PROPERTIES OF SUPERHEAVY ISOTOPES $Z = 120$ AND ISOTONES $N = 184$ WITHIN THE SKYRME–HFB MODEL*

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We study the nuclear properties of even–even superheavy $Z = 120$ isotopes and $N = 184$ isotones with the Skyrme–Hartree–Fock–Bogoliubov (HFB) approach. Within this model, we examine the deformation energy surfaces and two paths to fission: a reflection-symmetric path with elongated fission fragments (sEF) and a reflection-asymmetric path corresponding to elongated fission fragments (aEF). Furthermore, we explore the energy surfaces in the region of very large oblate deformations with toroidal nuclear density distributions. While the energy surfaces of toroidal $Z = 120$ isotopes and $N = 184$ isotones do not possess energy minima without angular momenta, local energy minima (toroidal high-spin isomeric states) appear for many of these superheavy nuclei with specific angular momenta about the symmetry axis. We have theoretically located the toroidal high-spin isomers (THSIs) of $^{302}\mathrm{Og}_{184}$, $^{302}120_{182}$, $^{306}120_{186}$, and $^{306}122_{184}$.

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1. Introduction

Nuclei with excessive charges have a tendency to distribute the density in the oblate configuration in the shape of a biconcave disks or a toroid [1–3]. The additional presence of a large angular momentum about the symmetry axis enhances the stability of a toroidal density distribution. Previously, we examined the energy surfaces of even–even superheavy $Z = 120$ isotopes and $N = 184$ isotones in extremely prolate and oblate shapes, without and with an angular momentum [2, 3]. We found that even though toroidal density distributions frequently appear in constraint HFB calculations, there is no

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local toroidal energy minimum when there is no angular momentum in this region of Z and A. However, under the constraint of an angular momentum about the symmetry axis, toroidal high-spin isomers (THSIs) show up as local energy minima in $^{304}120_{184}$ when it is endowed with specific angular momenta [3]. In the present contribution, we continue our examination of the energy surfaces of other even–even superheavy $Z = 120$ isotopes and $N = 184$ isotones, and search for THSIs under the constraint of angular momenta about the symmetry axis. We find many local energy minima, THSI states, in the even–even neighborhood of $^{304}120$, indicating the common presence of toroidal high-spin isomer in the superheavy region.

2. Skyrme–HFB Model

In our method, we use the constrained and/or cranked Skyrme–Hartree–Fock–Bogoliubov (HFB) approach, which is equivalent to minimization of the Skyrme energy density functional $E^{\rm tot}[\bar{\boldsymbol{\rho}}]$, with respect to the densities and currents under a set of constraints (see Ref. [3] and references cited therein). Taking the method of Lagrange multipliers, we solve an equality-constrained problem (ECP)

$$\begin{cases} \min\limits_{\bar{\boldsymbol{\rho}}} E^{\rm tot}[\bar{\boldsymbol{\rho}}] & \\ \text{subject to: } \left\langle \hat{N}_q \right\rangle = N_q\,, & (q=p,n)\,, \\ \qquad\qquad\quad \left\langle \hat{Q}_{\lambda\mu} \right\rangle = Q_{\lambda\mu}\,, & \\ \qquad\qquad\quad \left\langle \hat{J}_i \right\rangle = I_i\,, & (i=x,y,z)\,, \end{cases} \tag{1}$$

where the constraints are defined by the average values $N_{p,n}$ of the proton and neutron particle-number operators $\hat{N}_{p,n}$, the constrained values $Q_{\lambda\mu}$ of the mass multiple moment operators $\hat{Q}_{\lambda\mu}$, and the constraint components of the angular momentum vector I_i. The above ECP equations are solved using an augmented Lagrangian method [4] with the symmetry-unrestricted code HFODD [5], which uses the basis expansion method utilizing a three-dimensional Cartesian deformed harmonic oscillator (h.o.) basis. The basis was composed of the 1140 lowest states taken from the $N_0 = 26$ h.o. shells. In the particle–hole channel, the Skyrme SkM* force [6] was applied and a density-dependent *mixed* pairing [3] interaction in the particle–particle channel was used.

We use the constrained Skyrme–HFB approach when we try to establish the region of the quadrupole deformation with the spherical and toroidal density distributions. The cranked Skyrme–HF model (without pairing correlations) was applied to locate the THSIs under the constraint of an angular momentum about the symmetry axis.

