) copies are reached[2]

$$\Omega = \left\{ A^a_\mu;\, \partial A^a = 0,\, \mathcal{M}^{ab} > 0 \right\}\,. \tag{4}$$

The Gribov–Zwanziger action,

$$S_{\mathrm{GZ}} = S_{\mathrm{YM}} + S_{\mathrm{gf}} + \gamma^4 H(A) - \gamma^4 V D(N^2 - 1)\,, \tag{5}$$

implements such a constraint by introducing a Gribov mass γ that is fixed by a gap equation for the vacuum energy

$$\frac{\partial \mathcal{E}}{\partial \gamma} = 0 \rightarrow \langle H(A) \rangle_{\mathrm{1PI}} = V D \left(N^2 - 1 \right)\,, \tag{6}$$

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[1] For a review, *cf.* Ref. [2] and references therein.
[2] It has been shown that all physical configurations are represented within this region, so no physics is lost in the process of constraining the path integral.

where the horizon function is related to the inverse Faddeev–Popov operator[3]

$$H(A) = \int \mathrm{d}^4 p \int \mathrm{d}^q A_\mu^a \left(\mathcal{M}^{ab} \right)^{-1} A_\mu^b , \tag{7}$$

which is highly non-local, but can be put in local form by the use of auxiliary fields. At one-loop order, it is easy to show that the Gribov mass is a non-perturbative contribution: $\gamma \propto e^{-1/g^2}$.

It turns out that this theory is unstable against the formation of certain dimension two condensates, giving rise to the following infrared effective Lagrangian of the refined GZ (RGZ) theory[4]:

$$\mathcal{L}_{\mathrm{RGZ}} = \tfrac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + b^a \partial_\mu A_\mu^a + \bar{c}^a \partial_\mu D_\mu^{ab} c^b + \mathcal{L}_{\ell H} + \mathcal{L}_{\mathrm{R}} , \tag{8}$$

where $\mathcal{L}_{\ell H}$ is the localized horizon term

$$\mathcal{L}_{\ell H} = \bar{\varphi}_\mu^{ac} \mathcal{M}^{ab} \varphi_\mu^{bc} - \bar{\omega}_\mu^{ac} \mathcal{M}^{ab} \omega_\mu^{bc} + \gamma^2 g f^{abc} A_\mu^a \left(\varphi_\mu^{bc} + \bar{\varphi}_\mu^{bc} \right) - \gamma^4 D \left(N_c^2 - 1 \right) . \tag{9}$$

Here, $\bar{\varphi}, \varphi, \bar{\omega}, \omega$ are auxiliary localizing fields that acquire a non-zero dimension 2 condensate, giving rise, together with the A^2 gluon condensate, to a refinement contribution

$$\mathcal{L}_{\mathrm{R}} = \frac{m^2}{2} A_\mu^a A_\mu^a + M^2 \left(\bar{\varphi}_\mu^{ab} \varphi_\mu^{ab} - \bar{\omega}_\mu^{ab} \omega_\mu^{ab} \right) , \tag{10}$$

where the parameters m, M can, in principle, be determined self-consistently through a minimization of the respective effective potential.

The tree-level propagators in Eq. (8) describe very well the lattice data [8] and can be used to obtain consistent estimates of the masses of glueball states with different quantum numbers [9,10]. It is also renormalizable and reduces to perturbative Yang–Mills in the ultraviolet regime, while being written in terms of confined fields, in the sense that they do not allow for an asymptotic particle description, since their spectral representation violates the reflection positivity axiom.

3. A proposal for matter confinement

In Ref. [4], we have presented a confined matter model applicable for any type of field in the adjoint or fundamental representations of the non-Abelian

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[3] For a derivation of this function from Gribov's no-pole condition for the ghost propagator, *cf.* Ref. [7].

[4] We shall always assume Landau gauge, but this formalism can also be extended to other gauges (*cf. e.g.* [11]). For details and notations, the reader is referred to Ref. [2] and references therein.

gauge group in question. For QCD, our aim is then

$$\mathcal{L}_{\text{QCD}} \overset{\text{quantum non-pert. effects}}{\Longrightarrow} \mathcal{L}_{\text{IRQCD}} = \mathcal{L}_{\text{RGZ}} + \mathcal{L}_{\text{M}}\,, \tag{11}$$

where $\mathcal{L}_{\text{M}}$ is the Lagrangian associated with the effective dynamics of infrared, confined matter that we would like to obtain. For the action $\mathcal{L}_{\text{IRQCD}}$ to represent a good model of infrared QCD, it should be compatible with quark and gluon confinement, as well as with lattice data available in this regime. Moreover, the quark two-point function should display dynamical mass generation, compatible with the spontaneous breaking of chiral symmetry in the vacuum of QCD.

