

GAMMA TUNNELING IN NUCLEI*

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Tunneling along the γ path between prolate and oblate axial shapes of even–even nuclei, being in the ground state, is analyzed. Mixing of two 0^+ states and splitting of their energies are calculated in the quasi-classical approximation. Expressions for the E0 transition rates between these 0^+ levels are derived and compared with the experiment for Kr isotopes. Mixing of the prolate and oblate shapes in these nuclei is found to reach 50%. Tunneling between states of the nucleus with the same axial shape but with the angular momentum parallel and perpendicular to the symmetry axis is also studied. The hindrance factor for decay of the 25^+ isomeric level of ^{182}Os , provided by γ tunneling, is estimated.

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1. Introduction

The potential energy of many nuclei has several minima, corresponding to different shapes. In some cases, when the barriers separating such minima are sufficiently low, the shapes can be mixed. In particular, theoretical calculations [1] have shown that the energy surface of even–even isotopes of Kr, as a function of the quadrupole parameters β, γ , has minima at $\gamma = 0$ and $\gamma = \pi/3$, corresponding, respectively, to prolate and oblate axially symmetric shapes. The deformation parameter β remains almost the same in both potential wells. The barrier, separating them along the γ direction, occurs to be shallow, so that just the γ motion ensures mixing of the shapes and splitting of the 0^+ levels in the ground state of Kr nuclei. The electron conversion experiments [2–4], measuring intensities of the electric monopole transitions between these 0^+ levels, revealed their considerable enhancement, which was attributed to the prolate–oblate mixing.

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More extraordinary example of the γ tunneling is represented by the K isomer 25^+ of ^{182}Os with the energy of 7049.5 keV. It has a uniquely short half-life of 150 ns and decays with probability 6% into the ground band level $I^\pi = 24^+$, $K = 0$ [5]. The corresponding lifetime $\tau \approx 0.4 \times 10^{-5}$ s, while the Weisskopf estimation for M1 transition only gives 0.3×10^{-13} s. So the hindrance factor is $F_H \approx 10^{-8}$. Such swift decay was explained [6] by mixing of two states, corresponding to orientation of the total angular momentum along the symmetry axis and perpendicularly to it. The mixing is realized by a barrier tunneling in γ direction between the wells at $\gamma = 0$ and $\gamma = 2\pi/3$, where $K = 25$ and $K = 0$, respectively. The admixture of the state with $K = 0$ in the isomer wave function has been numerically calculated in [7], taking into account pairing correlations of nucleons.

We shall consider γ tunneling between the prolate and oblate shapes as well as two prolate shapes of ^{182}Os , directed perpendicularly to each other, and having the same spin $I = 25$ but different projections of the intrinsic angular momenta on the symmetry axis $K = 25$ and $K = 0$. We start from the collective Hamiltonian, derived in [8, 9], which formally coincides with the Bohr–Mottelson Hamiltonian. The corresponding Schrödinger equation is solved quasi-classically.

2. Collective kinetic energy

The relative motion of A nucleons in the center-of-mass system is described by the Jacobi vectors $\vec{\xi}_1, \vec{\xi}_2, \dots, \vec{\xi}_{A-1}$. In order to separate nuclear rotation, we introduce the body-fixed frame with axes x', y', z' directed along the principal axes of the inertia ellipsoid of the nucleus. Then the projections of $\vec{\xi}_i$ on these axes should satisfy the following conditions:

$$\sum_{i=1}^{A-1} \xi_{ix'} \xi_{iy'} = \sum_{i=1}^{A-1} \xi_{ix'} \xi_{iz'} = \sum_{i=1}^{A-1} \xi_{iy'} \xi_{iz'} = 0. \quad (1)$$

Orientation of the body-fixed system is determined by the Euler angles $\theta_1, \theta_2, \theta_3$.

