QUESTION OF γ -SOFTNESS OF A CORE AND POSSIBLE WOBBLING IN THE LIGHT OF RICH EXPERIMENTAL DATA ON ¹¹⁹I^{*}

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Analysis of rich experimental data on four negative parity bands of ¹¹⁹I in frame of the CQPC (Core-Quasiparticle Coupling) model with the $h_{11/2}$ proton coupled to a collective quadrupole core is presented. Confrontation of results obtained for two kinds of the even–even core: γ -soft and γ -rigid does not show distinct difference. This conclusion confirms hypothesis that a chiral structure in odd–odd nuclei can be equally well-explained with a rigid triaxial core as well with a soft triaxial core. Possible occurrence of a transverse wobbling motion is also briefly discussed.

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1. Introduction

To explain several phenomena in nuclear spectroscopy, one should assume triaxial quadrupole deformation of an atomic nucleus. Particularly, chirality and wobbling cannot exist in the case of axially symmetric rotor. There is still a discussion how can we describe both phenomena in the case of softness of a triaxial shape of an even–even core. The most reliable way of experimental verification of γ -softness or γ -rigidity is based on the sum rules method of obtaining values of appropriate invariants [1] (see also P. Napiorkowski talk at this conference). However, such a method requires extended knowledge of many E2 matrix elements, very often not available. In

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the case it is not possible to get all matrix E2 elements needed to measure triaxiality dispersion of the wave function, one can use a phenomenological way of testing features of a quadrupole even–even core by comparing predictions of various core models with experimental results. From our experience, a type of triaxiality of the even–even core — soft or rigid can be tested not only by taking into account data (a level energy pattern and B(E2) values) from an even–even nucleus but also from neighbouring odd–even nuclei.

Our calculations are based on the Core-Quasiparticle Coupling model (CQPC) [2]. The main assumption of the model is a quadrupole–quadrupole interaction Qq between collective even–even core (Q) and particle (quasiparticle) (q). The pairing interaction is also included. For the first time we used



Fig. 1. Negative parity bands in $^{119}{\rm I}$ [3] being a good ground for testing the properties of even–even core.

the model for interpretation of the structure of 127 Cs [4]. Later on, our group has interpreted the following odd–even nuclei: 111 Ru [5], 113 Ru [6], 119 I [3], 121 Cs [7], 123 Cs [7, 8], 125 Cs [8], 125 La [9], 127 La [9], 129 La [10], 131 La [11], 127 Pr [12]. For 111 Ru, 113 Ru properties of collective cores were calculated microscopically within the General Bohr Hamiltonian model without any fitting of parameters to specific experimental data. All others were described by the CQPC model using phenomenological collective models for core excitations.

In the present paper, we recall rich experimental data on E2 transition probabilities in four negative parity bands of ¹¹⁹I obtained in Ref. [3]. Together with a band structure [13] they are interpreted within the CQPC model to find information on γ -soft or γ -rigid features of the even–even core. The case of ¹¹⁹I is very well-suited to address the above question since:

- the pattern of low-lying negative-parity states in ¹¹⁹I presented in Refs. [3,13] (see Fig. 1) has a simple structure, based on the intruder $1h_{11/2}$ proton state placed high above the Fermi level. This means that the properties of the negative-parity states in ¹¹⁹I do not depend strongly on the position of the $h_{11/2}$ level, which is only approximately known from the Nilsson model calculation,
- for 4 negative bands, 31 level lifetimes and 39 values of B(E2) were established. It is one of the richest lifetime information available for odd nucleus.

2. Core–Quasiparticle Coupling Model

The Hamiltonian of the model including quadrupole–quadrupole and pairing interactions reads

$$H = H_{\rm sp} - \frac{1}{2}\chi Qq - \frac{1}{4}\sum_{t=n,p} G_t P_t^{\dagger} P_t \,. \tag{1}$$

The quadrupole–quadrupole interaction strength was taken as $\chi = -15$ MeV, and the pairing gap $\Delta = 135/A$ MeV. A single particle level scheme was obtained from the spherical Woods–Saxon potential. The position of the Fermi level was fitted to reproduce the proper number of valence protons in ¹¹⁹I.

Usually, real (A-1) and (A+1) nuclei can be treated as even-even cores. That is not the case for nuclei with Z or N close to a magic number. In such a case, nuclear properties change rapidly and the cores differ significantly from real adjacent nuclei, due to the polarisation effect. Such a situation is found for the ¹¹⁹I nucleus. Figure 2 shows that the properties of ¹¹⁸Te and ¹²⁰Xe are drastically different because of the proximity of the Z = 50 magic number. The energies of the decoupled band in ¹¹⁹I follow the energies of the ground-state band in ¹²⁰Xe rather than that in ¹¹⁸Te. It seems that the $1h_{11/2}$ proton, added to ¹¹⁸Te, changes the properties of the (A - 1) core, making it similar to ¹²⁰Xe. That is why, in the following, we will use the same data of ¹²⁰Xe for description of (A - 1) and (A + 1) cores.