3. Energy surfaces of $Z = 120$ isotopes and $N = 184$ isotones

Figures 1 and 2 present the total HFB energy of even–even superheavy $Z = 120$ isotopes with number of neutrons from $N = 160$ to 190. For each isotope except $^{300}120_{160}$, one can see two paths leading to fission: a reflection-symmetric path with the elongated fission fragments (sEF) (open circles) and a reflection-asymmetric path with the elongated fission fragments (aEF) (dashed curves). The axially symmetric sEF ($\gamma = 0°$) fission paths are marked by the dash-dotted thin curves. The differences between the total HFB energies calculated along sEF and sEF with $\gamma = 0°$ paths indicate dependence of fission barriers on triaxiality. Furthermore, Figs. 1 and 2 show that with increasing number of neutrons, barrier heights increase and reach a maximal value of ~ 10 MeV for $N = 180$ and 182.

For neutron-deficient isotopes with number of neutrons N from 160 to 166, the ground state lies in the region of $Q_{20} \sim -50$ b. This means that these nuclei are super-oblate-deformed, see Ref. [7]. For the next group of six nuclei, *i.e.* with number of neutrons from $N = 168$ to 178, there exist two minima, the ground state minimum in the region of $Q_{20} \sim -25$ b and the second one in the region of $Q_{20} \sim +25$ b. It indicates the transitional nature of the nuclei. For the next nucleus, with $N = 180$ neutrons, we have another situation, in which the barrier between two minima disappears. This flat-bottom spherical potential allows the mixture of oblate, spherical, and prolate shapes at the ground state, and can be viewed as the E(5) critical-point solution [8] in the interacting boson approximation. All of the next isotopes (with $N \geq 182$) have spherical shapes in the ground state.

Figure 3 presents the total HFB energy but for even–even superheavy $N = 184$ isotones with number of protons from $Z = 106$ (seaborgium) to 126. All of them have spherical ground state minima and the double-humped fission barriers (with inner and outer maxima). Moreover, one can see that with the increase of protons number Z, the inner-barrier height rises and for $Z = 114$ to 124, this barrier reaches the maximum value of ~ 10 MeV (taking into account the effect of the lowering of the barrier height due to triaxiality). Each of the examined isotones, except the seaborgium, exhibit two fission paths: sEF and aEF. For $Z \geq 116$, the aEF paths have substantially lower outer barriers, which means that these isotones favor the asymmetric fission path.

The total HFB energy surfaces in Figs. 1–3 are presented for the quadrupole moment $Q_{20} \geq -75$ b only. If the magnitude of oblate Q_{20} deformation increases, the oblate spheroidal shapes of nuclei alter to the biconcave disc shapes and for even greater oblate deformations, a new family of toroidal shapes emerges, see Fig. 2 in Ref. [3]. In Figs. 4 and 5, we explore the shape of the energy surface of even–even $Z = 120$ isotopes with $N = 160$ to 190 in the extremely oblate configuration by increasing the magnitude

