QUANTUM MECHANICAL DESCRIPTION OF PROCESSES WITH DELAYED CHOICES*

Marek Góźdź, Andrzej Góźdź

Faculty of Mathematics, Physics and Computer Science
Maria Curie-Skłodowska University
pl. Marii Curie-Skłodowskiej 1, 20-031 Lublin, Poland

(Received December 28, 2017)

We present a description of Wheeler's delayed choice experiment. Unlike the usual approach, we concentrate on the possible time structure of the process. We stress that both the test particle and the apparatus have non-trivial temporal parts of their wave functions, which opens a new channel of interaction. Our calculations show that the temporal overlap between the quantum states is enough to account for the observed results. The description is based on the formalism of the projection time evolution.

DOI:10.5506/APhysPolBSupp.11.29

1. Time in physical models and the delayed choice experiments

Almost all physical models treat time as a kind of background. We either assume that time is not needed to describe the physical system or that it is easily accessible and common for each of the subsystems. The only exception is the theory of general relativity which rests on the assumption that time is a coordinate. Being the fourth dimension, it forms, together with the three spatial coordinates, the space-time.

Even though the relativistic approach is well-established, it is not easy to incorporate its ideas in other models. Traditional quantum mechanics is a good example of a theory within which time cannot be described unless one rewrites it as a relativistic field theory. The difficulty was pointed out by Pauli who showed [1] that a self-adjoint time operator, canonically conjugate with the Hamiltonian H, should have a continuous spectrum even for discrete or semibounded H. He concluded that there is no possibility to build the time operator, hence time in quantum mechanics cannot be an observable. The assumptions used by Pauli are now believed to be too strong (see, *e.g.*, [2]) and work was done in order to consistently describe

* Presented at the XXIV Nuclear Physics Workshop "Marie and Pierre Curie", Kazimierz Dolny, Poland, September 20–24, 2017.

the time evolution of quantum systems [3,4]. This research is in compliance with some of the experiments which suggest that quantum objects are not only extended in space, but also in time [5].

Let us assume that a particle has to penetrate a barrier, with both of these objects being extended in space and time. An example profile of such a barrier, consisting of Gauss functions in the x and t directions, is shown in Fig. 1. The $t = 0$ path leads through the highest value of the blocking potential, but for a time-extended particle, the preferred path may cross the barrier at slightly "earlier" or "later" times, like $t = \pm 1$. This is, of course, impossible if we analyze the situation on infinitely thin time slices, *i.e.*, we travel in time without moving from $t = 0$. Unfortunately, both the theoretical and experimental research on the time structure of the elementary processes is difficult.

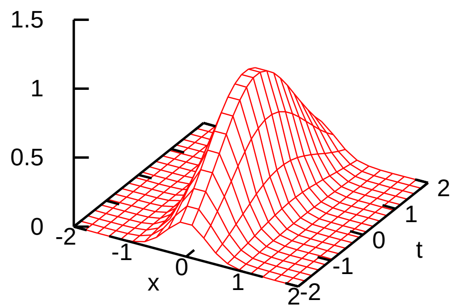

Fig. 1. An example profile of a space-time extended barrier in two dimensions. The preferred way of penetrating the barrier may lead through $t \neq 0$ paths. (Arbitrary units.)

One of the characteristic features of the quantum theory is that every object is represented by its wave function ψ and as a wave it is spatially extended. This spatial delocalization implies the possibility of detecting the particle in different places of the experimental setup. To prove it, the so-called delayed-choice experiments have been proposed and conducted. The main idea of such an experiment assumes that alterations to the experimental setup are introduced after the particle has (classically) passed this part of the apparatus. The question was: will the particle's quantum state change according to the alterations despite the fact that they should be causally disconnected? The answer was always positive and the explanation was always based on the fact that the particle's wave function ψ must have been spatially wide enough to "see" the setup change. A consistent approach should, however, take into account also the spread of ψ in the time direction and the possibility of interaction with the setup via this channel. We present such a description in the next section.

2. The Wheeler's Gedankenexperiment

In 1978, Wheeler proposed a delayed choice Gedankenexperiment [6]. This idea was realized experimentally [7] confirming that the particle respected the changes made to the setup.

