Vol. 11 (2018) *Acta Physica Polonica B Proceedings Supplement* No 3

ARBITRARY ω–ϕ MIXING FORM IN GELL-MANN–OKUBO QUADRATIC MASS RELATION CREATES THE SAME MIXING ANGLE θ VALUE*

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(Received August 7, 2018)

The inverse relations of the four independent couples of physically acceptable ω–ϕ mixing forms give expressions for ω_8 and ω_0 as functions of the unknown mixing angle θ and physical states ω and ϕ. Substituting for expressions obtained in such a way for ω_8 repeatedly into Gell-Mann–Okubo quadratic mass relation, which yields quadratic mass of $m^2_{\omega_8}$ as a combination of quadratic masses of $K^*(980)$ and $\rho^0(770)$ vector mesons, always determines the same value of mixing angle θ. Next, the same result is obtained also by using all physically non-acceptable ω–ϕ mixing forms.

DOI:10.5506/APhysPolBSupp.11.525

1. Introduction

In [1], we have demonstrated that there are generally eight possible ω–ϕ mixing forms

$$
\begin{array}{lll}
1. & \omega = \omega_8 \sin\theta + \omega_0 \cos\theta\,, & \phi = -\omega_8 \cos\theta + \omega_0 \sin\theta\,, \\
2. & \omega = -\omega_8 \sin\theta + \omega_0 \cos\theta\,, & \phi = -\omega_8 \cos\theta - \omega_0 \sin\theta\,, \\
3. & \omega = \omega_8 \sin\theta - \omega_0 \cos\theta\,, & \phi = \omega_8 \cos\theta + \omega_0 \sin\theta\,, \\
4. & \omega = -\omega_8 \sin\theta - \omega_0 \cos\theta\,, & \phi = \omega_8 \cos\theta - \omega_0 \sin\theta\,, \\
5. & \omega = \omega_8 \sin\theta + \omega_0 \cos\theta\,, & \phi = \omega_8 \cos\theta - \omega_0 \sin\theta\,,
\end{array}
$$

* Presented at “Excited QCD 2018”, Kopaonik, Serbia, March 11–15, 2018.

$$
\begin{aligned}
&6.\quad \omega = -\omega_8 \sin\theta + \omega_0 \cos\theta\,, && \phi = \omega_8 \cos\theta + \omega_0 \sin\theta\,,\\
&7.\quad \omega = \omega_8 \sin\theta - \omega_0 \cos\theta\,, && \phi = -\omega_8 \cos\theta - \omega_0 \sin\theta\,,\\
&8.\quad \omega = -\omega_8 \sin\theta - \omega_0 \cos\theta\,, && \phi = -\omega_8 \cos\theta + \omega_0 \sin\theta
\end{aligned}
\tag{1}
$$

from which only four, to be denoted by

$$
\begin{aligned}
1.\quad \omega &= \omega_8 \sin\theta + \omega_0 \cos\theta\,,\\
\phi &= -\omega_8 \cos\theta + \omega_0 \sin\theta\,,\\
4.\quad \omega &= -\omega_8 \sin\theta - \omega_0 \cos\theta\,,\\
\phi &= \omega_8 \cos\theta - \omega_0 \sin\theta\,,\\
5.\quad \omega &= \omega_8 \sin\theta + \omega_0 \cos\theta\,,\\
\phi &= \omega_8 \cos\theta - \omega_0 \sin\theta\,,\\
8.\quad \omega &= -\omega_8 \sin\theta - \omega_0 \cos\theta\,,\\
\phi &= -\omega_8 \cos\theta + \omega_0 \sin\theta
\end{aligned}
\tag{2}
$$

are physically acceptable.

That is one considerable result of our investigations.

Another one is demonstrated in this contribution and it concerns of a determination of the ω–ϕ mixing angle θ value by using the Gell-Mann–Okubo quadratic mass relation.

Further, it will be clearly exhibited that arbitrary ω–ϕ mixing form, physically acceptable or physically non-acceptable, to be applied in Gell-Mann–Okubo quadratic mass relation of 1^- vector mesons, creates for mixing angle θ the same value.

2. Gell-Mann–Okubo quadratic mass relation

An application of the SU(3) symmetry to a classification of mesons and baryons revealed an existence of the "quarks", and induced the idea that "mesons" are bound states of quarks and antiquarks, and "baryons" are compound of 3 quarks.