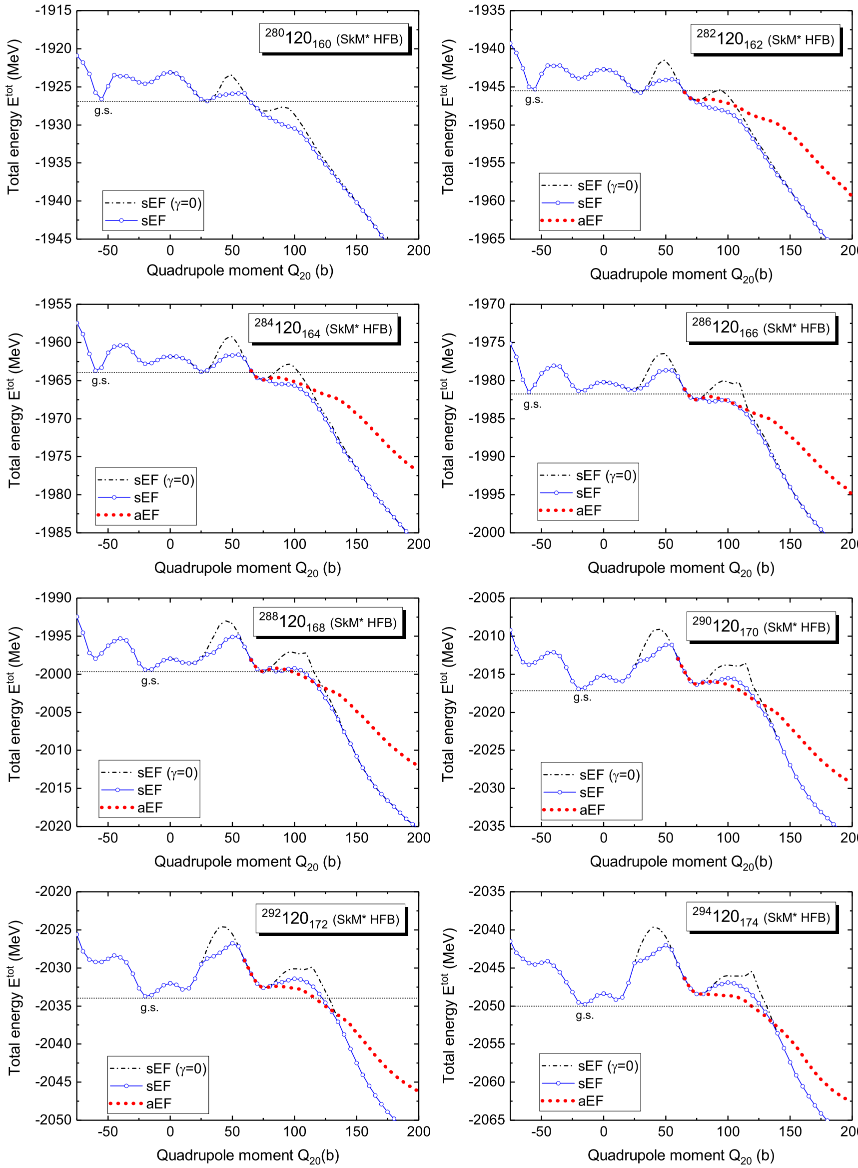

Fig. 1. (Color online) Total HFB energy as a function of the quadrupole moment for the even–even $Z = 120$ isotopes with number of neutrons from $N = 160$ to 174. The open circular points (blue) and short dashed (red) lines show the symmetric (sEF) and asymmetric (aEF) elongated fission pathways, respectively. The axially symmetric sEF ($\gamma = 0°$) fission pathways are marked by the dash-dotted curves.