We introduced the Euclidean space of the particle numbers with basis unit vectors $\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_{A-1}$ and vectors

$$\mathbf{A}_{x'} = \sum_{i=1}^{A-1} \xi_{ix'} \mathbf{e}_i, \quad \mathbf{A}_{y'} = \sum_{i=1}^{A-1} \xi_{iy'} \mathbf{e}_i, \quad \mathbf{A}_{z'} = \sum_{i=1}^{A-1} \xi_{iz'} \mathbf{e}_i. \quad (2)$$

Constraint (1) can be treated as an orthogonality condition of these vectors. Another three collective variables are introduced as lengths of the vectors $\mathbf{A}_{x', y', z'}$

$$a = \left(\sum_{i=1}^{A-1} \xi_{ix'}^2 \right)^{1/2}, \quad b = \left(\sum_{i=1}^{A-1} \xi_{iy'}^2 \right)^{1/2}, \quad c = \left(\sum_{i=1}^{A-1} \xi_{iz'}^2 \right)^{1/2}, \quad (3)$$

while the remaining $3A-9$ variables as generalized Euler angles, which define orientation of the vectors $\mathbf{A}_{x',y',z'}$ in the abstract space.

It is convenient to transform a, b, c to new variables ρ, β, γ

$$\begin{aligned} a &= \frac{\rho}{\sqrt{3}} \left[1 + \beta \cos \left(\gamma - \frac{2\pi}{3} \right) \right], \\ b &= \frac{\rho}{\sqrt{3}} \left[1 + \beta \cos \left(\gamma + \frac{2\pi}{3} \right) \right], \\ c &= \frac{\rho}{\sqrt{3}} [1 + \beta \cos \gamma]. \end{aligned} \quad (4)$$

Here,

$$\rho = (a^2 + b^2 + c^2)^{1/2} \quad (5)$$

denotes the hyperradius of the nucleus, and the coordinates β and γ define, respectively, the deformation and triaxiality of the inertia ellipsoid.

The kinetic energy operator of the nucleus has been expressed in these coordinates in [8]. Its collective part has the form of

$$\begin{aligned} \hat{T}_{\text{coll}} &= \hat{T}_\rho - \frac{\hbar^2}{2B(\rho)} \left[\frac{1}{\beta^4} \frac{\partial}{\partial \beta} \beta^4 \frac{\partial}{\partial \beta} + \frac{1}{\beta^2 \sin 3\gamma} \frac{\partial}{\partial \gamma} \sin 3\gamma \frac{\partial}{\partial \gamma} \right] \\ &+ \frac{\hbar^2}{8B(\rho)\beta^2} \left[\frac{\hat{I}_{x'}^2}{\sin^2(\gamma - 2\pi/3)} + \frac{\hat{I}_{y'}^2}{\sin^2(\gamma + 2\pi/3)} + \frac{\hat{I}_{z'}^2}{\sin^2 \gamma} \right], \end{aligned} \quad (6)$$

where $\hat{\mathbf{I}}$ is the total angular momentum operator,

$$\hat{T}_\rho = -\frac{\hbar^2}{2m} \frac{1}{\rho^{3A-4}} \frac{\partial}{\partial \rho} \rho^{3A-4} \frac{\partial}{\partial \rho} \quad (7)$$

is the kinetic energy operator for the monopole vibrations, the mass function is

$$B(\rho) = \frac{1}{2} m \rho^2, \quad (8)$$

and m is the mass of the nucleon.

For rigid volume vibrations, the function $B(\rho)$ may be replaced by $B = B(\rho_0)$, depending on the equilibrium value of ρ . Let us compare it with the hydrodynamical one $B_{\text{hydr}} = (3/8\pi) A m R_0^2$, where R_0 is the radius of the

nucleus. For a uniform nucleus with sharp quadrupole surface, ρ_0 and R_0 are related by

$$\rho_0^2 = A \langle r^2 \rangle \simeq 0.6 A R_0^2, \quad (9)$$

where $\langle r^2 \rangle^{1/2}$ is the mean-square radius of the nucleus. Then the mass parameter becomes [9]

$$B = 0.3 A m R_0^2 \approx 2.5 B_{\text{hydr}}. \quad (10)$$

3. Collective motion

Let us consider an even-even nonrotating nuclei ($I = 0$), whose wave function is governed by the equation

$$\begin{aligned} & -\frac{\hbar^2}{2B} \left\{ \frac{1}{\beta^4} \frac{\partial}{\partial \beta} \beta^4 \frac{\partial}{\partial \beta} + \frac{1}{\beta^2 \sin 3\gamma} \frac{\partial}{\partial \gamma} \sin 3\gamma \frac{\partial}{\partial \gamma} \right\} \Psi(\beta, \gamma) \\ & + W(\beta, \gamma) \Psi(\beta, \gamma) = E \Psi(\beta, \gamma). \end{aligned} \quad (11)$$

For every fixed γ , the potential energy $W(\beta, \gamma)$ *versus* the deformation parameter β reaches its local minimum at the point $\beta_0(\gamma)$. As γ varies from 0 to $\pi/3$, the curve $\beta_0(\gamma)$ traces the collective path along the valley connecting prolate and oblate minima. The ends of the path are defined by $\beta_1 = \beta_0(0)$ if $\gamma = 0$ and $\beta_2 = \beta_0(\pi/3)$ if $\gamma = \pi/3$.