Moreover, experimentally observed hadrons with similar properties are, according to irreducible representations of the SU(3) group, arranged into octuplets, decuplets, 27plets, 35plets *etc.*

As it is well-known, *e.g.* the nonet of 1^- vector mesons can be represented by 3×3 octet matrix and a singlet ω_0 of the form of

$$
V = \begin{pmatrix} \omega_8/\sqrt{6} + \rho^0/\sqrt{2} & \rho^+ & K^{*+} \\ \rho^- & \omega_8/\sqrt{6} - \rho^0/\sqrt{2} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -2\omega_8/\sqrt{6} \end{pmatrix}, \quad \omega_0\,. \tag{3}
$$

The mass of every vector meson from matrix (3) can be generally expressed through its quantum numbers, such as strangenes S and isospin I, which leads to the following Gell-Mann–Okubo quadratic mass relation:

$$m^2(\omega_8) = \frac{4\frac{m^2(K^{*0})+m^2(\bar{K}^{*0})}{2} - m^2\left(\rho^0\right)}{3} = (932.14\ \mathrm{MeV})^2\,, \tag{4}$$

which next is used for a determination of the ω–ϕ mixing angle θ value.

3. Determination of the ω–ϕ mixing angle value

As the unitary singlet is denoted by ω_0 and the unitary octet by ω_8, K^*, $\bar{K}^*$, ρ, then physically acceptable mixing forms between ω, ϕ and ω_8, ω_0 exist as they are presented by (2).

The reversed relations to the mixing forms (2) are obtained by solutions always of the two algebraic equations 1, 4, 5 and 8, with two unknowns, ω_8 and ω_0, and as a result, one gets

$$\begin{array}{rrcl}
1. & \omega_0 &=& \omega\cos\theta + \phi\sin\theta\,,\\
 & \omega_8 &=& \omega\sin\theta - \phi\cos\theta\,,\\
4. & \omega_0 &=& -\omega\cos\theta - \phi\sin\theta\,,\\
 & \omega_8 &=& -\omega\sin\theta + \phi\cos\theta\,,\\
5. & \omega_0 &=& \omega\cos\theta - \phi\sin\theta\,,\\
 & \omega_8 &=& \omega\sin\theta + \phi\cos\theta\,,\\
8. & \omega_0 &=& -\omega\cos\theta + \phi\sin\theta\,,\\
 & \omega_8 &=& -\omega\sin\theta - \phi\cos\theta\,.
\end{array} \tag{5}$$

If orthogonal states $|\omega\rangle$ and $|\phi\rangle$ are "eigenfunctions" of the quadratic mass operator $\mathcal{M}^2$, then the non-diagonal matrix elements are equal to zero

$$\langle\omega|\mathcal{M}^2|\phi\rangle = \langle\phi|\mathcal{M}^2|\omega\rangle = 0\,. \tag{6}$$

Then utilizing from the first relation of (5) for $|\omega_8\rangle$ the expression ω_8

1. a calculation of the mass squared of the ω_8 particle gives

$$\begin{aligned}
m^2(\omega_8) &= \langle\omega_8|\mathcal{M}^2|\omega_8\rangle\\
&= (\sin\theta\langle\omega| - \cos\theta\langle\phi|)\mathcal{M}^2(|\omega\rangle\sin\theta - |\phi\rangle\cos\theta)\\
&= \sin^2\theta\langle\omega|\mathcal{M}^2|\omega\rangle + \cos^2\theta\langle\phi|\mathcal{M}^2|\phi\rangle\\
&\quad - \sin\theta\cos\theta\langle\omega|\mathcal{M}^2|\phi\rangle - \cos\theta\sin\theta\langle\phi|\mathcal{M}^2|\omega\rangle
\end{aligned} \tag{7}$$

and exploiting (6), finally, one gets

$$m^2(\omega_8) = m^2(\omega)\sin^2\theta + m^2(\phi)\cos^2\theta\,. \tag{8}$$

If from the second relation of (5) for $|\omega_8\rangle$, the expression ω_8 is used

4. a calculation of the mass squared of the ω_8 particle gives

$$\begin{aligned} m^2(\omega_8) &= \langle\omega_8|\mathcal{M}^2|\omega_8\rangle \\ &= (-\sin\theta\langle\omega| + \cos\theta\langle\phi|)\mathcal{M}^2(-|\omega\rangle\sin\theta + |\phi\rangle\cos\theta) \\ &= \sin^2\theta\langle\omega|\mathcal{M}^2|\omega\rangle + \cos^2\theta\langle\phi|\mathcal{M}^2|\phi\rangle \\ &\quad -\sin\theta\cos\theta\langle\omega|\mathcal{M}^2|\phi\rangle - \cos\theta\sin\theta\langle\phi|\mathcal{M}^2|\omega\rangle \end{aligned} \tag{9}$$

and exploiting (6), finally, one gets

$$m^2(\omega_8) = m^2(\omega)\sin^2\theta + m^2(\phi)\cos^2\theta\,. \tag{10}$$

If from the third relation of (5) for $|\omega_8\rangle$, the expression ω_8 is used