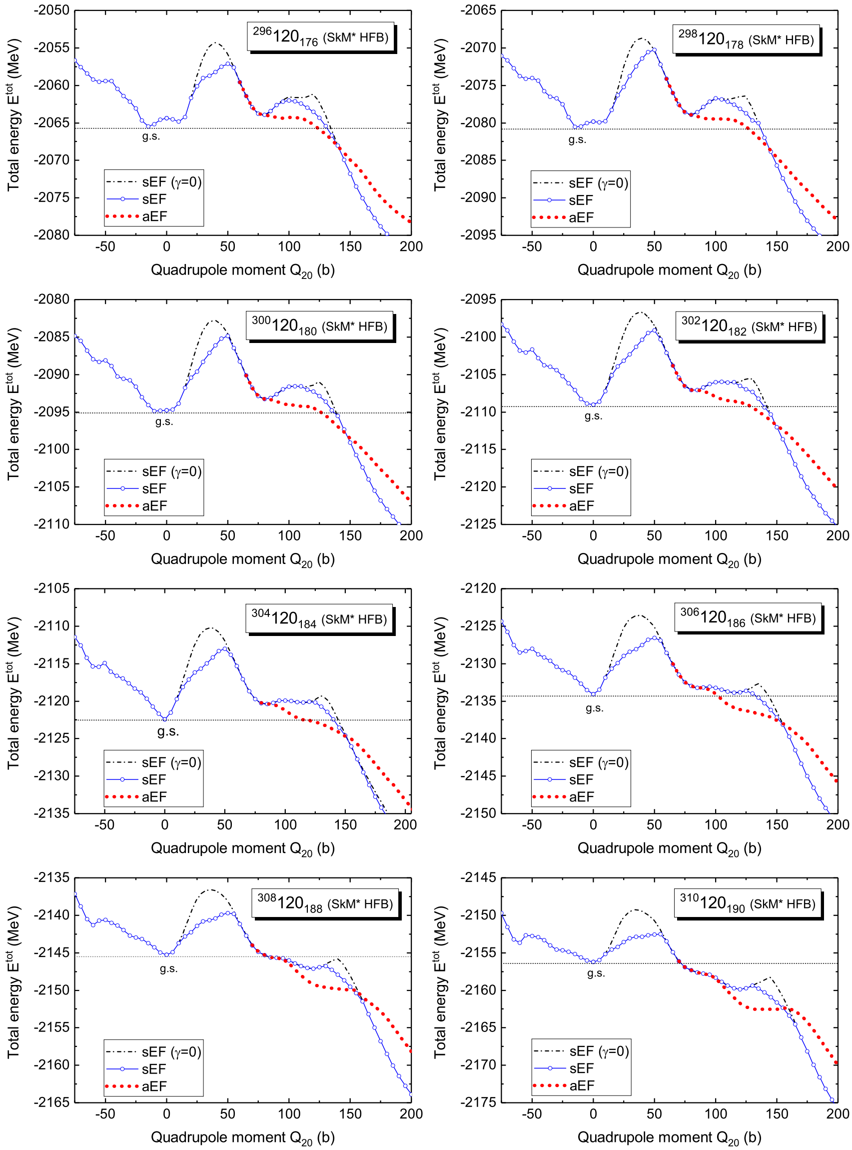

Fig. 2. (Color online) The same as Fig. 1, but for number of neutrons from $N = 176$ to 190.

of the constrained (negative) quadrupole moment ($Q_{20} \geq -300$ b). For all isotopes with $N \geq 166$, the figures show how the energy surfaces vary as the shape makes a transition from biconcave disc to the toroidal shape at the region of large oblate deformation $Q_{20} \lesssim -160$ b.

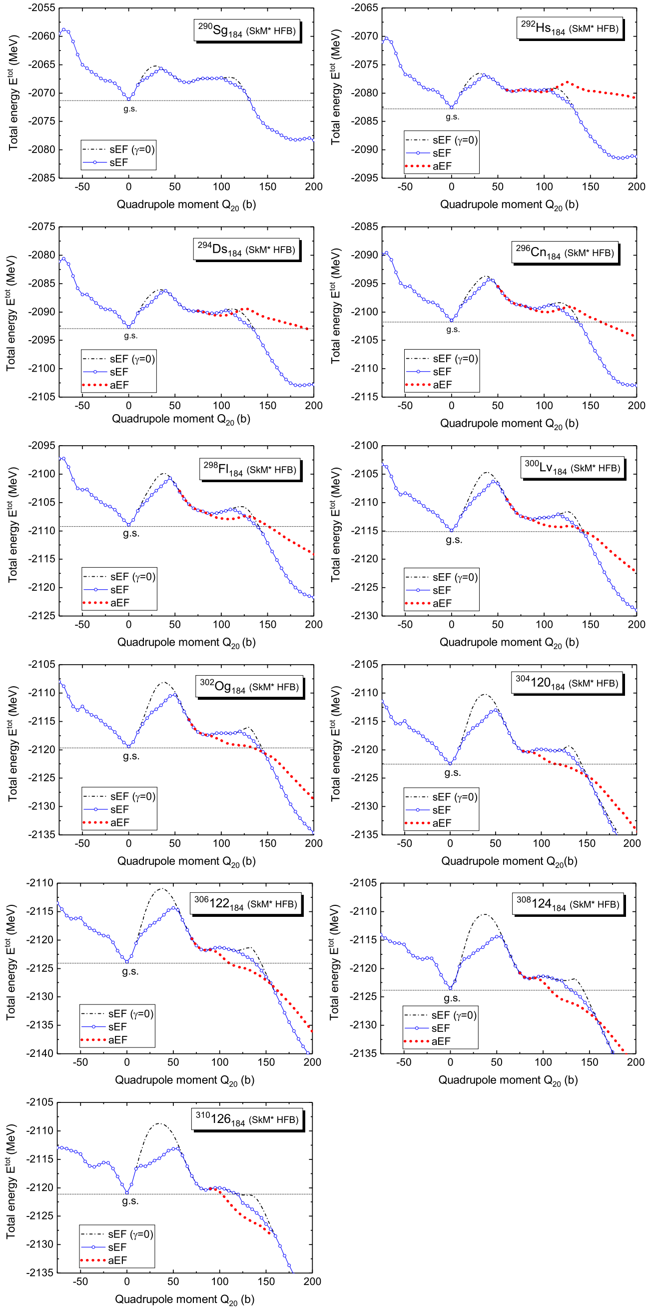

Fig. 3. (Color online) The same as Figs. 1 and 2, but for even–even $N = 184$ isotones with number of protons from $Z = 106$ to 126.

