

FLUID DYNAMICS FOR RELATIVISTIC SPIN-POLARIZED MEDIA*

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We briefly review the basic features of a new framework for relativistic perfect fluid hydrodynamics of polarized systems consisting of particles with spin one half. Using this approach, we numerically study the stability of a stationary vortex-like solution, representing global equilibrium of a rotating medium.

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1. Introduction

The recent observation of global spin polarization of Λ hyperons by the STAR Collaboration [1] has rekindled the interest in polarization and vorticity in ultrarelativistic heavy-ion collisions. In contrast to a multitude of classical effects [2, 3], the polarization of spin represents a first, rather clear, experimental manifestation of a pure quantum phenomenon in nucleus–nucleus collisions. A particularly appealing theoretical explanation of this effect invokes a direct coupling between the thermal vorticity and polarization, which is realized in the global equilibrium state of a rotating medium [4, 5]. Recently, a new framework for relativistic perfect fluid hydrodynamics of spin-polarized media was presented [6–9], which extends the work of Refs. [4, 5] to systems in local equilibrium. In this contribution, we discuss the approach presented in Refs. [6–9] and study its physical consequences.

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2. Local equilibrium distribution functions

Following Refs. [6–9], we consider a local equilibrium state of a relativistic system of particles (+) and antiparticles (−) with spin $1/2$ and mass m , whose phase-space distribution functions are given by the spin density matrices ($r, s = 1, 2$) [5]

$$f_{rs}^+(x, p) = \bar{u}_r(p) X^+ u_s(p), \quad f_{rs}^-(x, p) = -\bar{v}_s(p) X^- v_r(p). \quad (1)$$

Here, $u_r(p)$ and $v_r(p)$ are Dirac bispinors and $X^\pm$ are four-by-four matrices

$$X^\pm = \exp \left[\pm \xi(x) - \beta_\mu(x) p^\mu \pm \frac{1}{2} \omega_{\mu\nu}(x) \Sigma^{\mu\nu} \right], \quad (2)$$

where $\xi \equiv \mu/T$, $\beta^\mu \equiv u^\mu/T$, and T , μ and u^μ denote the temperature, baryon chemical potential and four-velocity of the fluid, respectively. The quantity $\omega_{\mu\nu}$ is the spin polarization tensor and $\Sigma^{\mu\nu} \equiv \frac{1}{2} \sigma^{\mu\nu} = \frac{i}{4} [\gamma^\mu, \gamma^\nu]$ is the spin operator. For later convenience, it is useful to define the quantity $\zeta \equiv \frac{1}{2\sqrt{2}} \sqrt{\omega_{\mu\nu} \omega^{\mu\nu}}$, assuming that $\epsilon_{\alpha\beta\gamma\delta} \omega^{\alpha\beta} \omega^{\gamma\delta} = 0$.

3. Thermodynamics of the spin-polarized medium

With the distribution functions (1) being defined, it is straightforward to obtain the basic thermodynamic variables describing the system in question. In particular, using definitions from Refs. [5, 10], one finds the following familiar expressions for the baryon current and the energy-momentum tensor:

$$N^\mu = \kappa \int \frac{d^3 p}{2E_p} p^\mu [\text{tr}_4(X^+) - \text{tr}_4(X^-)] = n u^\mu, \quad (3)$$

$$T^{\mu\nu} = \kappa \int \frac{d^3 p}{2E_p} p^\mu p^\nu [\text{tr}_4(X^+) + \text{tr}_4(X^-)] = (\varepsilon + P) u^\mu u^\nu - P g^{\mu\nu}. \quad (4)$$

In Eqs. (3)–(4), the factor $\kappa \equiv g/(2\pi)^3$ accounts for internal degrees of freedom excluding spin, tr_4 denotes the trace in the spinor space, and $g^{\mu\nu} = \text{diag}(+1, -1, -1, -1)$ is the metric tensor. Generalizing the Boltzmann expression for the entropy current, one also finds

$$S^\mu = -\kappa \int \frac{d^3 p}{2E_p} p^\mu \{ \text{tr}_4[X^+(\ln X^+ - 1)] + \text{tr}_4[X^-(\ln X^- - 1)] \} = s u^\mu. \quad (5)$$

