THE LARGE-N_c MASSES OF LIGHT SCALAR MESONS FROM QCD SUM RULES FOR LINEAR RADIAL SPECTRUM*

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We discuss a calculation of large-N_c masses of light scalar mesons from the QCD sum rules. Two methods based on the use of linear radial Regge trajectories are presented. We put a special emphasis on the appearance of pole near 0.5 GeV in the scalar–isoscalar channel which emerges in both methods and presumably corresponds to the scalar sigma meson.

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1. Introduction

It is widely known that the physics of non-perturbative strong interactions is enciphered in the values of hadron masses. This intricate physics is especially pronounced in the hadrons consisting of u and d quarks as their masses $m_{u,d}$ are much less than the non-perturbative scale $\Lambda_{\rm QCD}$. At the same time, the given hadrons shape the surrounding world. Aside from the nucleons and pions, an important role is played by the scalar sigma meson which is responsible for the main part of the nucleon attraction potential. In the particle physics, the given resonance is identified as $f_0(500)$ [1] and is indispensable for description of the chiral symmetry breaking in many phenomenological field models describing the strong interactions. The scalar sector below and near 1 GeV is perhaps the most difficult for traditional approaches in the hadron spectroscopy. The usual quark model faces serious problems in explaining the existence and properties of light scalar mesons, perhaps due to a strong admixture of glueball component. Despite the recent progress in description of these states by dispersive methods [1, 2], the scalar sector still remains puzzling.

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The physical characteristics of hadrons are encoded in various correlation functions of corresponding hadron currents. Perhaps the most important characteristics is the hadron mass. The calculation of a hadron mass from first principles consists in finding the relevant pole of two-point correlator $\langle JJ\rangle$, where the current J is built from the quark and gluon fields, and interpolates the given hadron. For instance, if the scalar–isoscalar state f_0 represents an ordinary light non-strange quark–antiquark meson, its current should be interpolated by the quark bilinear $J=\bar{q}q$, where q stays for the u or d quark. In the real QCD, the straightforward calculations of correlators are possible only in the framework of lattice simulations which are still rather restricted.

It is usually believed that confinement in QCD leads to approximately linear radial Regge trajectories (see, *e.g.*, [3]). The most important quantity in this picture is the slope of trajectories. The slope is expected to be nearly universal as arising from flavor-independent non-perturbative gluodynamics which thereby sets a mass scale for light hadrons.

Among the phenomenological approaches to the hadron spectroscopy, the method of spectral sum rules [4] is perhaps the most related with QCD. In many cases, it permits to calculate reliably the masses of ground states on the radial trajectories. This method exploits some information from QCD via the Operator Product Expansion (OPE) of correlation functions [4]. On the other hand, one assumes a certain spectral representation for a correlator in question. Typically, the representation is given by the Ansatz "one infinitely narrow resonance + perturbative continuum". Such an approximation is very rough but in many cases works phenomenologically well. Theoretically, the zero-width approximation arises in the large-N_c limit of QCD [5]. In this limit, the only singularities of the two-point correlation function of a hadron current J are one-hadron states. For instance, the two-point correlator for $J^S=\bar{q}q$ has the following form to lowest order in $1/N_c$ (in the momentum space):

$$\Pi_S\left(q^2\right)=\left\langle J^S(q)J^S(-q)\right\rangle=\sum_n\frac{G_n^2M_S^2(n)}{q^2-M_S^2(n)}\,, \tag{1}$$

where the residues appear from the definition of the matrix element $\langle 0|J^S|n\rangle = G_nM_S(n)$. The OPE of correlator (1) in the large-N_c limit and to the lowest order in the perturbation theory reads [6]

$$\Pi_S\left(Q^2\right)=\frac{3Q^2}{16\pi^2}\log\frac{Q^2}{\mu^2}+\frac{3}{2Q^2}m_q\langle\bar{q}q\rangle-\frac{\alpha_s}{16\pi}\frac{\langle G^2\rangle}{Q^2}-\frac{11}{3}\pi\alpha_s\frac{\langle\bar{q}q\rangle^2}{Q^4}+\dots\,, \tag{2}$$

where $\langle G^2\rangle$ and $\langle\bar{q}q\rangle$ denote the gluon and quark vacuum condensate, respectively. According to the main assumption of classical QCD sum rules [4],

these vacuum characteristics are universal, *i.e.*, their values do not depend on the quantum numbers of a hadron current J (the method is not applicable otherwise).