In addition, Figs. 4 and 5 present the energy difference, ΔE, between the ground state and the energy point where two topological solutions (biconcave and toroidal) have the same energy. The value of ΔE increases with the increasing number of neutrons until the isotope $^{302}120_{182}$, with the value $\Delta E = 54.6$ MeV, and then it starts to slowly decrease.

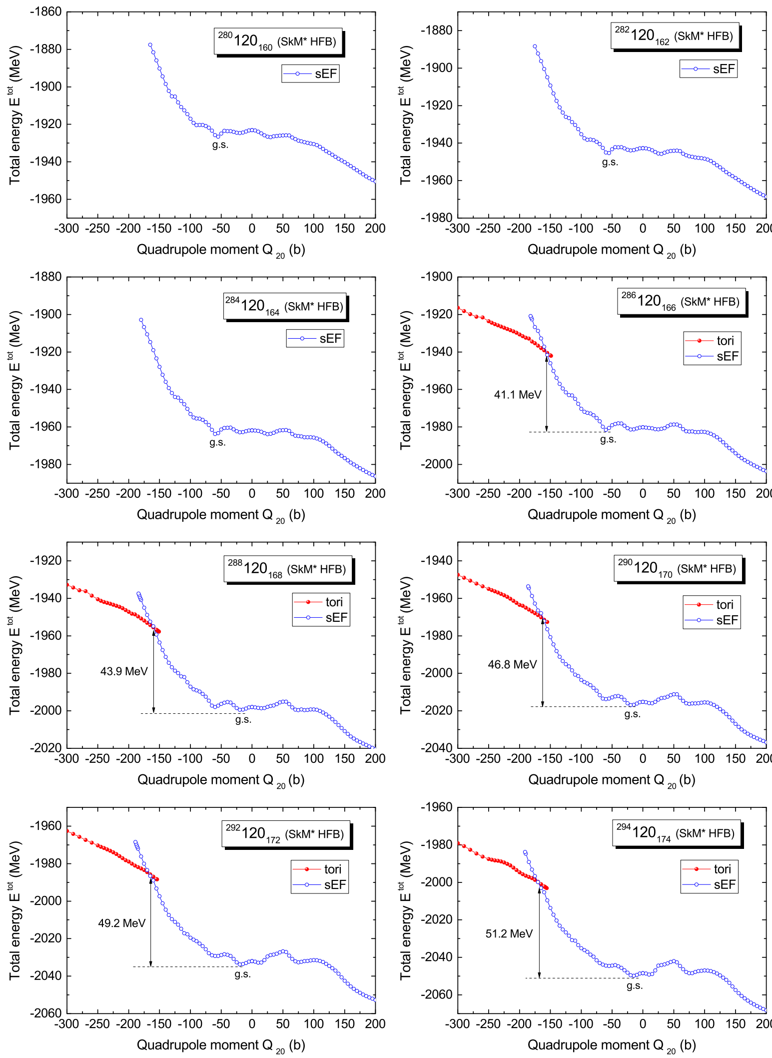

Fig. 4. (Color online) Total HFB energy of even–even $Z = 120$ isotopes with $N = 160$ to 174 as a function of the quadrupole moment. The open circular points (black/blue) show the symmetric elongated fission (sEF) pathways. The nuclear matter density distributions with toroidal shapes appear at the region of large oblate deformation $Q_{20} \lesssim -160$ b (solid grey/red circles).

