ON THE DYNAMIC GENERATION OF KINETIC TERMS FOR MESONIC BOUND STATES*

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Dynamical bosonisation within the functional renormalisation group is used to describe mesons as quark–antiquark bound states. Employing for simplicity the two-flavour quark–meson model, it is exemplified how the kinetic terms for pseudoscalar and scalar mesons are generated from the quark kinetic term upon lowering the renormalisation group scale. Relating this method to the Dyson–Schwinger–Bethe–Salpether approach, one can identify the momentum-dependent Yukawa three-point function in the limit of vanishing renormalisation group scale with the Bethe–Salpeter amplitude. This, in turn, might allow for a systematic comparison of the impact of truncations in the Dyson–Schwinger–Bethe–Salpether approach on the one hand and the functional renormalisation group with dynamical bosonisation on the other hand. This paper is concluded by an outlook on a respective on-going investigation.

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1. Introduction

Hadrons are composite objects built from valence quarks, a quark–antiquark sea, and gluonic fields. Although they can be described within QCD, the related calculations are very challenging and computationally demanding. A prime example for this is provided by the fact that hadron spectroscopy obtained from lattice QCD calculations became only precise after many technical obstacles have been overcome, see, *e.g.*, [1] and references therein. The impressive success of these investigations left no doubt that QCD is the correct theory of Strong Interactions also on sub-GeV scales.

Despite all the merits of these investigations, there are some qualitative questions that can hardly or not at all be answered by lattice calculations. In this paper, we will focus on one of them: With the quarks and gluons

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being the fundamental degrees of freedom, how do mesons as composite objects become propagating particles? To this end, we will apply functional methods, or more precisely two versions of them: First, we will employ the Functional Renormalisation Group (FRG) augmented with scale-dependent ("dynamical") bosonisation (DB), see [2, 3] and references therein. Second, we will also use the Dyson–Schwinger and Bethe–Salpeter equations (DSE–BSE) approach which has reached over the last years a considerable amount of sophistication, for a recent review on the successes and open problems within this approach, see the recent review [4] and references therein.

In this paper, we will also report on the status of an on-going investigation which intends to critically compare the FRG–DB with the DSE–BSE approach. Although, at a purely formal level, they were to yield the same results for observables if treated exactly, it is evident that a mapping of truncations beyond the most simple ones (*cf.* [5]) is a non-trivial task. On a purely technical level, the decisive question boils down to: In which of the two approaches an apparent level of convergence of results upon increasing the complexity of the employed truncation can be reached with less effort? As usual, in functional approaches, the computational complexity is much more dominated by the size of the kernels to be used when solving the equations than by the amount of needed CPU time, see *e.g.*, [6] as well as references therein for a recent discussion of technical issues and practical considerations in the DSE–BSE approach.

Last but not least, as in lattice QCD, also within functional approaches all calculations are performed within Euclidean quantum field theory. In the DSE–BSE approach, Minkowski time-like momenta (which are needed when considering massive bound states) can be included via analytical continuation performed within the equations to obtain then Green's functions dependent on complex momenta, see *e.g.* [4, 7–9] and references therein. The corresponding procedure within the FRG approach is dubbed real-time calculations and has been developed only recently, see *e.g.* [10–13]. An access to bound state properties is then provided by calculating the correlation functions of suitably chosen composite operators for complex values of momenta. Hereby, the pole masses and decay widths of the hadronic resonances are, at least in principle, encoded in the analytic properties of the propagators of the effective hadronic degrees of freedom. The recent progress on extracting decay widths in this manner within the DSE–BSE progress is described in [14] as well as in [15].

2. Dynamical hadronisation

Dynamical hadronisation [16] is a technique based on combining bosonisation with the FRG. The FRG equation within the investigation reported here is a suitably truncated version of the Wetterich equation [17]

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \, \frac{1}{\Gamma_k^{(2)} + R_k} \partial_t R_k \, , \tag{1}$$

where k is the Renormalisation Group (RG) scale, $\partial_t = k\partial_k$, and $\Gamma_k^{(n)}$ is the n^{th} field derivative of the effective action with respect to the bosonic and fermionic fields of the theory. The regulator $R_k(p^2)$ suppresses all quantum fluctuations below the scale k, and the trace sums over different fields, their internal indices and integrates four-momenta. In the next step, the set of flow equations is obtained by expanding the effective action in vertex functions, which are then, in turn, truncated to obtain a closed system. In addition, one might apply additional approximations to these vertex functions for numerical convenience, see [2] for a corresponding discussion. The resulting set of equations is then solved self-consistently. The physical information is extracted for vanishing RG scale $k \to 0$.

As the full complexity of QCD is not needed for the point we want to make here, we exemplify it within an NJL-like effective theory. One might motivate the use of this theory by "integrating out the gluons" [18] which is possible due to the infrared suppression of the gluon propagator, see [19] and references therein. Thus, we consider an effective theory with a four-fermion interaction which can be mapped via bosonisation techniques to the quark–meson model [18] containing quark–meson Yukawa interactions.

Within the FRG, the flow of the fermionic four-point function in this model is non-vanishing and thus the four-fermi interaction is regenerated. Applying in every RG step a Hubbard–Stratonovich (HS) transformation to reshuffle the four-fermion term into the Yukawa couplings is called dynamical hadronisation [16, 20, 21]. (NB: It has also been successfully applied to two-flavour QCD in the Landau gauge [22–24], and thus a generalisation of the arguments provided in the remainder of this paper to QCD is on the basis of these calculations straightforward.) Note that dynamical hadronisation provides also a significant computational advantage: The four-point Green's function is described by a meson exchange, simplifying thus the tensor structures without any loss of information. Such feature is a key point to establish a proper comparison with four-point functions used in the DSE–BSE approach in which the reduction of the corresponding DSE (sometimes then also called inhomogeneous BSE in this context) to the linear BSE provides a similar simplification.

