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HYDRODYNAMICS OF QCD MATTER*

PASI HUOVINEN

Institute of Theoretical Physics, University of Wrocław
pl. M. Borna 9, 50-204 Wrocław, Poland

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In ultrarelativistic collisions of heavy ions, we have seen behaviour which can be interpreted as a formation of locally thermalised system expanding as a fluid. I discuss the use of hydrodynamics to model the expansion of the collision system and what such a modelling has taught us about the properties of QCD matter.

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1. Fluid dynamics

In ultrarelativistic heavy-ion collisions, we want to create strongly interacting matter–matter in a sense that the thermodynamical concepts like temperature and pressure apply. Thus, it is reasonable to try to use fluid dynamics to describe how the system formed in a collision expands and cools. If the density of conserved charges is zero, the equations of motion are the conservation laws for energy and momentum

$$\partial_\mu T^{\mu\nu} = 0\,, \qquad \text{where} \qquad T^{\mu\nu} = (\epsilon + P + \Pi)u^\mu u^\nu - (P + \Pi)g^{\mu\nu} + \pi^{\mu\nu}\,,$$

and ϵ is energy density in the rest frame of the fluid, P equilibrium pressure, and Π bulk pressure. The fluid 4-velocity is denoted by u^μ, $g^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ the metric tensor and $\pi^{\mu\nu}$ the shear-stress tensor. These four equations contain eleven unknowns. To make this set of equations solvable, we need an equation of state (EoS) connecting equilibrium pressure to energy density, $P = P(\epsilon)$, and constitutive equations for shear stress and bulk pressure. A relativistic generalisation of Navier–Stokes equations, where the dissipative quantities are directly proportional to the gradients of flow velocity, leads to non-causal behaviour. Therefore, heavy-ion collisions are modelled using so-called Israel–Stewart, a.k.a. transient, fluid dynamics where $\pi^{\mu\nu}$ and Π are dynamical variables relaxing to their Navier–Stokes values on characteristic relaxation times τ_π and τ_Π.

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Once the equation of state and constitutive equations are fixed, the expansion dynamics is uniquely defined, but the actual solution depends on the boundary conditions: The initial distribution of matter and the criterion for the end of evolution. Hydrodynamics does not provide either of these, but they have to be supplied by other models. The end of evolution is usually taken to be a hypersurface of constant temperature or energy density, where the fluid is converted to particles (particlization). In pure hydrodynamical models, all interactions are assumed to cease at this point and particle distributions freeze out. In so-called hybrid models, particles formed at the end of fluid dynamical evolution are fed into a hadron cascade describing the late hadronic stage.

2. Azimuthal anisotropies of final particle distribution

In the primary nucleon–nucleon collisions, particle production is azimuthally isotropic, but the distribution of observed particles in $A+A$ collisions is not. The anisotropy originates from the rescatterings of the produced particles: In a non-central collision, the collision zone has an elongated shape. If a particle is heading to a direction where the collision zone is thick, it has a larger probability to scatter and change its direction than a particle heading to a direction where the collision zone is thin. Thus, more particles end up in the direction where the edge of the collision zone is near. Or, in a hydrodynamical language, the pressure gradient between the center of the system and the vacuum is larger in the "thin" direction, the flow velocity is thus larger in that direction and more particles are emitted in that direction.

The anisotropy is quantified in terms of a Fourier expansion of the azimuthal distribution. The coefficients of this expansion, v_n, and the associated event angles, ψ_n, are defined as

$$v_n = \langle \cos\left[n\left(\phi - \psi_n\right)\right] \rangle \qquad \text{and} \qquad \psi_n = \frac{1}{n} \arctan \frac{\langle p_{\mathrm{T}} \sin(n\phi) \rangle}{\langle p_{\mathrm{T}} \cos(n\phi) \rangle} \,.$$

Of these coefficients v_1 is called directed, v_2 elliptic, and v_3 triangular flow.

Elliptic flow of charged hadrons at RHIC was measured to be quite large and to increase with decreasing centrality [1], as expected if it has geometric origin as described. Thus, an $A+A$ collision is not just a sum of independent *pp* collisions, but there must be rescatterings among the particles formed in the collision. The measured values of elliptic flow were also very close to the hydrodynamically calculated values [2], which is a strong indication of hydrodynamical behaviour of the matter.