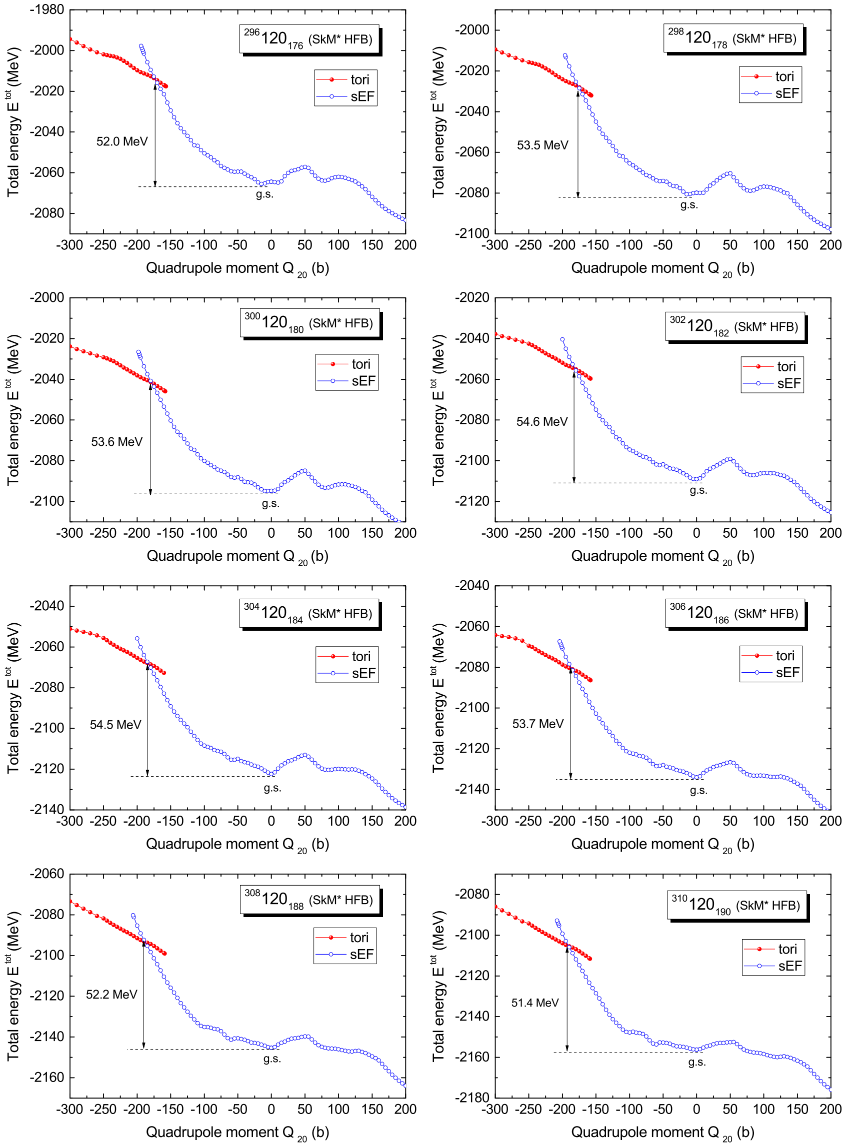

Fig. 5. (Color online) The same as Fig. 4, but for number of neutrons from $N = 176$ to 190.

Figure 6 shows the same as Figs. 4 and 5, but for even–even $N = 184$ isotones with number of protons from $Z = 106$ to 126. One can see that when the number of protons increases the energy difference, ΔE, rapidly decreases, from 78.1 to 38.7 MeV, and furthermore, the toroidal energy curves become oriented more horizontally.

