

NOT ALL POSSIBLE ω - ϕ MIXING FORMS ARE PHYSICALLY ACCEPTABLE*

CYRIL ADAMUSCIN, ERIK BARTOS, STANISLAV DUBNICKA

Institute of Physics, Slovak Academy of Sciences, Bratislava, Slovakia

ANNA ZUZANA DUBNICKOVA

Department of Theoretical Physics, Comenius University, Bratislava, Slovakia

(Received August 7, 2018)

Starting from the symmetric 2×2 mass matrix in the ω_8, ω_0 space and, subsequently, by its diagonalisation into physical vector meson states $\omega(782)$, $\phi(1020)$ by means of the orthogonal matrix in the most general form, all possible forms of the ω - ϕ mixing are found. Taking into account the quark structure of the ω_8 , ω_0 states and exploiting the ideal mixing angle value $\theta = 35.3^\circ$, it is demonstrated that only four of the found mixing forms are physically acceptable, as only they are in conformity with experimentally observed decays of the $\omega(782)$ and $\phi(1020)$ vector mesons.

DOI:10.5506/APhysPolBSupp.11.519

1. Introduction

The Eightfold way, by using the $SU(3)$ symmetry, classifies the vector meson nonet, similarly to the pseudoscalar mesons, into an octet matrix and a singlet [1] as

$$V = \begin{pmatrix} \omega_8/\sqrt{6} + \rho^0/\sqrt{2} & \rho^+ & K^{*+} \\ \rho^- & \omega_8/\sqrt{6} - \rho^0/\sqrt{2} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -2\omega_8/\sqrt{6} \end{pmatrix}, \quad \omega_0, \quad (1)$$

* Presented at “Excited QCD 2018”, Kopaonik, Serbia, March 11–15, 2018.

where quark representation of the corresponding vector mesons is given as

$$\begin{aligned}\rho^0 &= \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), & \rho^+ &= u\bar{d}, & \rho^- &= d\bar{u}, \\ K^{*+} &= u\bar{s}, & K^{*-} &= s\bar{u}, & \bar{K}^{*0} &= \bar{d}s, & K^{*0} &= d\bar{s}, \\ \omega_8 &= \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}), & \omega_0 &= \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}).\end{aligned}\quad (2)$$

However, in the case of the vector meson nonet, an inconsistency of the bare vector meson state ω_8 mass arises with existing experimental situation. Actually, its mass can be evaluated by the Gell-Mann–Okubo quadratic mass formula

$$m^2 = a_0 + a_1 S + a_2 \left(I(I+1) - \frac{1}{4} S^2 \right), \quad (3)$$

where S is a strangeness of a corresponding meson and I is its isospin. An application of this formula to ω_8, ρ^0 and $K^{*0}, \bar{K}^{*0}$ pair gives

$$\begin{aligned}m_{\omega_8}^2 &= a_0, \\ m_{\rho^0(770)}^2 &= a_0 + 2a_2, \\ m_{K^{*0}(892)}^2 &= a_0 - a_1 + a_2/2, \\ m_{\bar{K}^{*0}(892)}^2 &= a_0 + a_1 + a_2/2,\end{aligned}\quad (4)$$

from where one gets

$$m_{\omega_8}^2 = \frac{4 \frac{m_{K^{*0}}^2 + m_{\bar{K}^{*0}}^2}{2} - m_{\rho}^2}{3} \cong (933 \text{ MeV})^2. \quad (5)$$

This result indicates that ω_8, ω_0 do not correspond to the known physical states $\omega(782), \phi(1020)$. The latter problem has been solved by Sakurai [2] suggesting to apply for an ω – ϕ mixing some value of the mixing angle θ .

2. Specification of ω – ϕ mixing forms

It may seem that the so-called ω – ϕ mixing is a well-established concept and commonly it appears as integral part of many textbooks of the particle physics. However, one finds *e.g.* that textbooks of [1, 3, 4] have different definitions of the ω – ϕ mixing and so, there are at least three different incompatible versions of them.

Further, we show that there are actually eight possible versions of such mixing.

