

## THE TOPOLOGICAL SUSCEPTIBILITY VIA THE GRIBOV HORIZON?\*

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We analyze the SU(2) and SU(3) topological susceptibility  $\chi^4$ , in a BRST invariant fashion, using Padé approximation and the Refined Gribov–Zwanziger gluon propagator.

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### 1. Introduction

Since Gribov [1], we already know that the Faddeev–Popov construction is not valid at the non-perturbative level. In this regime, we have multiple intersections of the gauge orbits with the hypersurface corresponding to a given gauge condition  $f(A) = 0$ , so-called Gribov copies. To work around this problem, Gribov’s proposal was to restrict the domain of integration in the path integral to a certain region  $\Omega$  in field space, called the Gribov region, which is free from infinitesimal Gribov copies. In the linear covariant gauge and taking into account the dimension two condensates,  $\langle A_\mu^a A_\mu^a \rangle$  and  $\langle \bar{\varphi}_\mu^{ab} \varphi_\mu^{ab} - \bar{\omega}_\mu^{ab} \omega_\mu^{ab} \rangle$ , the (BRST invariant) restriction to the Gribov region is achieved by the action

$$S_{\text{lc}} = S_{\text{YM}} + S_{\text{GF}} + S_{\text{RGZ}} + S_\tau, \quad (1)$$

where  $S_{\text{YM}}$  is the Yang–Mills action,  $S_{\text{GF}}$  is the Faddeev–Popov gauge-fixing in linear covariant gauges, *i.e.*

$$S_{\text{GF}} = \int d^4x \left( \frac{\alpha}{2} b^a b^a + i b^a \partial_\mu A_\mu^a + \bar{c}^a \partial_\mu D_\mu^{ab}(A) c^b \right), \quad (2)$$

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with  $\alpha$  being the gauge parameter,  $S_{\text{RGZ}}$  is the Refined Gribov–Zwanziger (RGZ) action given by [2]

$$\begin{aligned}
 S_{\text{RGZ}} = & \int d^4x \left( -\bar{\varphi}_\nu^{ac} \mathcal{M}^{ab} (A^h) \varphi_\nu^{bc} + \bar{\omega}_\nu^{ac} \mathcal{M}^{ab} (A^h) \omega_\nu^{bc} \right. \\
 & + \gamma^2 g f^{abc} (A^h)_\mu^a (\varphi_\mu^{bc} + \bar{\varphi}_\mu^{bc}) + \frac{m^2}{2} \int d^4x (A^h)_\mu^a (A^h)_\mu^a \\
 & \left. + M^2 \int d^4x (\bar{\varphi}_\mu^{ab} \varphi_\mu^{ab} - \bar{\omega}_\mu^{ab} \omega_\mu^{ab}) \right), \tag{3}
 \end{aligned}$$

where  $\mathcal{M}^{ab}(A^h)$  is the Hermitian, gauge invariant operator  $\mathcal{M}^{ab}(A^h) = -\delta^{ab}\partial^2 + g f^{abc}(A^h)_\mu^c \partial_\mu$ , while  $\gamma$  is the Gribov parameter, dynamically fixed by the gap equation [3]

$$\left\langle f^{abc} A_\mu^{h,a} (\varphi_\mu^{bc} + \bar{\varphi}_\mu^{bc}) \right\rangle = 2d(N^2 - 1) \gamma^2 / g^2. \tag{4}$$

The configuration  $A_\mu^h$  is a non-local power series in the gauge field, obtained by minimizing the functional  $f_A[u]$  along the gauge orbit of  $A_\mu$  [4], with

$$f_A[u] \equiv \min_{\{u^\dagger\}} \text{Tr} \int d^4x A_\mu^u A_\mu^u, \quad A_\mu^u = u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u. \tag{5}$$

One finds that a local minimum is given by

$$\begin{aligned}
 A_\mu^h &= \left( \delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} \right) \phi_\nu, \quad \partial_\mu A_\mu^h = 0, \\
 \phi_\nu &= A_\nu - ig \left[ \frac{1}{\partial^2} \partial A, A_\nu \right] + \frac{ig}{2} \left[ \frac{1}{\partial^2} \partial A, \partial_\nu \frac{1}{\partial^2} \partial A \right] + O(A^3). \tag{6}
 \end{aligned}$$

Following [2, 5], we set

$$A_\mu^h = (A^h)_\mu^a T^a = h^\dagger A_\mu^a T^a h + \frac{i}{g} h^\dagger \partial_\mu h, \tag{7}$$

while  $h = e^{ig \xi^a T^a}$ . The local invariance of  $A_\mu^h$  under a gauge transformation  $u \in \text{SU}(N)$  is clear from

$$h \rightarrow u^\dagger h, \quad h^\dagger \rightarrow h^\dagger u, \quad A_\mu \rightarrow u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u. \tag{8}$$

The term

$$S_\tau = \int d^4x \tau^a \partial_\mu (A^h)_\mu^a \tag{9}$$

implements, through the Lagrange multiplier  $\tau$ , the transversality of the composite operator  $(A^h)_\mu^a$ ,  $\partial_\mu (A^h)_\mu^a = 0$ .