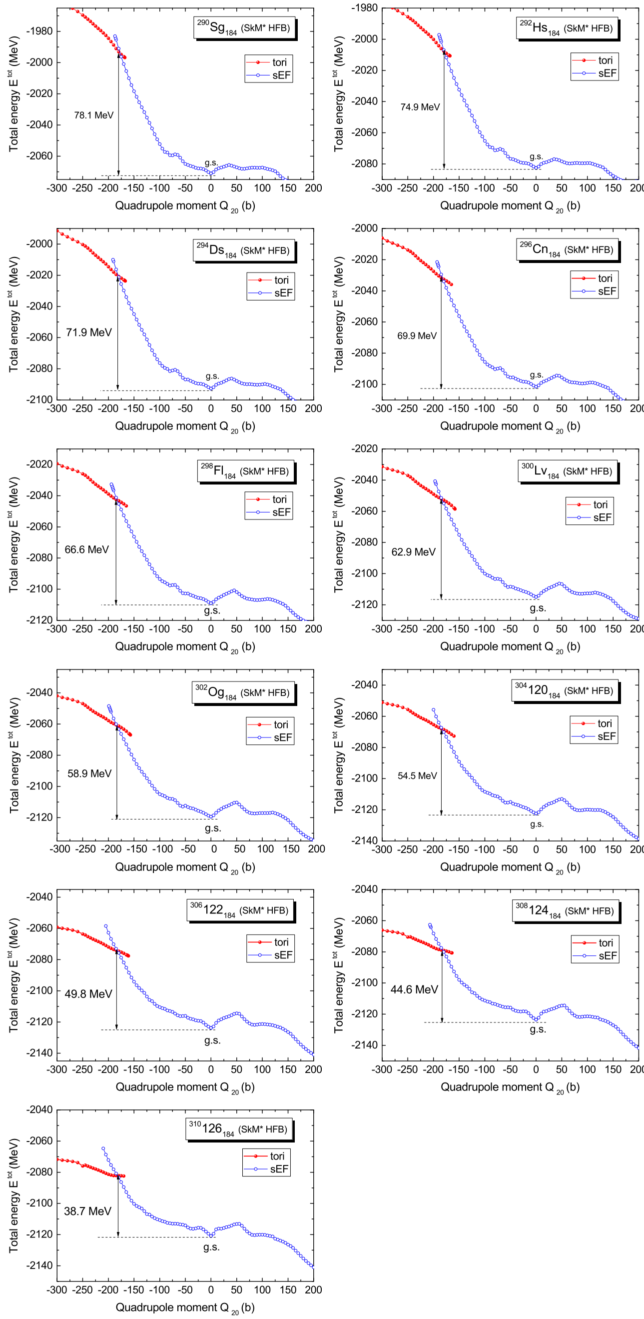

Fig. 6. (Color online) The same as Figs. 4 and 5, but for even–even $N = 184$ isotones with number of protons from $Z = 106$ to 126.

4. Toroidal high-spin isomers

From the results shown in Figs. 4–6 for the $Z = 120$ isotopes and the $N = 184$ isotones under consideration, it is important to note that the total energy curves in the toroidal configurations lie on a slope as a function of Q_{20}, and there is no energy minimum there. So without an angular momentum, these toroidal configurations are unstable and have a tendency to return to a sphere-like geometry. However, the stability of a toroidal nucleus may be enhanced if it possesses an angular momentum about the symmetry axis I_z. Because a quantal axially-symmetric toroid cannot collectively rotate about its symmetry axis, only non-collective rotations, such as those arising from particle–hole excitations are possible in quantum mechanics.

There are two equivalent ways to construct a high-spin state with spin aligned along the symmetry axis, as presented previously in [3], where we found THSI nucleus $^{304}120_{184}$ with $I_z = 81$ and $208\,\hbar$. Using similar methods to explore neighboring even–even nuclei in the (Z, A) space, we obtain the energy curves of toroidal density distributions with different angular momentum components I_z aligned along the symmetry axis. In Fig. 7, we show sections of the deformation energy surfaces of isotones $^{302}\mathrm{Og}_{184}$ and $^{306}122_{184}$ as a function of quadrupole moment Q_{20} for different aligned angular momenta.

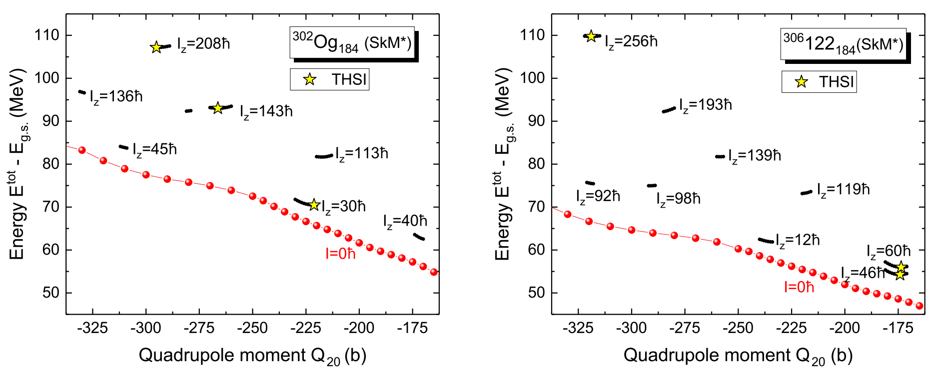

Fig. 7. (Color online) The deformation energies of $^{302}\mathrm{Og}_{184}$ (left panel) and $^{306}122_{184}$ (right panel) as a function of the quadrupole moment Q_{20} for different aligned angular momenta $I = I_z$. The locations of the toroidal high-spin isomers (THSIs) $^{302}\mathrm{Og}_{184}(I_z = 30, 143, 208\,\hbar)$ and $^{306}122_{184}(I_z = 46, 60, 256\,\hbar)$ are indicated by star symbols. All deformation energies are measured relative to the energy of the spherical ground state.

We find that while many energy curves for different aligned angular momenta do not possess an energy minimum, there are specific aligned angular momenta for which the energy curves show localized energy minima. These energy minima are indicated by the star symbols in Fig. 7 for THSI $^{302}\mathrm{Og}_{184}$ with $I_z = 30$, 143, 208 $\hbar$, and $^{306}122_{184}$ with $I_z = 46$, 60, 256 $\hbar$. Figure 8 represents the same situation as Fig. 7, but for isotopes $^{302}120_{182}$ and $^{306}120_{186}$ for which THSIs exist for $I_z = 111\,\hbar$ and $I_z = 117$, 218 $\hbar$, respectively.