In Eqs. (3)–(5), the energy density, $\varepsilon = 4 \cosh(\zeta) \cosh(\xi) \varepsilon_{(0)}(T)$, the pressure, $P = 4 \cosh(\zeta) \cosh(\xi) P_{(0)}(T)$, the baryon density, $n = 4 \cosh(\zeta) \sinh(\xi) n_{(0)}(T)$, and the entropy density, $s = 4 \cosh(\zeta) \cosh(\xi) s_{(0)}(T)$, are all related by the thermodynamic relation $\varepsilon + P = sT + \mu n + \Omega w$. Here, we introduced a new

variable, playing the role of a spin chemical potential $\Omega \equiv \zeta T$, and, related to it, a new charge, $w = 4 \sinh(\zeta) \cosh(\xi) n_{(0)}(T)$. The thermodynamic potentials are expressed in terms of those corresponding to an auxiliary system of spin-0 particles

$$\begin{aligned} n_{(0)}(T) &= \langle (u \cdot p) \rangle_0 = \frac{\kappa}{2\pi^2} T^3 \hat{m}^2 K_2(\hat{m}) , \\ \varepsilon_{(0)}(T) &= \langle (u \cdot p)^2 \rangle_0 = \frac{\kappa}{2\pi^2} T^4 \hat{m}^2 \left[3K_2(\hat{m}) + \hat{m} K_1(\hat{m}) \right] , \\ P_{(0)}(T) &= -\frac{1}{3} \langle [p \cdot p - (u \cdot p)^2] \rangle_0 = T n_{(0)}(T) , \end{aligned}$$

where $s_{(0)}(T) = \frac{1}{T} [\varepsilon_{(0)}(T) + P_{(0)}(T)]$ and $\hat{m} \equiv m/T$.

4. Fluid dynamics equations

In fluid dynamics, the space-time-dependent quantities $\mu(x)$, $T(x)$, $u^\mu(x)$ and $\omega^\mu(x)$ in Eq. (2) play a role of Lagrange multipliers, whose form should follow from the evolution equations. In particular, the requirement of energy and momentum conservation, $\partial_\mu T^{\mu\nu} = 0$, when projected onto directions orthogonal to the fluid flow, yields the three Euler equations

$$(\varepsilon + P)\dot{u}^\mu = \partial^\mu P - u^\mu \dot{P} , \quad (6)$$

where $\theta \equiv \partial \cdot u$ is the expansion scalar and $(\dot{}) \equiv u \cdot \partial$ denotes the comoving derivative. On the other hand, by projecting the energy and momentum conservation equation onto the fluid four velocity $u^\mu(x)$, and using the differentials of the pressure $P = P(T, \mu, \Omega)$, one obtains

$$T \partial_\mu (s u^\mu) + \mu \partial_\mu (n u^\mu) + \Omega \partial_\mu (w u^\mu) = 0 . \quad (7)$$

By requiring that the first two terms vanish due to the conservation of entropy and baryon number, respectively, Eq. (7) yields three separate conditions

$$\partial_\mu S^\mu = \dot{s} + s \theta = 0 , \quad (8)$$

$$\partial_\mu N^\mu = \dot{n} + n \theta = 0 , \quad (9)$$

$$\partial_\mu W^\mu = \partial_\mu (w u^\mu) = \dot{w} + w \theta = 0 . \quad (10)$$

5. Polarization dynamics

Equations (6), (8), (9) and (10) form a closed set of six differential equations, which allow one to determine time evolution of $\mu(x)$, $T(x)$, the three independent components of $u^\mu(x)$ and $\Omega(x) \equiv \frac{T}{2\sqrt{2}} \sqrt{\omega_{\mu\nu} \omega^{\mu\nu}}$. We

are thus left with four independent components of the polarization tensor, whose evolution does not influence the hydrodynamic background. The equations of motion for the polarization tensor follow from the angular momentum conservation law $\partial_\alpha J^{\alpha,\beta\gamma} = 0$, with $J^{\alpha,\beta\gamma} = L^{\alpha,\beta\gamma} + S^{\alpha,\beta\gamma}$, where $L^{\alpha,\beta\gamma} = x^\beta T^{\gamma\alpha} - x^\gamma T^{\beta\alpha}$ is the orbital angular momentum tensor and $S^{\alpha,\beta\gamma}$ is the spin tensor. Since $T^{\mu\nu}$ given in Eq. (4) is symmetric, one has [11]

$$\partial_\alpha S^{\alpha,\beta\gamma} = 0. \quad (11)$$

For the internal consistency of the approach, we assume that the spin tensor has the following form [4]:

$$S^{\lambda,\mu\nu} = \kappa \int \frac{d^3p}{2E_p} p^\lambda \text{tr}_4 [(X^+ - X^-) \Sigma^{\mu\nu}] = \frac{wu^\lambda}{4\zeta} \omega^{\mu\nu}. \quad (12)$$