Fig. 2. Part of the level schemes of the ¹¹⁹I and neighbouring nuclei with N = 66. Decoupled bands built on the $11/2^-$ state for odd-A nuclei are presented.

To test properties of ¹²⁰Xe, two different phenomenological cores were used:

- the γ -soft model of Wilets–Jean-type [14],
- the γ -rigid model of Davydov–Filippov [15].

The Wilets–Jean model was used in an extended version given in Dobaczewski, Rohoziński and Srebrny model [16]. The following approximations were made:

- the rotational inertial functions $B_x = B_y = B_z = B = \text{const}$,
- the vibrational inertial functions $B_{\beta\beta} = \text{const}, B_{\beta\gamma} = 0, B_{\gamma\gamma} = B$,

— the γ -independent potential energy surface was described by the formula

$$V(\beta) = \frac{1}{2}C_2\beta^2 + C_8\beta^8 + G\left(\exp\left(-\frac{\beta^2}{\alpha^2}\right) - 1\right).$$
 (2)

The values of parameters of the model are: $B = 110 \hbar^2/\text{MeV}$, $B_{\beta\beta} = 1000 \hbar^2/\text{MeV}$, $C_2 = 0.01$ MeV, $C_8 = 1.5 \times 10^4$ MeV, $\alpha = 0.18$. With these parameters, the equilibrium deformation β_0 is equal to 0.276 and the depth of potential D = 7.6 MeV. Plot of the potential is given in Fig. 3.



Fig. 3. β -deformation dependence of the γ -independent potential V given by Eq. (2).

To investigate the possibility of describing the ¹¹⁹I nucleus using the γ -rigid core model, the Davydov–Filippov model with different sets of parameters was applied. Model parameters, namely β_0 and γ_0 , and the energy $E(2^+_1)$ of the first 2⁺ state are listed in Table I. The electromagnetic properties of these cores are compared with experimental data for ¹²⁰Xe in Table II. The following three variants of the Davydov–Filippov model were considered:

- DF-A the properties of the core taken from the experimental data of 120 Xe,
- DF-B $\gamma = 30^{\circ}$ corresponds to $\langle \gamma \rangle = 30^{\circ}$ in the γ -soft core, the rest of model parameters were adjusted to reproduce ¹¹⁹I,
- DF-C all model parameters were adjusted to reproduce properties of ¹¹⁹I, particularly transition probabilities in band 9.

TABLE I

Variant	DF-A	DF-B	DF-C
$ \begin{array}{c} \gamma_0 \\ \beta_0 \\ E(2^+) \text{ [keV]} \end{array} $	23.4° 0.29 322.4	$30^{\circ} \\ 0.31 \\ 200$	$37^{\circ}\ 0.36\ 130$

Parameters for rigid cores.

TABLE II

Reduced transition probabilities B(E2) in ¹²⁰Xe. The experimental values are compared with calculated ones in framework of the γ -soft (WJ) and γ -rigid (DF, different γ_0 according to A, B, C variants) models. B(E2) values are given in e^2b^2 units.

Transition	WJ	exp	$\begin{array}{c} \text{DF-A} \\ \gamma_0 = 23.4^{\circ} \end{array}$	$\begin{array}{c} \text{DF-B} \\ \gamma_0 = 30^{\circ} \end{array}$	$\begin{array}{c} \text{DF-C} \\ \gamma_0 = 37^{\circ} \end{array}$
$\begin{array}{c} 2^+ \rightarrow 0^+ \\ 4^+ \rightarrow 2^+ \\ 6^+ \rightarrow 4^+ \end{array}$	$\begin{array}{c} 0.31 \\ 0.45 \\ 0.54 \end{array}$	$\begin{array}{c} 0.36 \pm 0.03 \\ 0.42 \pm 0.04 \\ 0.44 \pm 0.05 \end{array}$	$\begin{array}{c} 0.36 \\ 0.51 \\ 0.61 \end{array}$	$\begin{array}{c} 0.39 \\ 0.54 \\ 0.68 \end{array}$	$0.43 \\ 0.64 \\ 0.81$

3. Discussion of the results

The results of model calculation for ¹¹⁹I are compared with experimental energy levels in Fig. 4 and with B(E2) transition probabilities in Fig. 5.