Wheeler proposed to use a Mach–Zender interferometer, depicted in the left panel of Fig. 2, in which the second beamsplitter (BS2) can be inserted or removed in the region X after the particle has moved past the first beamsplitter (BS1). In the case the region X is empty during this experiment, the particle should behave classically — it will go either along the 1 or 2 path, directed randomly by the 50/50 BS1. If there is a second beamsplitter in X, the particle will travel between BS1 and BS2 along both ways, and there will be an interference between these two states in BS2. Both situations can be distinguished by looking at the outcome from the detectors. What will happen, however, if BS2 is inserted or removed after the particle passes BS1? Will the particle change its state before it reaches one of the detectors? The experimental verification showed that the particle indeed changes its behavior according to the changes introduced in the setup.

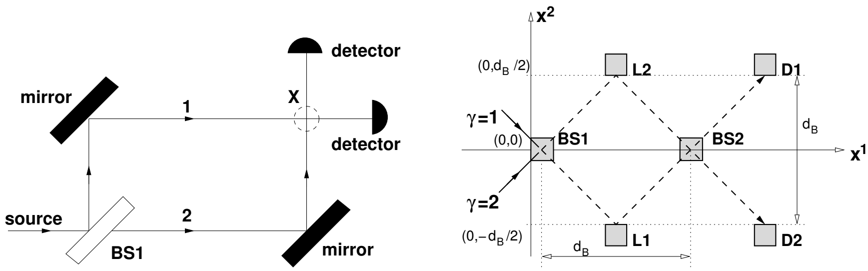

Fig. 2. Left: Wheeler's delayed choice experiment setup. BS1 is a 50/50 beamsplitter, the mirrors are 100% reflective. The region X may be influenced by inserting and removing a second beamsplitter (BS2) from it. Right: The geometrical scheme of the setup. γs denote two possible entrance channels, L1 and L2 are the mirrors, and D1 and D2 denote the detectors.

Let us now describe Wheeler's experiment focusing on the temporal part. We represent the particle's state by its density matrix $\rho(\tau, \nu)$, where ν is a set of quantum numbers and τ numbers the subsequent steps of the evolution of ρ. The parameter τ should not be mistaken with time, as it is not a physical quantity but merely a counter that labels the steps of the evolution

$$\rho(\tau_0, \nu_0) \to \rho(\tau_1, \nu_1) \to \cdots \to \rho(\tau_n, \nu_n) \,. \tag{1}$$

The evolution itself is driven by the family of operators $\mathbb{E}$ such that if the initial density matrix is $\rho(\tau_0, \nu_0)$, the subsequent states are constructed in the following way [4]:

$$\rho(\tau_1,\nu_1) \;=\; \frac{\mathbb{E}_1\rho(\tau_0,\nu_0)\mathbb{E}_1^\dagger}{\mathrm{Tr}\left[\mathbb{E}_1\rho(\tau_0,\nu_0)\mathbb{E}_1^\dagger\right]}\,, \tag{2}$$

$$\rho(\tau_2,\nu_2) \;=\; \frac{\mathbb{E}_2\rho(\tau_1,\nu_1)\mathbb{E}_2^\dagger}{\mathrm{Tr}\left[\mathbb{E}_2\rho(\tau_1,\nu_1)\mathbb{E}_2^\dagger\right]} = \frac{\mathbb{E}_2\mathbb{E}_1\rho(\tau_0,\nu_0)\mathbb{E}_1^\dagger\mathbb{E}_2^\dagger}{\mathrm{Tr}\left[\mathbb{E}_2\mathbb{E}_1\rho(\tau_0,\nu_0)\mathbb{E}_1^\dagger\mathbb{E}_2^\dagger\right]}\,, \tag{3}$$

$$\rho(\tau_n,\nu_n) \;=\; \frac{\mathbb{E}_n\mathbb{E}_{n-1}\ldots\mathbb{E}_1\rho(\tau_0,\nu_0)\mathbb{E}_1^\dagger\ldots\mathbb{E}_n^\dagger}{\mathrm{Tr}\left[\mathbb{E}_n\mathbb{E}_{n-1}\ldots\mathbb{E}_1\rho(\tau_0,\nu_0)\mathbb{E}_1^\dagger\ldots\mathbb{E}_n^\dagger\right]}\,. \tag{4}$$

Here, the denominators are normalization factors. As one can see, the evolution of the quantum state may be realized by projections of the previous state onto the space of all possible new states, with $\mathbb{E}$ being the projection operators. This procedure introduces randomness in the choice of the next state. With the specific choice of the $\mathbb{E}$ operators, the unitary Schrödinger as well as the Dirac and Klein–Gordon evolutions can be obtained. For the particle's initial wave function ψ,

$$\psi(x) = \psi\left(x^0,\vec{x}\right) = \xi\left(x^0\right)\;\eta\left(\vec{x}\right)\,, \tag{5}$$

the density matrix reads $\rho(\tau_0,\nu_0) = |\psi\rangle\langle\psi|$.