Let us expand the potential energy in the Taylor series

$$W(\beta, \gamma) = W(\beta_0(\gamma), \gamma) + \frac{1}{2} C(\gamma) (\beta - \beta_0(\gamma))^2 + \dots, \quad (12)$$

where the coefficient

$$C(\gamma) = \left(\frac{\partial^2 W}{\partial \beta^2} \right)_{\beta=\beta_0(\gamma)} \quad (13)$$

determines stiffness of the local β vibrations. We assume that the local softness parameter

$$\mu(\gamma) = \frac{\beta_{00}(\gamma)}{\beta_0(\gamma)} \ll 1, \quad (14)$$

where the oscillator length

$$\beta_{00}(\gamma) = \sqrt{\frac{\hbar}{B \omega_\beta(\gamma)}} \quad (15)$$

and frequency

$$\omega_\beta(\gamma) = \sqrt{C(\gamma)/B}. \quad (16)$$

This allows us to write the wave function as a product

$$\Psi(\beta, \gamma) = \beta^{-2} \chi_{n_\beta}(\xi_\beta) \psi(\gamma), \quad (17)$$

where the function $\chi_{n_\beta}(\xi_\beta)$ describes harmonic vibrations about local equilibrium position $\beta_0(\gamma)$

$$\xi_\beta = (\beta - \beta_0(\gamma)) / \beta_{00}(\gamma), \quad (18)$$

and $n_\beta(\gamma)$ is the number of β phonons.

Another factor $\psi(\gamma)$ satisfies the equation

$$\left\{ -\frac{\hbar^2}{2M(\gamma)} \frac{1}{\sin 3\gamma} \frac{\partial}{\partial \gamma} \sin 3\gamma \frac{\partial}{\partial \gamma} + U(\gamma) - E \right\} \psi(\gamma) = 0, \quad (19)$$

where

$$M(\gamma) = B\beta_0^2(\gamma) \quad (20)$$

is the effective mass parameter, varying along the collective path,

$$U(\gamma) = W(\beta_0(\gamma), \gamma) + \frac{\hbar^2}{M(\gamma)} + \hbar\omega_\beta(\gamma) \left(n_\beta + \frac{1}{2} \right)$$

is the effective potential.

By substitution

$$\psi(\gamma) = (\sin 3\gamma)^{-1/2} \varphi(\gamma), \quad (21)$$

we transform the Schrödinger equation (19) for γ motion to

$$\left\{ -\frac{\hbar^2}{2M(\gamma)} \frac{\partial^2}{\partial \gamma^2} + V(\gamma) - E \right\} \varphi(\gamma) = 0 \quad (22)$$

with the new effective potential

$$V(\gamma) = U(\gamma) - \frac{9\hbar^2}{8M(\gamma)} \left(1 + \frac{1}{\sin^2 3\gamma} \right). \quad (23)$$

The function $\varphi(\gamma)$ obeys the boundary condition

$$\varphi(0) = \varphi(\pi/3) = 0. \quad (24)$$

4. WKB approximation

We solve Eq. (22) in the quasi-classical approximation (the turning points are denoted by a and b). It is not applicable near the points $\gamma = 0$ and $\pi/3$. Therefore, following [10], we omit the divergent tails of $V(\gamma)$ in these points. For simplicity, we assume that β_0 does not change along the path. Besides, the potential in the region $0 < \gamma < a$ is approximated by the parabola

$$V(\gamma) = \frac{M\omega_\gamma^2}{2}\gamma^2, \quad (25)$$

where the mass parameter $M = B\beta_0^2$, whereas at $b < \gamma < \pi/3$ by

$$V(\gamma) = \frac{M\omega_\gamma^2}{2}(\gamma - \pi/3)^2 + \Delta V_0, \quad (26)$$