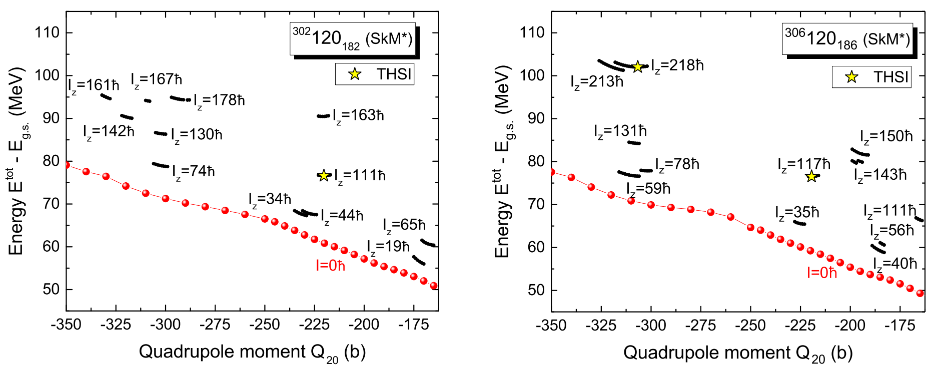

Fig. 8. (Color online) The same as Fig. 7, but for $^{302}120_{182}(I_z = 111\,\hbar)$ (left panel) and $^{306}120_{186}(I_z = 117, 218\,\hbar)$ (right panel) THSIs.

The properties of found THSI states are tabulated in Table I, where $\hbar\omega_z$ is the Lagrange multiplier (cranking frequency) and E^* denotes the energy measured relative to the energy of the spherical ground state.

TABLE I

Properties of the toroidal high-spin isomers (THSIs) of $^{302}\mathrm{Og}_{184}$, $^{306}122_{184}$, $^{302}120_{182}$, and $^{306}120_{186}$.

Z	N	$I_z = I_z^{\mathrm{proton}} + I_z^{\mathrm{neutron}}$ [$\hbar$]	Q_{20} [b]	$\hbar\omega_z$ [MeV]	E^* [MeV]
118	184	$30 = 15 + 15$	-221.2	0.05	70.5
		$143 = 59 + 84$	-266.3	0.20	93.0
		$208 = 79 + 129$	-295.1	0.30	107.2
122	184	$46 = 14 + 32$	-174.1	0.10	54.3
		$60 = 14 + 46$	-173.6	0.15	56.0
		$256 = 98 + 158$	-318.9	0.32	109.7
120	182	$111 = 44 + 67$	-220.3	0.20	76.6
120	186	$117 = 40 + 77$	-219.4	0.20	76.5
		$218 = 79 + 139$	-306.7	0.30	102.0

5. Conclusions and discussions

We examine here properties of the superheavy nuclei of even–even $Z = 120$ isotopes and $N = 184$ isotones. The obtained results for $Z = 120$ isotopes show the change of position of the ground state with the increase of the number of neutrons from $N = 160$ to 190. The most neutron-deficient isotopes form the group of the super-oblate-deformed nuclei, for $N = 168$ to 178, there exists the transitional region of nuclei with two oblate and prolate minima separated by small barrier. This barrier disappears for $N = 180$ and the flat-bottom potential allows the mixture of oblate, spherical, and prolate shapes at the ground state. Finally, all next isotopes with $N \geq 182$ are spherical in their ground states. In the case of $N = 184$ isotones with spherical ground states and the double-humped fission barriers, it is found that with the increase of the number of protons, the inner-barrier height rises and reaches the value of ~ 10 MeV for $Z = 114$ to 124. Moreover, for majority of the $Z = 120$ isotopes and $N = 184$ isotones, one can observe competition between two paths leading to fission: one with the symmetric elongated fragments (sEF) and the second with asymmetric elongated fragments (aEF).

Focusing our attention on the extremely oblate region, we search for toroidal high-spin isomers in the neighborhood of $^{304}120_{184}$ where toroidal high-spin isomers have been located previously in theoretical calculations [3]. We find that the neighboring even–even $N = 184$ isotone with $Z = 118$ and 122, as well as the $Z = 120$ isotopes with $N = 182$ and 186, also possess toroidal high-spin isomers at various angular momentum components and quadrupole moments. The occurrence of toroidal high-spin isomers may appear to be rather common in the superheavy nuclei region.

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