Let us denote the mixing configuration generally by means of the matrix relation

$$\begin{pmatrix} \phi \\ \omega \end{pmatrix} = R^T \begin{pmatrix} \omega_8 \\ \omega_0 \end{pmatrix}, \quad (6)$$

where R is an orthogonal 2×2 matrix in the most general forms

$$R_1 = \begin{pmatrix} -p & q \\ q & p \end{pmatrix} \quad \text{or} \quad R_2 = \begin{pmatrix} p & q \\ -q & p \end{pmatrix}, \quad (7)$$

with

$$p^2 + q^2 = 1. \quad (8)$$

Then determinants of matrices (7) are

$$\det R_1 = \begin{vmatrix} -p & q \\ q & p \end{vmatrix} = -1 \quad \text{and} \quad \det R_2 = \begin{vmatrix} p & q \\ -q & p \end{vmatrix} = +1. \quad (9)$$

Next, by means of a diagonalisation of the 2×2 vector meson mass matrix in the ω_8, ω_0 space, by orthogonal matrices (7), we derive a quadratic mass relation among $m_{\omega_8}^2$, m_ϕ^2 and m_ω^2 , which together with condition (8), produce just eight possible different forms of ω - ϕ mixing, from which not all are physically acceptable.

The quadratic vector meson masses are defined as matrix elements of the quadratic mass operator $\mathcal{M}^2$

$$\begin{aligned} m^2(\omega_8) &= \langle \omega_8 | \mathcal{M}^2 | \omega_8 \rangle, \\ m^2(\omega_0) &= \langle \omega_0 | \mathcal{M}^2 | \omega_0 \rangle, \\ m^2(\omega) &= \langle \omega | \mathcal{M}^2 | \omega \rangle, \\ m^2(\phi) &= \langle \phi | \mathcal{M}^2 | \phi \rangle. \end{aligned} \quad (10)$$

If $|\omega\rangle$ and $|\phi\rangle$ are eigenfunctions of the operator $\mathcal{M}^2$, then the non-diagonal matrix elements disappear

$$\langle \omega | \mathcal{M}^2 | \phi \rangle = \langle \phi | \mathcal{M}^2 | \omega \rangle = 0. \quad (11)$$

The symmetric 2×2 vector meson mass matrix in the ω_8, ω_0 space takes then the following form:

$$\mathcal{M} = \begin{pmatrix} m_{\omega_8}^2 & m_{08}^2/2 \\ m_{08}^2/2 & m_{\omega_0}^2 \end{pmatrix}, \quad (12)$$

and its diagonalisation by means of the orthogonal matrix R_1 from (7) leads to three algebraic equations for unknown p and q

$$\begin{aligned} p^2 m_{\omega_8}^2 - 2pq m_{08}^2 + q^2 m_{\omega_0}^2 &= m_\phi^2, \\ q^2 m_{\omega_8}^2 + 2pq m_{08}^2 + p^2 m_{\omega_0}^2 &= m_\omega^2, \\ -pq m_{\omega_8}^2 + (q^2 - p^2) m_{08}^2 + pq m_{\omega_0}^2 &= 0. \end{aligned} \quad (13)$$

Expressing m_{08}^2 from the third equation in (13), one obtains

$$m_{08}^2 = \frac{pq(m_{\omega_8}^2 - m_{\omega_0}^2)}{(q^2 - p^2)}, \quad (14)$$

and a substitution of the latter into first two equations in (13) leads to the following solutions:

$$\begin{aligned} m_{\omega_8}^2 &= p^2 m_\phi^2 + q^2 m_\omega^2, \\ m_{\omega_0}^2 &= q^2 m^2 m_\phi^2 + p^2 m_\omega^2. \end{aligned} \quad (15)$$

The diagonalisation of matrix (12) by means of the orthogonal matrix R_2 from (7) leads to the same three algebraic equations (13) and their solution (15).