A consistent extension of the infrared effective Lagrangian to include matter fields should encode a non-perturbative transmission of the effects of the Gribov copies to the matter sector. The Faddeev–Popov operator (or rather its inverse) is omnipresent in the construction of RGZ theories and it is a strictly non-perturbative quantity whose existence is guaranteed inside the Gribov region. Even though the restriction of the measure of integration in the path integral to the Gribov region Ω is fully implemented in the gauge sector of the RGZ framework, it represents a sharp boundary condition on a quantum operator that carries colour indices and shall affect all possible correlation functions involving it. In this sense, it is reasonable to assume that this non-perturbative constraint will indeed affect all fields that present colour indices in the theory. This property is an essential assumption of our proposal for a picture of colour confinement emerging from Gribov physics: the inverse Faddeev–Popov operator shall play a central role as a mediator of non-perturbative effects to all coloured fields. The general gauge matter infrared action reads

$$S_{\text{IR}} = S_{\text{YM}} + \sum_F M_F \mathcal{H}_F\,, \tag{12}$$

where $\mathcal{H}_F$ is a universal term of the form of the Gribov horizon,

$$\mathcal{H}_F = g^2 \int\limits_{x,y} \bar{\Psi}^i(x)\,(T^a)^{ij}\left[\mathcal{M}^{-1}(x,y)\right]^{ab}\Psi^k(y)\left(T^b\right)^{jk} \tag{13}$$

that communicates confinement to the quark fields Ψ^i via the infrared parameter M_F. In contrast to the Gribov parameter γ^2, that is fixed by a gap equation, the matter horizon parameter M_F is free. In principle, it could be computed by the consistent construction and minimization of an effective potential, but this remains to be explicitly proven.

Interesting features of the proposal Eq. (12) are: *(i)* a local[5] and renormalizable action that allows for predictive power; *(ii)* the reduction to QCD in the ultraviolet regime. In particular, all infrared parameters M_F have renormalization factors that are fully fixed by the original UV theory. They are thus not new independent parameters, but rather compatible with dynamically generated scales; *(iii)* tree-level propagators that are compatible [4] with lattice data, signalling that the model captures well the non-perturbative background. A good fit of lattice data was performed using the analytic form predicted by our confinement model, namely

$$\langle \psi^i(k)\bar{\psi}^j(-k)\rangle = \delta^{ij}\frac{i\slashed{k} + \mathcal{A}\left(k^2\right)}{k^2 + \mathcal{A}^2\left(k^2\right)}\,, \qquad \mathcal{A}\left(k^2\right) = m_\psi + \frac{g^2 \Gamma_\psi C_{\mathrm{F}}}{k^2 + \mu_\psi^2}\,, \qquad (14)$$

where the quark mass function $\mathcal{A}(k^2)$ depends on the current quark mass m_ψ and the refinement condensate mass μ_ψ; *(iv)* absence of coloured fields from the spectrum. The particle interpretation of the excitation of a given field relies on the existence of a Källén–Lehmann representation with a positive spectral function. This is, however, not achievable when reflection positivity is violated, so that a field with this property cannot be present in the physical spectrum of the theory. This feature has been consistently verified for gluons in the RGZ framework. The case of matter fields displaying a non-Abelian charge was investigated in [4], with positivity violation occurring in the tree-level propagator of our model when the values of the parameters that fit lattice quark data are used, as was shown in [3].

The thermodynamics of a confining quark model — defined by the leading-order approximation of the model here discussed — has been investigated in Ref. [5]. The approximation adopted is very simple, keeping only the quark degrees of freedom under the influence of a non-trivial gluon background encoded in a non-local propagator. Nevertheless, the results of this lowest-order model show non-trivial thermodynamic behaviour.

Since this confining quark model model is quadratic, all thermodynamic quantities may be computed exactly (for details, *cf.* [5]). In Fig. 1, we show the pressure as a function of the temperature at zero chemical potential. We normalize the result by the pressure of free, massless fermions and compare to the pressure of massless quarks subject to a constant negative *bag pressure* $p_{\mathrm{Bag}} = -B = -(0.145 \text{ GeV})^4$. A smooth but fast rise of the pressure as temperature increases is observed, indicating a drastic change in the number of thermal degrees of freedom between low- and high-temperature systems.

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[5] The localization of the matter horizon Eq. (13) may be undertaken in analogy to the localization of the Gribov one, presented above in the (R)GZ action. This introduces new localizing auxiliary fields that may also give rise to a refinement in the matter sector due to dimension 2 condensates.

A similar behaviour is seen for the thermal crossover in lattice QCD simulations. In contrast to the bag model, our model is capable of describing a large temperature range, indicating that the non-perturbative background encoded in it may contain information from the low-energy, confined phase. For very low temperatures, pressure assumes extremely small, but negative and unphysical values [12], and this lowest-order approximation remains to be improved by the inclusion of further interactions.
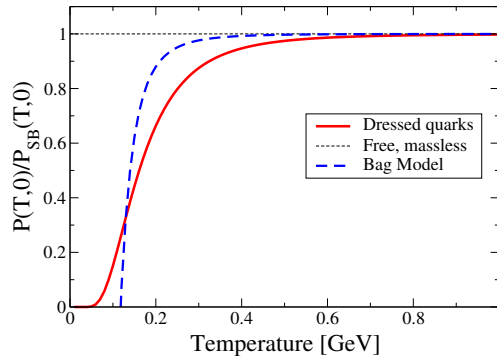

Fig. 1. Pressure as a function of temperature for $\mu = 0$, normalized by the free gas limit for non-local quarks and for the MIT bag model with bag constant $B = (0.145 \text{ GeV})^4$.

Overall the results of the model show that there is a clear change of behaviour of all thermodynamic quantities at $T \simeq 0.15\text{GeV}$, the typical temperature scale of the chiral or deconfined phase crossover.

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