Our problem is three-dimensional, with two spatial coordinates x^1 and x^2, and one time coordinate x^0 (see Fig. 2, right panel). Without the loss of generality, we assume that the whole setup is geometrically symmetric, which means that it takes the particle the same time to move through each segment of the setup. We denote this time by T_0 and we expect the particle to appear in the detectors around the time $3T_0$.

The evolution operator representing the 50/50 beamsplitter, neglecting all possible phase changes, should randomly choose one of the two outputs, creating in the quantum case an output state in the form of an interference between both channels. The operator is assumed to act on the spatial part of the density matrix only and reads

$$M = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\,. \tag{6}$$

Unlike the original proposition, we assume that BS2 is always present but we can manipulate the BS1, which is further away from the detectors. Each beamsplitter can either be missed, in which case the particle leaves the setup and does not reach any of the detectors, or hit. In the latter case, the particle is being transmitted or reflected, both with 50% probability. We introduce functions $\psi_m^{(\gamma)}(x)$ which describe all possible situations

$$\psi_1^{(1)}(x) = 0\,, \tag{7}$$

$$\psi_2^{(1)}(x) = 0\,, \tag{8}$$

$$\psi_3^{(2)}(x) = 0\,, \tag{9}$$

$$\psi_4^{(\gamma)}(x) = \frac{N_D}{\sqrt{2}}\left[1 - \chi_{B_1^T+3T_0}(t)\right]\ \xi(t-3T_0)\ \eta_{D_\gamma^x}(\vec{x}\,)\,, \tag{10}$$

$$\psi_5^{(\gamma)}(x) = \frac{N_D}{2}\chi_{B_1^T+3T_0}(t)\ \xi(t-3T_0)\ \eta_{D_\gamma^x}(\vec{x}\,)\,, \tag{11}$$

$$\psi_6^{(\gamma)}(x) = (-1)^\gamma\frac{N_D}{2}\chi_{B_1^T+3T_0}(t)\ \xi(t-3T_0)\ \eta_{D_\gamma^x}(\vec{x}\,)\,, \tag{12}$$

where N_D are the normalization factors and $\chi_\alpha(t)$ denotes the projection on the time interval α. In our case, B_1^T is the time interval in which BS1 is present. Discretizing time $t = n\delta_T$, with $n \in Z$, and denoting by $D_{\gamma n}$ the space-time interval occupied by the detector Dγ, the probability pev(γ, n) of detecting the particle by the γ detector in the time interval n is given by

$$\mathrm{pev}(\gamma,n) \;=\; \frac{1}{\mathcal{N}}\int\limits_{D_{\gamma n}} \mathrm{d}^3x \left|\sum_{m=1}^{6}\psi_m^{(\gamma)}(x)\right|^2\,, \tag{13}$$

$$\mathcal{N} \;=\; \sum_{n=-\infty}^{\infty}\ \sum_{\gamma=1,2}\ \int\limits_{D_{\gamma n}} \mathrm{d}^3x \left|\sum_{m=1}^{6}\psi_m^{(\gamma)}(x)\right|^2\,. \tag{14}$$

Using (7)–(12), the general formula can be rewritten as

$$\mathrm{pev}(\gamma,n) = \frac{1}{2}\int\limits_{(n-\frac{1}{2})\delta_T}^{(n+\frac{1}{2})\delta_T} \mathrm{d}t\left[1 + (-1)^\gamma\chi_{B_1^T+3T_0}(t)\right]\ |\xi(t-3T_0)|^2\,. \tag{15}$$

One sees that due to assumption (5) about the separability of the wave function, the spatial part does not play any role in our description, with the interference taking place in the time domain only.