The action  $S_{\text{lc}}$  enjoys an exact nilpotent local BRST invariance,  $sS_{\text{lc}} = 0$  [2].

For the current work, we are mostly interested in the general form of the gluon propagator [5]

$$D_{\mu\nu}(p) = D(p)P_{\mu\nu}(p) + L(p)\frac{p_\mu p_\nu}{p^2}, \tag{10}$$

with the transverse form factor  $D(p)$

$$D(p) = \frac{p^2 + M^2}{p^4 + (M^2 + m^2)p^2 + M^2m^2 + \lambda^4} \tag{11}$$

containing all non-trivial information, next to  $L(p) = \alpha/p^2$ , with

$$P_{\mu\nu}(p) = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}, \quad L_{\mu\nu}(p) = \frac{p_\mu p_\nu}{p^2} \tag{12}$$

the transversal and longitudinal projectors.

## 2. The topological susceptibility

The topological susceptibility is linked to the  $\eta'$  mass via [6]

$$m_{\eta'}^2 = \frac{4N_f}{f_\pi^2} \chi_{\theta=0, N_f=0}^4 = \mathcal{O}(1/N), \tag{13}$$

where  $\theta$  is the vacuum angle and  $f_\pi$  the pion decay constant. Witten and Veneziano suggested that the vacuum topology fluctuations can be captured by the occurrence of an unphysical mass pole [6], the Veneziano ghost, in the topological current correlator

$$p_\mu p_\nu \langle K_\mu K_\nu \rangle_{p=0} \neq 0, \tag{14}$$

whereby  $K_\mu$  is the topological Chern–Simons current

$$K_\mu = \frac{g^2}{16\pi^2} \epsilon_{\mu\nu\rho\sigma} A_{\nu,a} \left( \partial_\rho A_\sigma^a + \frac{g}{3} f^{abc} A_\rho^b A_\sigma^c \right). \tag{15}$$

Following the Euclidean conventions of [7], we have

$$\chi^4 = - \lim_{p^2 \rightarrow 0} p_\mu p_\nu \langle K_\mu K_\nu \rangle \geq 0. \tag{16}$$

For the Källén–Lehmann spectral density of the current correlator, we have

$$\begin{aligned} \langle K_\mu(p)K_\nu(-p) \rangle &= \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \mathcal{K}_\perp(p^2) + \frac{p_\mu p_\nu}{p^2} \mathcal{K}_\parallel(p^2) \\ &\equiv \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \int_0^\infty d\tau \frac{\rho_\perp(\tau)}{\tau + p^2} \\ &\quad + \frac{p_\mu p_\nu}{p^2} \int_0^\infty d\tau \frac{\rho_\parallel(\tau)}{\tau + p^2} \end{aligned} \tag{17}$$

based on Euclidean invariance. Then, we already find that

$$-\chi^4 = \lim_{p^2 \rightarrow 0} p^2 \mathcal{K}_\parallel(p^2) = \lim_{p^2 \rightarrow 0} p^2 \int_0^\infty d\tau \frac{\rho_\parallel(\tau)}{\tau + p^2}. \tag{18}$$

From dimensional analysis, it is clear that we need 2 subtractions ( $\rho_\parallel(\tau) \sim \tau$  for  $\tau \rightarrow \infty$ ), so we actually have

$$-\chi^4 = \lim_{p^2 \rightarrow 0} p^6 \int_0^\infty d\tau \frac{\rho_\parallel(\tau)}{(\tau + p^2)\tau^2}. \tag{19}$$

The spectral density associated with the Källén–Lehmann representation of the physical part of the  $K_\mu$  correlation function is given by [8]

$$\rho_\parallel(\tau) = -2A_+ A_- \frac{g^4 (N^2 - 1)}{2^{2d+5} \pi^{7/2} \Gamma\left(\frac{d-1}{2}\right)} \frac{(\tau^2 - 4b^2 - 4a\tau)^{(d-1)/2}}{\tau^{d/2}} \tag{20}$$

for  $\tau \geq \tau_c = 2(a + \sqrt{a^2 + b^2})$ , where  $a = M_2^2/2$  and  $b = \sqrt{M_3^4 - M_2^4}/4$ .