where

$$\Delta V_0 = U(\pi/3). \quad (27)$$

The WKB wave function at $0 \leq \gamma < a$ has the form of

$$\varphi(\gamma) = \frac{c_1}{\sqrt{k(\gamma)}} \sin \left(\int_0^\gamma k(\gamma') d\gamma' \right), \quad (28)$$

where

$$k(\gamma) = \sqrt{2M[E - V(\gamma)]}/\hbar. \quad (29)$$

Applying standard matching rules near the turning point a , one finds the wave function under the barrier ($a < \gamma < b$)

$$\begin{aligned} \varphi(\gamma) = & \frac{c_1}{\sqrt{|k(\gamma)|}} \left\{ \cos \left(\phi_1 - \frac{\pi}{4} \right) e^A \exp \left(- \int_\gamma^b |k(\gamma')| d\gamma' \right) \right. \\ & \left. + \frac{1}{2} \sin \left(\phi_1 - \frac{\pi}{4} \right) e^{-A} \exp \left(\int_\gamma^b |k(\gamma')| d\gamma' \right) \right\}, \end{aligned} \quad (30)$$

where the action

$$A = \int_a^b |k(\gamma)| d\gamma \quad (31)$$

and the angles

$$\phi_1 = \int_0^a k(\gamma) d\gamma, \quad \phi_2 = \int_b^{\pi/3} k(\gamma) d\gamma. \quad (32)$$

Inserting (25), (26) into (32), one has

$$\phi_1 = \frac{\pi E}{2\hbar\omega_\gamma}, \quad \phi_2 = \frac{\pi(E - \Delta V_0)}{2\hbar\omega_\gamma}. \quad (33)$$

Approximating also the barrier in the region $a < \gamma < b$ by the inverse parabola

$$V(\gamma) \simeq -\frac{M\omega_B^2}{2} \left(\gamma - \frac{\pi}{6} \right)^2, \quad (34)$$

one finds the well-known expression

$$A = \frac{\pi W}{\hbar\omega_B}, \quad (35)$$

where the barrier height

$$W = V(\pi/6) - E. \quad (36)$$

The wave function in the oblate well ($b < \gamma < \pi/3$) becomes

$$\varphi(\gamma) = \frac{c_1}{\sqrt{k(\gamma)}} \left\{ C_1 \sin \left(\int_\gamma^{\pi/3} k(\gamma') d\gamma' \right) + C_2 \cos \left(\int_\gamma^{\pi/3} k(\gamma') d\gamma' \right) \right\}, \quad (37)$$

where the coefficients are

$$C_1 = 2 \cos \left(\phi_1 - \frac{\pi}{4} \right) \sin \left(\phi_2 - \frac{\pi}{4} \right) e^A + \frac{1}{2} \sin \left(\phi_1 - \frac{\pi}{4} \right) \cos \left(\phi_2 - \frac{\pi}{4} \right) e^{-A}, \quad (38)$$

and

$$C_2 = 2 \cos \left(\phi_1 - \frac{\pi}{4} \right) \cos \left(\phi_2 - \frac{\pi}{4} \right) e^A - \frac{1}{2} \sin \left(\phi_1 - \frac{\pi}{4} \right) \sin \left(\phi_2 - \frac{\pi}{4} \right) e^{-A}. \quad (39)$$

From the boundary condition $\varphi(\pi/3) = 0$, it follows that $C_2 = 0$, *i.e.*,

$$4 \cot \left(\phi_1 - \frac{\pi}{4} \right) \cot \left(\phi_2 - \frac{\pi}{4} \right) = e^{-2A}. \quad (40)$$

Then the wave function takes the form of

$$\varphi(\gamma) = \frac{c_2}{\sqrt{p}} \sin \left(\frac{1}{\hbar} \int_{\pi/3}^{\gamma} p \, d\gamma \right), \quad (41)$$

where the amplitude for the oblate component of the wave function is

$$c_2 = \frac{1}{2} \frac{\sin(\phi_1 - \pi/4)}{\cos(\phi_2 - \pi/4)} e^{-A} c_1. \quad (42)$$

Let the barrier have small transparency, *i.e.*, $e^{-2A} \ll 1$. In the case of $e^{-2A} = 0$, Eq. (40) reduces to the Bohr–Sommerfeld quantization rule

$$\phi_{1(2)}(\epsilon) = (n_{1(2)} + 3/4)\pi, \quad (n_i = 0, 1, 2, \dots), \quad (43)$$

which determines the unperturbed energies ϵ_1, ϵ_2 in both wells.