In the FRG calculations presented below, a full-momentum-dependent dynamical hadronisation has been performed, and meson self-interactions up to the 12^{th} power in the meson field have been taken into account.

3. Comparison to Dyson–Schwinger–Bethe–Salpeter approach

Observables constitute the main comparable objects not only between different functional methods but also with any other non-perturbative approach. The procedure of obtaining properties of bound states within the FRG approach will be published elsewhere [2].

In the present work, we will focus on the relations and differences between the functional methods. Although the compatibility between them has been proven, at least in principle [5], there are features of the system that cannot be observed in the DSE–BSE approach, and there is some additional information which can be extracted. Furthermore, the three-point function from the FRG and the Bethe–Salpeter amplitude should coincide.

4. Preliminary results, conclusions and outlook

It is instructive to analyse the wave-function renormalisation functions $Z_{k,i}(p^2)$. In the HS transformation, we include pseudoscalar–isovector $\vec{\pi}$ and scalar–isoscalar σ fields, and as no kinetic term for them is generated in the very first bosonisation step, an appropriate choice is to set $Z_{k=UV,\pi} = Z_{k=UV,\sigma} = 0$. Furthermore, defining

$$Z_{k,\mathrm{sum}} \equiv \sqrt{Z_{k,q}^{-2} + Z_{k,\phi}^2}\,, \tag{2}$$

with $Z_{k,\phi}$ being a particular isospin-weighted combination of boson terms, probability conservation requires $Z_{k,\mathrm{sum}} = 1$ at every scale k. (NB: We employ $Z_{k,q}^{-2}$ to be in agreement with the usual notation.) Our results for $Z_{k,i}(p^2 = 0)$ are displayed in Fig. 1.

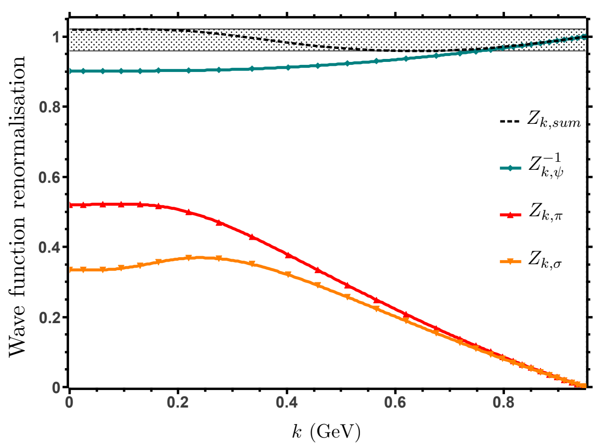

Fig. 1. Plot of scale evolution of the wave-function renormalisations at zero momentum in terms of the scale.

This plot shows the explicit generation of kinetic terms for the pseudoscalar and scalar mesons caused by the RG flow. Furthermore, the probability amplitude is preserved along the flow with a relative error less than 5% which we believe is mostly caused by the missing contributions of higher-lying mesons. Note that the employed model is not confining, and thus the quark stays propagating also in the infrared. It is nevertheless illustrative to see how the quark kinetic term first generates and then feeds the *ab initio* absent kinetic terms of the composite states, the mesons. A complete analysis and discussion will be published elsewhere [25].

In addition, we obtained the full-momentum-dependent quark–meson three-point function. Setting a different set of initial conditions such that infrared values agree with experimental data, they should agree with the correspondingly determined Bethe–Salpeter amplitude from the DSE–BSE approach. In Fig. 2, we plot the renormalised three-point function at zero relative momentum. It is displayed together with its values at negative p^2 obtained by performing an analytical continuation.

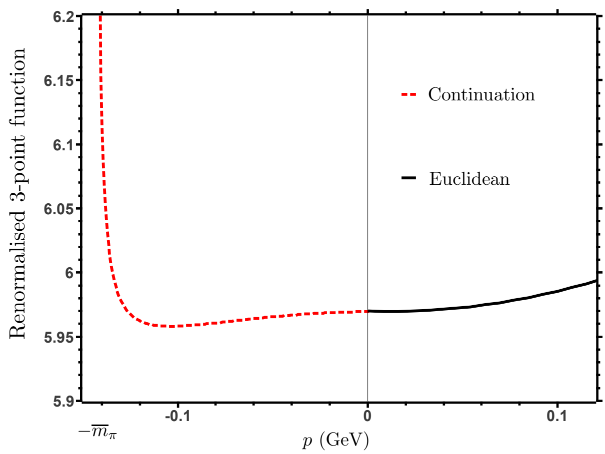

Fig. 2. Plot of the renormalised three-point function with 0 relative momenta and angle, for positive and negative values of $p = \sqrt{p^2}$.

It shows the expected behaviour for time-like momenta and possesses a pole exactly at $p^2 = -\bar{m}_\pi^2$. In an on-going investigation [25], we aim at a systematic comparison of this three-point function in increasingly more complete truncations to the correponding Bethe–Salpeter amplitude obtained in different truncation schemes in order to elucidate the relation between these two functional approaches after suitable truncations have been applied.

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