5. a calculation of the mass squared of the ω_8 particle gives

$$\begin{aligned} m^2(\omega_8) &= \langle\omega_8|\mathcal{M}^2|\omega_8\rangle \\ &= (\sin\theta\langle\omega| + \cos\theta\langle\phi|)\mathcal{M}^2(|\omega\rangle\sin\theta + |\phi\rangle\cos\theta) \\ &= \sin^2\theta\langle\omega|\mathcal{M}^2|\omega\rangle + \cos^2\theta\langle\phi|\mathcal{M}^2|\phi\rangle \\ &\quad +\sin\theta\cos\theta\langle\omega|\mathcal{M}^2|\phi\rangle + \cos\theta\sin\theta\langle\phi|\mathcal{M}^2|\omega\rangle \end{aligned} \tag{11}$$

and exploiting (6), finally, one gets

$$m^2(\omega_8) = m^2(\omega)\sin^2\theta + m^2(\phi)\cos^2\theta\,. \tag{12}$$

If from the fourth relation of (5) for $|\omega_8\rangle$, the expression ω_8 is used

8. a calculation of the mass squared of the ω_8 particle gives

$$\begin{aligned} m^2(\omega_8) &= \langle\omega_8|\mathcal{M}^2|\omega_8\rangle \\ &= (-\sin\theta\langle\omega| - \cos\theta\langle\phi|)\mathcal{M}^2(-|\omega\rangle\sin\theta - |\phi\rangle\cos\theta) \\ &= \sin^2\theta\langle\omega|\mathcal{M}^2|\omega\rangle + \cos^2\theta\langle\phi|\mathcal{M}^2|\phi\rangle \\ &\quad +\sin\theta\cos\theta\langle\omega|\mathcal{M}^2|\phi\rangle + \cos\theta\sin\theta\langle\phi|\mathcal{M}^2|\omega\rangle \end{aligned} \tag{13}$$

and exploiting (6), finally, one gets

$$m^2(\omega_8) = m^2(\omega)\sin^2\theta + m^2(\Phi)\cos^2\theta\,. \tag{14}$$

If the masses of $\omega_8, \omega(782), \phi(1020)$ are taken from [2]

$$\begin{aligned} m(\omega_8) &= 932.14\ \text{MeV}\,, &(15)\\ m(\omega) &= 782.65\ \text{MeV}\,, &(16)\\ m(\phi) &= 1019.462\ \text{MeV}\,, &(17) \end{aligned}$$

then from four previous identical relations, by means of the expression

$$\sin^2\theta = \frac{m^2(\phi) - m^2(\omega_8)}{m^2(\phi) - m^2(\omega)}\,, \tag{18}$$

one obtains

$$\theta = \sin^{-1} 0.63192 = 39.19^\circ$$

and, in this way, we have demonstrated that the ω–ϕ mixing angle θ value does not depend on the physically acceptable ω–ϕ mixing forms.

In a similar way, one can convince himself that the θ-value does not even depend on the physically non-acceptable ω–ϕ mixing forms and is also equal to $\theta = \sin^{-1} 0.63192 = 39.19^\circ$.

There is a question: In which physical "circumstances" the physically acceptable forms of ω–ϕ mixing and physically non-acceptable forms of ω–ϕ mixing will produce different results.

This question will be a subject of our further investigations.

4. Conclusions

Starting from the physically acceptable ω–ϕ mixing forms, calculating their reversed relations and exploiting the Gell-Mann–Okubo quadratic mass formula for 1^- octet of vector mesons, the ω–ϕ mixing angle $\theta = 39.19^\circ$ has been determined.

However, subsequently it was verified that this result is valid without any specification to physically acceptable, or physically non-acceptable, mixing forms.

The problem is, however, arisen in the calculation of the mixing angle θ' for the first excited states of vector mesons, where a substitution of the concerned particle masses into (18) gives $\sin^2\theta' = 1.011 > 1$.

In the evaluation of θ'', the following value $\sin^2\theta'' = 0.9223$ is found, from which the value $\theta'' = \sin^{-1} 0.96 = 73.81^\circ$ is determined.

The work was supported by the VEGA grant No. 2/0153/17.

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