For the MOM renormalized gluon propagator, we must use

$$D(p^2) = Z \frac{p^2 + M_1^2}{p^4 + M_2^2 p^2 + M_3^4}, \quad Z = \frac{1}{\mu^2} \frac{\mu^4 + M_2^2 \mu^2 + M_3^4}{\mu^2 + M_1^2}. \tag{21}$$

For the dynamical mass parameters, rather than attempting to solve their gap equations (4), we will resort to estimates obtained from fitting (21) to lattice propagator data [9].

For SU(3), we approximated (19) with the [3,1] Padé rational function in variable  $p^2$ , around  $p^2 = P^2$ . This scale  $P^2$  should be not too small, so that we can trust the perturbative regime and not too large so that we are taking into account sizable non-perturbative effects and perform a sensible

extrapolation of the approximant to zero momentum to get an estimate for  $\chi^4$ . In the l.h.s. of Fig. 1, we show  $\chi(\mu^2, P^2 = 5 \text{ GeV}^2)$ , and we clearly observe an optimal value of  $\mu_*^2 = 3.330 \text{ GeV}^2$ . From the r.h.s. of Fig. 1, we are unable to extract an optimal scale from where to start the Padé approximation over a reasonable interval of  $P^2$ -values,  $P^2 = 3 \dots 7 \text{ GeV}^2$ . Unfortunately, we cannot make a definite, in the sense of optimal, prediction for  $\chi$  from the right graph of Fig. 1. A natural choice would then be to set  $P^2 = \mu_*^2$ , since the MOM renormalization scale  $\mu^2$  is subject to the same assumptions as  $P^2$  when used to renormalize lattice data. This gives  $\chi \approx 142 \text{ MeV}$ , a bit below the lattice ballpark of  $\chi \sim 200 \text{ MeV}$  [10].

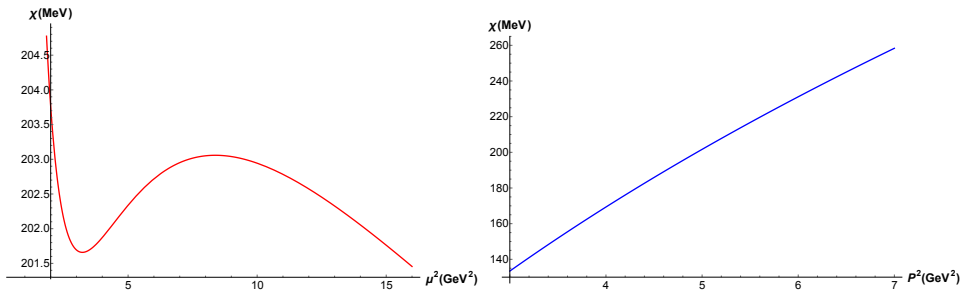


Fig. 1. Topological susceptibility  $\chi$  for variable  $\mu^2$  and fixed  $P^2 = 5 \text{ GeV}^2$  (left) and for variable  $P^2$  and fixed  $\mu^2 = 3.330 \text{ GeV}^2$  (right)  $\mu^2$  (SU(3) case).

For  $N = 2$ , following the same procedure as for  $N = 3$ , we get the graphs of Fig. 2 in the  $N = 2$  case. We observe that there is now neither an optimal  $\mu^2$  nor  $P^2$ . For more details about the calculus and references, see [8].

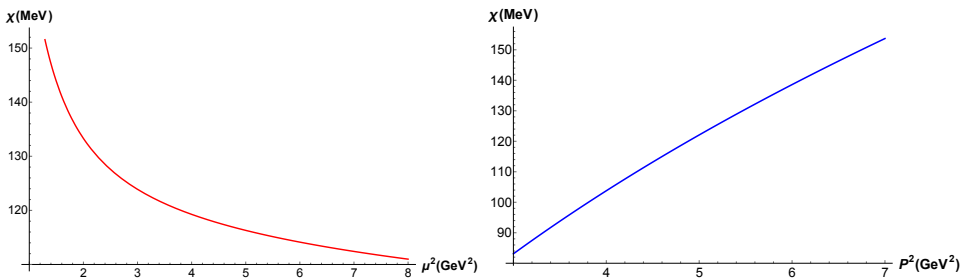


Fig. 2. Topological susceptibility  $\chi$  for variable  $\mu^2$  and fixed  $P^2 = 5 \text{ GeV}^2$  (left) and for variable  $P^2$  and fixed  $\mu^2 = 3.330 \text{ GeV}^2$  (right)  $\mu^2$  (SU(2) case).

### 3. Conclusion

Although we cannot present a precise value for the topological susceptibility, we did obtain rough estimates for it, qualitatively compatible with lattice data. In order to improve upon this crude estimation, we would have to include the next order correction in future work. However, this will be computationally challenging, because of the enlarged set of vertices in the now considered Refined Gribov–Zwanziger action for the linear covariant gauge.

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