For the physical (perturbed) energies from Eq. (40), one finds

$$E^{\pm} = \frac{1}{2}(\epsilon_1 + \epsilon_2) \pm \frac{1}{2}\sqrt{\Delta^2 + 4v^2}, \quad (44)$$

where the notations are

$$\Delta = \epsilon_1 - \epsilon_2, \quad v = 2v_0, \quad v_0 = \frac{\hbar\omega_{\gamma}}{2\pi} e^{-A}. \quad (45)$$

The parameter v_0 means the tunneling strength in the case of the potential $V(x)$, having two minima and tending to $+\infty$ when $x \rightarrow \pm\infty$ [8]. However, for γ motion in the finite interval $0 \leq \gamma \leq \pi/3$ it doubles, $v_0 \rightarrow v = 2v_0$.

Setting the energy of the ground state 0_1^+ to be zero, one finds the energy of the first excited 0_2^+ state

$$E_{0_2^+} = \sqrt{\Delta^2 + 4v^2}. \quad (46)$$

The wave functions of this ground 0^+ doublet can be written as a superposition of well-known functions $\varphi_{1(2)}^{(0)}(\gamma)$, which describe γ vibrations in two isolated potential wells:

$$\varphi_{\pm}(\gamma) = c_1^{\pm} \varphi_1^{(0)}(\gamma) + c_2^{\pm} \varphi_2^{(0)}(\gamma). \quad (47)$$

The ratio of their amplitudes is given by

$$R_{\pm} = c_2^{\pm}/c_1^{\pm} = -\frac{2v}{\Delta \pm \sqrt{\Delta^2 + 4v^2}}. \quad (48)$$

For $v \ll |\Delta|$,

$$c_1^+ \approx 1, \quad |c_2^{\pm}| \approx \frac{v}{|\Delta|}. \quad (49)$$

5. Electric monopole transitions

Now, we shall analyze the E0 transitions between the levels of the doublet 0_2^+ and 0_1^+ . For the nucleus, treated as an uniformly charged drop with a quadrupole deformation, the E0 transition operator reads [11]

$$M(\text{E0}) = \frac{3Z}{4\pi} \left(\beta^2 + \frac{5\sqrt{5}}{21\sqrt{\pi}} \beta^3 \cos 3\gamma \right), \quad (50)$$

where Z is the nuclear charge number. The E0 transition strength from the initial state 0_2^+ to the final one 0_1^+ is given by

$$B(\text{E0}; 0_2^+ \rightarrow 0_1^+) = |M_{fi}(\text{E0})|^2. \quad (51)$$

Inserting here the wave functions, one finds [9]

$$B(\text{E0}; 0_2^+ \rightarrow 0_1^+) = q B_{\text{max}}(\text{E0}; 0_2^+ \rightarrow 0_1^+), \quad (52)$$

where the factor

$$q = \frac{4}{(1 + R_+^2)(1 + R_-^2)} \quad (53)$$

varies from $q = 0$ in the case of pure shape to $q = 1$ in the case of complete mixing of shapes; the maximal strength

$$B_{\text{max}}(\text{E0}; 0_2^+ \rightarrow 0_1^+) = \left(\frac{3Z}{8\pi} \right)^2 \times \left\{ \beta_1^2 - \beta_2^2 + \frac{5\sqrt{5}}{21\sqrt{\pi}} \left[\beta_1^3 + \beta_2^3 + \frac{3}{2}(\beta_1 + \beta_2) \left(\frac{\hbar}{B\omega_\beta} - \frac{3}{2} \frac{\hbar}{B\omega_\gamma} \right) \right] \right\}^2. \quad (54)$$

Here, we neglected small terms $\sim (\hbar/B\omega_{\gamma(\beta)})^2$ inside the square brackets.