As an example, let us assume that the temporal part of the photon has the form of a Gauss function. In the first case (Fig. 3), the photon interacts with BS1. The superposition of states going along the $\gamma = 1$ and $\gamma = 2$ paths reaches BS2 and interferes destructively (constructively) in the D1 (D2) direction. The maximum detection probability is at $n = 15$, which numerically corresponds to the time $3T_0$.

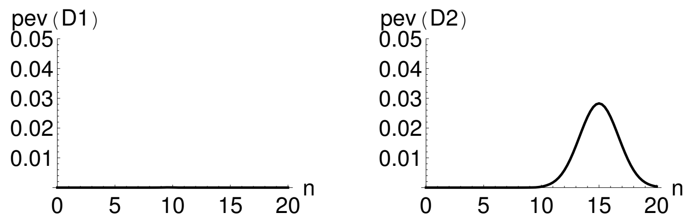

Fig. 3. BS1 has a time overlap with the photon. Both beamsplitters work creating a constructive interference in D2 and a destructive one in D1.

The second case shows the situation in which the photon does not interact with BS1 due to the lack of an overlap between the time intervals occupied by the particle and the beamsplitter. The only interaction comes from BS2, which uniformly distributes the photon to both detectors. Two equal probability distributions can be seen in Fig. 4.

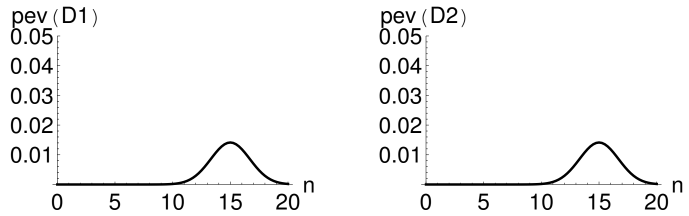

Fig. 4. BS1 has no time overlap with the photon. Only BS2 works redirecting the photon to both detectors.

Figure 5 presents a situation in which the photon's wave function is wide in the temporal direction. BS1 is being switched on for a short period. The shape of the probability distributions clearly shows that the photon changes its state, reacting to the manipulations with BS1.

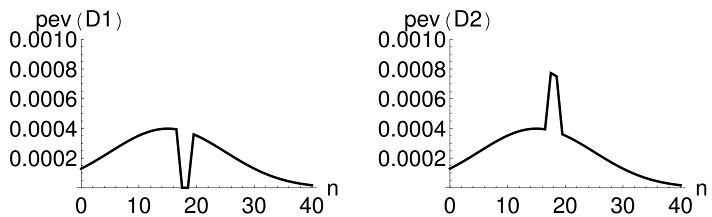

Fig. 5. The photon occupies a wide time interval, BS1 is switched on for a short period. The change of photon's behavior is readily visible.

REFERENCES

[1] W. Pauli, *Handbuch der Physik*, vol. V/1, Springer-Verlag, Berlin 1926.

[2] E.A. Galapon, *Proc. R. Soc. Lond. Ser. A* **458**, 451 (2002).

[3] J.G. Muga, R. Sala Mayato, I.L. Egusquiza (Eds.), *Time in Quantum Mechanics*, *Lect. Notes Phys.* **734**, Springer-Verlag, 2008; J.G. Muga, A. Ruschhaupt, A. del Campo (Eds.), *Time in Quantum Mechanics — Vol. 2*, *Lect. Notes Phys.* **789**, Springer-Verlag, 2009.

[4] A. Góźdź, M. Dębicki, K. Stefańska, *Phys. At. Nucl.* **71**, 1 (2008); A. Góźdź, K. Stefańska, *J. Phys.: Conf. Ser.* **104**, 012007 (2008); M. Dębicki, A. Góźdź, K. Stefańska, *Int. J. Mod. Phys. E* **16**, 616 (2007); A. Góźdź, M. Dębicki, *Phys. At. Nucl.* **70**, 529 (2007); M. Dębicki, A. Góźdź, *Int. J. Mod. Phys. E* **15**, 437 (2006).

[5] U. Hauser, W. Neuwirth, N. Thesen, *Phys. Lett. A* **49**, 57 (1974); F. Lindner *et al.*, *Phys. Rev. Lett.* **95**, 040401 (2005).

[6] J.A. Wheeler in: A.R. Marlow (ed.), *Mathematical Foundations of Quantum Theory*, Academic Press, 1978.

[7] A.G. Manning *et al.*, *Nature Phys.* **11**, 539 (2015).