The main conclusion is that for three bands: band 8 (yrast), band 7 and band 6, it is very hard to distinguish which model of the core (γ -soft or γ -rigid) better reproduces the energy levels and transition probabilities. Only for the band 9, the γ -rigid version DF-A gives worse results than other variants of the core models, the same is for the band 7 and DF-B version. For B(E2) in the band 8 (yrast band) γ -rigid DF-C version is a little bit worse than other core models. A small indication of the γ -softness of the core properties are the following features of band 9:

- the regular $\Delta I = 2$ energy level structure observed in the experiment is well-reproduced in the theoretical calculation with γ -soft core,
- the $11/2 \rightarrow 9/2$ and $9/2 \rightarrow 7/2$ transitions that should exist according to model calculations with γ -rigid core (see Fig. 4) are not observed in the experiment,
- the very small energy distance between the $11/2^-$ and $9/2^-$ level of the band 9 (according to model with γ -soft core) explains the lack of transitions mentioned above.



Fig. 4. Negative-parity states in ¹¹⁹I. The experimental level scheme (bands 6–9) [13] is compared with the results of the CQPC model calculations. The $9/2^{-}$ levels not seen in experiment are shown as dashed lines.



Fig. 5. Experimental and theoretical values of B(E2) (in e^2b^2 units) for the negative-parity states in ¹¹⁹I according to results presented in Table 5 of [3].

Detailed discussion of band 9 features is given in [3] (see particularly Fig. 16 of [3]). It is worth to add that such kind of band is hardly visible in other nuclei from the region of 52 < Z, N < 80.

4. Question of existence of a wobbling band in ¹¹⁹I

The first experimental identification of the wobbling band was given in high spin region $(18-32\hbar)$ in ¹⁶³Lu [17,18]. Such high spin levels are difficult to excite and to identify spin values and transition probabilities. In the last years, there appeared experimental data and a theoretical interpretation of low-spin wobbling in ¹³⁵Pr [19] (wobbling band with $\Delta I = 2$, $17/2^-$ to $29/2^-$ — see Fig. 6).



Fig. 6. Partial levels scheme of 135 Pr [19] with indication of the wobbling band.

This is a reason that in the presented paper the case of ¹¹⁹I is recalled. The band 6 (see Fig. 1) for spins from $13/2^-$ to $29/2^-$ can be treated as a wobbling band with $\Delta I = 1$ transitions to yrast band 8. Equation (1) from Ref. [19] defines the wobbling energies as

$$E_{\rm wob}(I) = E(I, n_{\omega} = 1) - \frac{E(I-1, n_{\omega} = 0) + E(I+1, n_{\omega} = 0)}{2}, \quad (3)$$

where n_{ω} is the wobbling quantum number.

Table III contains results of Eq. (3) applied to ¹³⁵Pr and ¹¹⁹I. In the case of ¹¹⁹I, n_{ω} equal to 0 and 1 were applied to bands 6 and 8, respectively.

According to Ref. [19], one of the characteristic features of the wobbling band is that The wobbling energy decreases with angular momentum — this is the hallmark of transverse wobbling. It is seen from Table III that such a hallmark similar to ¹³⁵Pr is visible in ¹¹⁹I. The other hallmark, mainly E2 multipolarity of $\Delta I = 1$ transitions from band 6 to band 8, is not excluded by angular distribution ratios R measured in [3].

TABLE III

Energy of the transverse wobbling in 135 Pr and 119 I according to Eq. (3).

Ι	$E_{\rm wob}$	[MeV]
	$^{135}\mathrm{Pr}$	119 I
17/2	0.417	0.602
21/2	0.386	0.556
25/2	0.255	0.495
29/2	0.172	0.437

5. Summary

Rich experimental data on band structure and B(E2) transition probabilities give a very good ground for testing various theoretical interpretations. When the CQPC model is applied, an energy levels pattern and transition probabilities of an odd-even nuclei are sensitive to E2 matrix elements of an even-even core. Detailed analysis of four negative parity bands of ¹¹⁹I (especially band 8, band 7 and band 6) does not show an important difference between application of γ -soft or γ -rigid structure of an even-even core. It indicates that at the first order approximation, only the average value $\langle \cos 3\gamma \rangle$ and not a γ -softness or a γ -rigidity is crucial for collective core characterisation. It exactly coincides with conclusion of Droste *et al.* [20] in the case of a chiral structure with the S-symmetry features: The results of calculations for the two different cores are compared. The properties of the nucleus with the rigid, maximally triaxial ($\gamma = 30^{\circ}$) and with the entirely γ -soft core are qualitatively very similar

A new feature of ¹¹⁹I negative parity spectrum appears with a tentative interpretation of band 6 as a transverse wobbling band with $\Delta I = 1$ transitions with a decreasing wobbling energy from band 6 to band 8. Acknowledgements are due to Chrystian Droste for his enlightening discussion, explaining details of the CQPC model and careful reading of the manuscript. This project was supported in part by the National Science Centre, Poland (NCN) grant No. 2013/10/M/ST2/00427.

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