It is useful to rewrite the factor q as

$$q = 4c^2 (1 - c^2) = 4 \left(v/E_{0_2^+} \right)^2, \quad (55)$$

where $c^2 = (c_2^-)^2$ is the weight of the oblate shape in the ground state 0_1^+ . Inverting this equation, one can find the parameters c^2 and v :

$$c^2 = 0.5 \left(1 \pm \sqrt{1 - q} \right), \quad v = 0.5\sqrt{q}E_{0_2^+}. \quad (56)$$

These formulas allow us to extract the parameters c^2 , v , and Δ from experimental values of transition strengths $B(E0)$ between the levels of the ground 0^+ doublet in the Kr isotopes [2–4]. Everywhere the mass parameter B was calculated by means of Eq. (10) with the nuclear radius $R_0 = 1.2A^{1/3}$ fm. For ^{74}Kr , where the energy $E_{0_2^+} = 0.5$ MeV, the deformation parameters $\beta_1 \approx 0.38$, $\beta_2 \approx 0.32$ and phonon energies $\hbar\omega_\gamma = 1.689$ MeV, $\hbar\omega_\beta \simeq 1.5$ MeV, one finds $B_{\max}(E0) = 0.074$.

From experimental data $0.065 < B_{\exp}(E0; 0_2^+ \rightarrow 0_1^+) < 0.105$ [4], we get

$$c^2 = 50 \pm 17\%, \quad v = 0.23 \pm 0.01 \text{ MeV}, \quad 0 \leq |\Delta| \leq 0.2 \text{ MeV}. \quad (57)$$

Note that Petrovici *et al.* [12] predicted for ^{74}Kr the weight $c^2 = 30\%$ and 47% with $\Delta = -0.071$ MeV, but their $B(E0) < 0.037$ contradicts the experiment.

For ^{76}Kr with $E_{0_2^+} = 0.76$ MeV and $\beta_1 \approx 0.37$, $\beta_2 \approx 0.30$, we obtained $B_{\max}(E0) = 0.079$. Comparing with the experimental data $0.07 < B_{\exp}(E0; 0_2^+ \rightarrow 0_1^+) < 0.09$ [13], we found

$$c^2 = 50 \pm 17\%, \quad v = 0.37 \pm 0.01 \text{ MeV}, \quad 0 \leq |\Delta| \leq 0.25 \text{ MeV}. \quad (58)$$

For ^{78}Kr , where $E_{0_2^+} = 1.0$ MeV, $\beta_1 \approx 0.34$ and $\beta_2 \approx 0.30$ [1], comparison of $B_{\max}(E0) = 0.038$ with the data $0.035 < B_{\exp}(E0; 0_2^+ \rightarrow 0_1^+) < 0.060$ [13], gives

$$c^2 = 50 \pm 14\%, \quad v = 0.49 \pm 0.01 \text{ MeV}, \quad 0 \leq |\Delta| \leq 0.28 \text{ MeV}. \quad (59)$$

6. Gamma tunneling in K -isomer ^{182}Os

We estimate now the hindrance factor F_H for decay of the 25^+ isomer of ^{182}Os , using the above formulas for mixing of the states $\varphi_1^{(0)}(\gamma)$ with $K = 25$ and $\varphi_2^{(0)}(\gamma)$ with $K = 0$. The angular momentum $I^\pi = 25^+$ is conserved during the mixing. In both cases, when $\gamma = 0$ and $2\pi/3$, the nucleus ^{182}Os is axially symmetric and prolate. At $\gamma = 0$, it is oriented along the axis z' of the body-fixed frame, while at $\gamma = 2\pi/3$, along the axis x' .

The decay probability per unit time from such a mixed state $\varphi_i(\gamma)$ into the level $|f\rangle = |I_f^\pi = 24^+, K = 0\rangle$ level is determined by

$$P_{i \rightarrow f} = c_2^2 w_{i \rightarrow f}^{(2)}, \quad (60)$$

where $w_{i \rightarrow f}^{(2)}$ is the decay rate from the pure state $\varphi_2^{(0)}$, so that the hindrance factor $F_H = c_2^2$. Then from Eq. (49), one finds the hindrance factor

$$F_H = \left(\frac{\hbar\omega_\gamma}{\pi\Delta} \right) \exp \left(-\frac{2\pi W}{\hbar\omega_B} \right).$$

According to Fig. 1 of [7], $2\Delta = \hbar\omega_\gamma = 0.4$ MeV, $W = 2.152$ MeV. Putting also $\hbar\omega_B = 0.4$ MeV, we found the factor $F_H \approx 10^{-15}$, whereas Bengtsson *et al.* [7] obtained $F_H \approx 10^{-6}$ – 10^{-9} . At the same time, they indicated that F_H lowers by 5–6 orders if K -isomer has the same pairing gap as in the ground state.

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