In the present paper, we will demonstrate how all these ideas can be used for calculation of large-N_c masses of light scalar mesons.

2. Scalar sum rules: Some results

We will assume the linear radial spectrum with universal slope

$$M_S^2(n) = \Lambda^2 \left(n + m_s^2\right) , \qquad n = 0, 1, 2, \dots , \tag{3}$$

and (for consistency with the OPE): $G_n = G$. Here, M_S denotes the mass of a scalar state and m_s^2 is the intercept parameter of scalar trajectory. With the linear Ansatz (3) for the radial mass spectrum, expression (1) can be summed analytically, expanded at large $Q^2 = -q^2$ and compared with the corresponding OPE in QCD. Thus, one obtains a set of sum rules. Similar large-N_c sum rules were considered many times in the past for vector, axial, scalar and pseudoscalar channels (see, *e.g.*, references in [7]).

As *a priori* we do not know reliably the radial Regge behavior of scalar masses, two simple possibilities can be considered: (I) The ground $n = 0$ state lies on the linear trajectory (3); (II) The state $n = 0$, below called σ, is not described by the linear spectrum (3). The second assumption looks more physical. Within the latter assumption, the mass of σ meson can be derived as a function of the intercept parameter m_s^2 (we refer to Ref. [8] for details, the chiral limit is considered)

$$M_\sigma^2 = \frac{\frac{1}{16\pi^2}\Lambda^6 m_s^2 \left(m_s^2 + \frac{1}{2}\right)\left(m_s^2 + 1\right) + \frac{11}{3}\pi\alpha_s\langle\bar{q}q\rangle^2}{\frac{3}{32\pi^2}\Lambda^4\left(m_s^4 + m_s^2 + \frac{1}{6}\right) + \frac{\alpha_s}{16\pi}\langle G^2\rangle} . \tag{4}$$

Substituting the physical values of vacuum condensates and numerical value for slope Λ^2 obtained from a solution of QCD sum rules, the mass function (4) is displayed in Fig. 1 [8].

The mass of the first radially excited state $M_S(1)$ is rather stable and seems to reproduce the mass of $a_0(1450)$-meson, $M_{a_0(1450)} = 1474 \pm 19$ MeV [1]. Its isosinglet partner (the candidates is $f_0(1370)$) should be degenerate with $a_0(1450)$ in the planar limit.

The plot in Fig. 1 demonstrates that the actual prediction for M_σ is rather sensitive to the intercept of scalar linear trajectory, though initially M_σ is not described by the linear spectrum (3). And *vice versa*, the expected value of M_σ (around 0.5 GeV [1]) imposes a strong bound on the allowed values of intercept m_s^2. The plot in Fig. 1 shows that m_s^2 is likely close to zero.

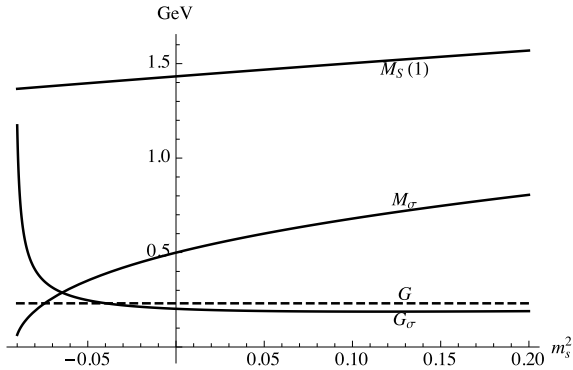


Fig. 1. The values of M_σ, G_σ, G, and the first state on the scalar trajectory $M_{\rm S}(1)$ as a function of dimensionless intercept $m_{\rm s}^2$.

Thus, interpolating the scalar states by the simplest quark bilinear current, we predict a light scalar resonance with mass about 500 ± 100 MeV which is a reasonable candidate for the scalar sigma meson $f_0(500)$ [1].