Taking the masses in the first equation of (15) from [5] and (5), and combining it with (8), one finds four solutions $p = \pm 0.63192$, $q = \pm 0.77500$ for p and q . By substituting all combinations of these solutions into R_1 and R_2 , from the relation (6), one gets

$$\begin{aligned} 1. \quad & \omega = 0.63192\omega_8 + 0.77500\omega_0, \quad \phi = -0.77500\omega_8 + 0.63192\omega_0, \\ 2. \quad & \omega = -0.63192\omega_8 + 0.77500\omega_0, \quad \phi = -0.77500\omega_8 - 0.63192\omega_0, \\ 3. \quad & \omega = 0.63192\omega_8 - 0.77500\omega_0, \quad \phi = 0.77500\omega_8 + 0.63192\omega_0, \\ 4. \quad & \omega = -0.63192\omega_8 - 0.77500\omega_0, \quad \phi = 0.77500\omega_8 - 0.63192\omega_0, \\ 5. \quad & \omega = 0.63192\omega_8 + 0.77500\omega_0, \quad \phi = 0.77500\omega_8 - 0.63192\omega_0, \\ 6. \quad & \omega = -0.63192\omega_8 + 0.77500\omega_0, \quad \phi = 0.77500\omega_8 + 0.63192\omega_0, \\ 7. \quad & \omega = 0.63192\omega_8 - 0.77500\omega_0, \quad \phi = -0.77500\omega_8 - 0.63192\omega_0, \\ 8. \quad & \omega = -0.63192\omega_8 - 0.77500\omega_0, \quad \phi = -0.77500\omega_8 + 0.63192\omega_0. \end{aligned} \quad (16)$$

By using a correspondence $0.63192 = \sin \theta$ and $0.77500 = \cos \theta$ with $\theta = 39.19^\circ$, one obtains generally 8 possible canonic ω - ϕ mixing forms

$$\begin{aligned} 1. \quad & \omega = \omega_8 \sin \theta + \omega_0 \cos \theta, \quad \phi = -\omega_8 \cos \theta + \omega_0 \sin \theta, \\ 2. \quad & \omega = -\omega_8 \sin \theta + \omega_0 \cos \theta, \quad \phi = -\omega_8 \cos \theta - \omega_0 \sin \theta, \\ 3. \quad & \omega = \omega_8 \sin \theta - \omega_0 \cos \theta, \quad \phi = \omega_8 \cos \theta + \omega_0 \sin \theta, \\ 4. \quad & \omega = -\omega_8 \sin \theta - \omega_0 \cos \theta, \quad \phi = \omega_8 \cos \theta - \omega_0 \sin \theta, \\ 5. \quad & \omega = \omega_8 \sin \theta + \omega_0 \cos \theta, \quad \phi = \omega_8 \cos \theta - \omega_0 \sin \theta, \\ 6. \quad & \omega = -\omega_8 \sin \theta + \omega_0 \cos \theta, \quad \phi = \omega_8 \cos \theta + \omega_0 \sin \theta, \\ 7. \quad & \omega = \omega_8 \sin \theta - \omega_0 \cos \theta, \quad \phi = -\omega_8 \cos \theta - \omega_0 \sin \theta, \\ 8. \quad & \omega = -\omega_8 \sin \theta - \omega_0 \cos \theta, \quad \phi = -\omega_8 \cos \theta + \omega_0 \sin \theta. \end{aligned} \quad (17)$$

3. Physically acceptable ω - ϕ mixing forms

Further, we demonstrate that some of the ω - ϕ mixing forms in (17) are physically non-acceptable. With this aim, quark representation of the ω_8 and ω_0 mesons from (2) are taken into account and ideal mixing angle value $\theta = 35.3^\circ$ is used. Substituting all this into Eqs. (17), one gets an explicit quark representation of the ϕ and ω mesons as follows:

$$\begin{aligned}
 1. \quad \omega &= +\frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}) , & \phi &= +s\bar{s} , \\
 2. \quad \omega &= +\frac{1}{3\sqrt{2}} (u\bar{u} + d\bar{d}) + \frac{4}{3\sqrt{2}} s\bar{s} , & \phi &= -\frac{2}{3} (u\bar{u} + d\bar{d}) + \frac{1}{3} s\bar{s} , \\
 3. \quad \omega &= -\frac{1}{3\sqrt{2}} (u\bar{u} + d\bar{d}) - \frac{4}{3\sqrt{2}} s\bar{s} , & \phi &= +\frac{2}{3} (u\bar{u} + d\bar{d}) - \frac{1}{3} s\bar{s} , \\
 4. \quad \omega &= -\frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}) , & \phi &= -s\bar{s} , \\
 5. \quad \omega &= +\frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}) , & \phi &= -s\bar{s} , \\
 6. \quad \omega &= +\frac{1}{3\sqrt{2}} (u\bar{u} + d\bar{d}) + \frac{4}{3\sqrt{2}} s\bar{s} , & \phi &= +\frac{2}{3} (u\bar{u} + d\bar{d}) - \frac{1}{3} s\bar{s} , \\
 7. \quad \omega &= -\frac{1}{3\sqrt{2}} (u\bar{u} + d\bar{d}) - \frac{4}{3\sqrt{2}} s\bar{s} , & \phi &= -\frac{2}{3} (u\bar{u} + d\bar{d}) + \frac{1}{3} s\bar{s} , \\
 8. \quad \omega &= -\frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}) , & \phi &= +s\bar{s} ,
 \end{aligned} \tag{18}$$

from where it is transparent to see that only

$$\begin{aligned}
 1. \quad \omega_0 &= \omega \cos \theta + \phi \sin \theta , \\
 \omega_8 &= \omega \sin \theta - \phi \cos \theta , \\
 4. \quad \omega_0 &= -\omega \cos \theta - \phi \sin \theta , \\
 \omega_8 &= -\omega \sin \theta + \phi \cos \theta , \\
 5. \quad \omega_0 &= \omega \cos \theta - \phi \sin \theta , \\
 \omega_8 &= \omega \sin \theta + \phi \cos \theta , \\
 8. \quad \omega_0 &= -\omega \cos \theta + \phi \sin \theta , \\
 \omega_8 &= -\omega \sin \theta - \phi \cos \theta
 \end{aligned} \tag{19}$$

are physically acceptable.

This result is also directly confirmed from experimentally observed PDG-booklet decays of the ω and ϕ vector mesons

$$\begin{aligned}
 \omega &\rightarrow \pi^+\pi^-\pi^0 \quad (89.2\%), \\
 &\rightarrow \pi^0\gamma \quad (8.28\%), \\
 &\rightarrow \pi^+\pi^- \quad (1.53\%), \\
 \phi &\rightarrow K^+K^- \quad (48.9\%), \\
 &\rightarrow K_L^0K_S^0 \quad (34.2\%), \\
 &\rightarrow \rho\pi + \pi^+\pi^-\pi^0 \quad (15.32\%),
 \end{aligned}
 \tag{20}$$

$$\tag{21}$$

where one finds ω vector meson decaying dominantly into states with the u and d quark content and ϕ vector meson decaying dominantly into K mesons with the strange quark content.

4. Conclusions

Generally, 8 possible ω - ϕ mixing forms have been found, starting from the mass matrix in the ω_8, ω_0 space and carrying out its diagonalisation by means of the orthogonal matrix in two most general forms.

By exploiting quark structure of ω_8, ω_0 and the value of the ω - ϕ mixing angle to be “ideal” 35.3° , it was demonstrated that only four, *i.e.* 1, 4, 5, and 8, are ω - ϕ mixing forms physically acceptable. Crucial in such a classification is quark structure of ω and ϕ vector mesons which is also confirmed by experimental data of their dominant decays.

The work was supported by VEGA grant No. 2/0153/17.

REFERENCES

- [1] M. Gourdin, *Unitary Symmetries and Their Application to High Energy Physics*, North-Holland, 1967.
- [2] J.J. Sakurai, *Currents and Mesons*, Chicago Lectures in Physics, 1969.
- [3] F.E. Close, *An Introduction to Quarks and Partons*, Academic Press, 1979.
- [4] S. Gasiorowicz, *Elementary Particle Physics*, Springer-Verlag, 1966.
- [5] C. Patrignani *et al.* [Particle Data Group], *Chin. Phys. C* **40**, 100001 (2016).