By introducing the rescaled spin polarization tensor, $\bar{\omega}^{\mu\nu} = \omega^{\mu\nu}/(2\zeta)$ and using Eq. (10), one arrives at the formula

$$\dot{\bar{\omega}}^{\mu\nu} = 0. \quad (13)$$

Equation (13) states that the scaled components of the polarization tensor are conserved in the comoving frame.

6. Stability of the stationary vortex solution

In Refs. [6, 8], it was shown that Eqs. (6), (8), (9) and (10) have the following stationary vortex-like solution, representing a global equilibrium state with rotation [4, 5]:

$$u^\mu = \gamma \left(1, -\tilde{\Omega} y, \tilde{\Omega} x, 0 \right), \quad (14)$$

$$T = T_0 \gamma, \quad \mu = \mu_0 \gamma, \quad \Omega = \Omega_0 \gamma, \quad (15)$$

where $\gamma = 1/\sqrt{1 - \tilde{\Omega}^2 r^2}$ is the Lorentz factor, $r = \sqrt{x^2 + y^2}$ and T_0 , μ_0 , and Ω_0 are arbitrary constants. The corresponding polarization tensor is in this case either zero or has a form where the only non-vanishing component is $\omega_{xy} = -\omega_{yx} = \tilde{\Omega}/T_0 = 2\Omega_0/T_0$.

One may notice that, due to the limiting speed of light, the stationary solution may be realized only within a cylinder of a finite radius $R < 1/\tilde{\Omega}$. It is difficult to imagine how a corresponding boundary condition could be implemented in Nature. Using the approach presented above, we explore the evolution of such a vortex, with more realistic boundary conditions numerically. To that end, we set up the initial conditions for the system in such a way that it reproduces the stationary vortex solution (dash-dotted blue

lines) within a central region (small r), and departs from it at the edges, as illustrated in Fig. 1 (solid red lines). By letting the system evolve in time, we find that relaxing the boundary conditions causes the system to depart from the global equilibrium solution. Thus, our results indicate that, with realistic boundary conditions, the vortex solution is unstable.

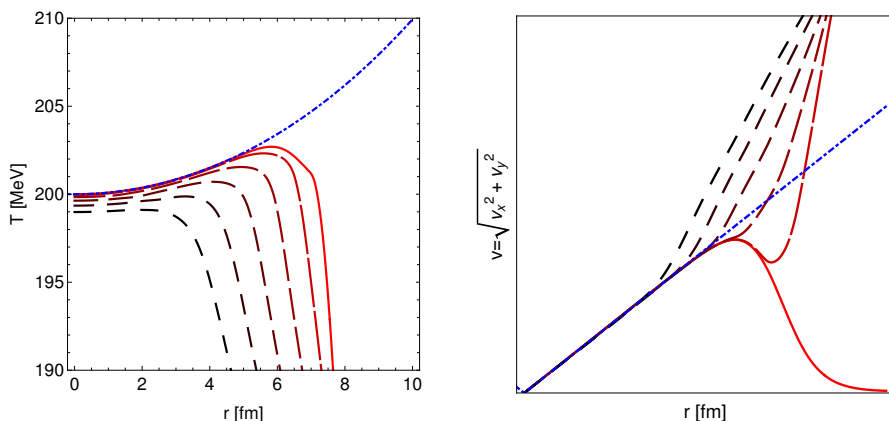


Fig. 1. (Color online) Space (r) dependence of the temperature, and velocity magnitude $v = \sqrt{v_x^2 + v_y^2}$ at times: $t = 0.1, 2, 4, 6, 8, 10$ fm (color of the lines changing from red to black as the time increases).

7. Summary

Using the framework of perfect fluid hydrodynamics of particles with spin $1/2$, we studied the stability of the stationary vortex-like solution representing the global equilibrium state of such a system. We find that with more realistic boundary conditions, the stationary solution is unstable. Consequently, it is rather unlikely that such a stationary state is realized in Nature.

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