3. Borelized scalar sum rules: Some results

The original QCD sum rules made use of the Borel transformation [4]

$$L_M \Pi\left(Q^2\right) = \lim_{\substack{Q^2,n\to\infty \\ Q^2/n=M^2}} \frac{1}{(n-1)!}\left(Q^2\right)^n \left(-\frac{d}{dQ^2}\right)^n \Pi\left(Q^2\right) . \tag{5}$$

The borelized version has a number of advantages and can be applied to our large-$N_{\rm c}$ case. The details are contained in Ref. [9]. In short, the mass of ground scalar meson $m_0 \equiv M_{\rm S}(0)$ as a function of Borel parameter is shown in Fig. 2. It is seen that there are two solutions with "Borel window" extending to infinity. The corresponding asymptotic values are given by

$$M_{\rm S}^2(0) = \frac{\Lambda^2}{2} \pm \frac{1}{2}\sqrt{\frac{\Lambda^4}{3} - 64\pi^2\left(m_q\langle\bar{q}q\rangle + \frac{\alpha_{\rm s}}{24\pi}\langle G^2\rangle\right)} . \tag{6}$$

The heavier state corresponds to the ground scalar mass in the standard QCD sum rules. Normalizing this mass to the value $m_{f_0} = 1.00 \pm 0.03$ GeV extracted from these canonical sum rules [6], we predict the value of slope for the scalar trajectory, $\Lambda_{f_0}^2 = 1.38 \pm 0.07$ GeV^2, which is used in Fig. 2. We obtain then the mass of the lightest scalar state, $M_\sigma \approx 0.62$ GeV.

We arrive thus at the conclusion that our method predicts two parallel scalar trajectories. The ground state on the first trajectory can be identified with $f_0(980)$ and on the second one with $f_0(500)$ [1]. The existence of

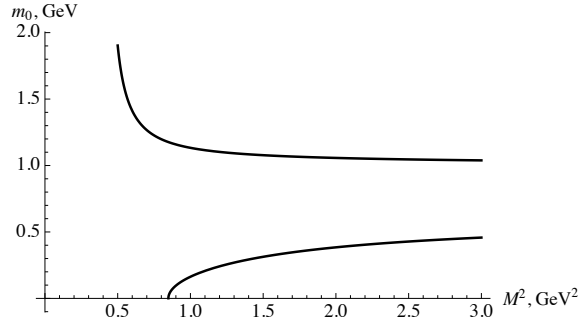


Fig. 2. The mass of ground scalar meson $m_0 \equiv M_S(0)$ as a function of Borel parameter at $\Lambda^2 = 1.38\,\mathrm{GeV}^2$ [9].

two parallel radial scalar trajectories seems to agree with the experimental data [3]. The masses of predicted radial states and a tentative comparison with the observed scalar mesons for two trajectories are displayed in Tables I and II, correspondingly.

TABLE I

The radial spectrum of the first f_0-trajectory for the slope $\Lambda^2 = 1.38 \pm 0.07\,\mathrm{GeV}^2$. The first 5 predicted states are tentatively assigned to the resonances $f_0(980)$, $f_0(1500)$, $f_0(2020)$, $f_0(2200)$, and $X(2540)$ [1].

n	0	1	2	3	4
m_{f_0} (th 1)	1000 ± 30	1540 ± 20	1940 ± 40	2270 ± 50	2560 ± 50
m_{f_0} (exp 1)	990 ± 20	1504 ± 6	1992 ± 16	2189 ± 13	$2539 \pm 14^{+38}_{-14}$

TABLE II

The radial spectrum of the second f_0-trajectory for the slope $\Lambda^2 = 1.38 \pm 0.07\,\mathrm{GeV}^2$. The first 5 predicted states are tentatively assigned to the resonances $f_0(500)$, $f_0(1370)$, $f_0(1710)$, $f_0(2100)$, and $f_0(2330)$ [1].

n	0	1	2	3	4
m_{f_0} (th 2)	620	1330 ± 30	1780 ± 40	2130 ± 50	2430 ± 60
m_{f_0} (exp 2)	400–550	1200–1500	1723^{+6}_{-5}	2101 ± 7	2300–2350

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