3. η/s has very low minimum

The anisotropy coefficients are sensitive to fluid's equation of state and dissipative coefficients. Shear viscosity strongly reduces v_2 [3], and thus extracting the ratio of the shear viscosity coefficient to entropy density, η/s, from the data is easy in principle: One needs to calculate the p_{T}-averaged v_2 of charged hadrons using various values of η/s and choose the value of η/s which reproduces the data. Unfortunately, our ignorance of the initial state makes this approach way more complicated. The values of v_2 calculated using non-zero value of η/s fit the data best [4], but the preferred value depends on how the initial state of hydrodynamic evolution is chosen: Whether one uses the so-called MC-Glauber [5] or MC-KLN [6] model causes a factor of two difference in the preferred value ($\eta/s = 0.08$–0.16) [4].

The calculations have been improved since [4] by a better treatment of the hadronic phase (see, *e.g.*, [7]), but uncertainty remains the same. This uncertainty can be reduced by studying the higher flow coefficients ($v_n, n > 2$). Because of the fluctuations of the positions of nucleons in the nuclei, the initial collision region has an irregular shape which fluctuates event-by-event, see figure 1, and thus all the coefficients v_n are finite [8]. As illustrated in figure 2, the larger the n, the more sensitive the coefficient v_n is to viscosity [9]. This provides a possibility to distinguish between different initialisations.

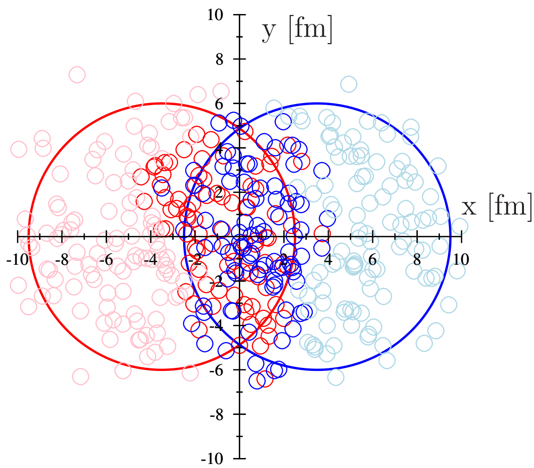

Fig. 1. An example of the positions of interacting nuclei in the MC-Glauber model. Figure is from [10], and reprinted with permission.

On the other hand, in event-by-event studies, it is not sufficient to reproduce only the average values of v_n, but the fluctuations of the flow coefficients should be reproduced as well. The distributions of these fluctuations provide

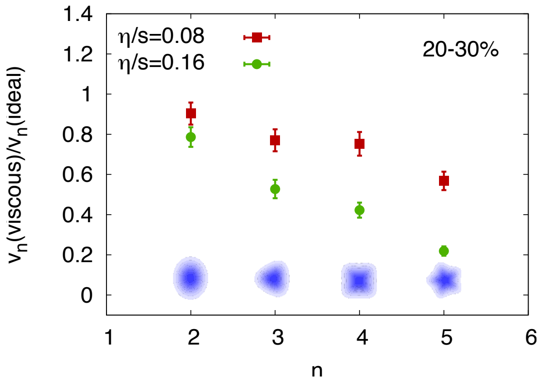

Fig. 2. Ratio of the anisotropy coefficients of charged hadrons in viscous calculation to the coefficients in ideal fluid calculation [9]. Figure is from [11], courtesy to Bjoern Schenke.

a way to constrain the fluctuation spectrum of initial-state models independently of the dissipative properties of the fluid. As shown in figure 3, once the average v_n has been scaled out, the distributions of these fluctuations, *i.e.*, $(v_n - \langle v_n \rangle)/\langle v_n \rangle$ or $v_n/\langle v_n \rangle$, are almost independent of viscosity. The independence extends to other details of the evolution to such an extent that the distributions of the fluctuations of initial anisotropies are good approximations of the measured distributions of v_n [12], and thus it is sufficient to compare the fluctuations of initial shape, ϵ_n, to the observed fluctuations of v_n. Neither the MC-Glauber nor MC-KLN model seems to be able to reproduce the measured fluctuations [13], whereas the recent calculations using so-called IP-Glasma [14] and EKRT [15] initialisations reproduce both the fluctuations and the average values of v_2, v_3 and v_4 [15, 16].

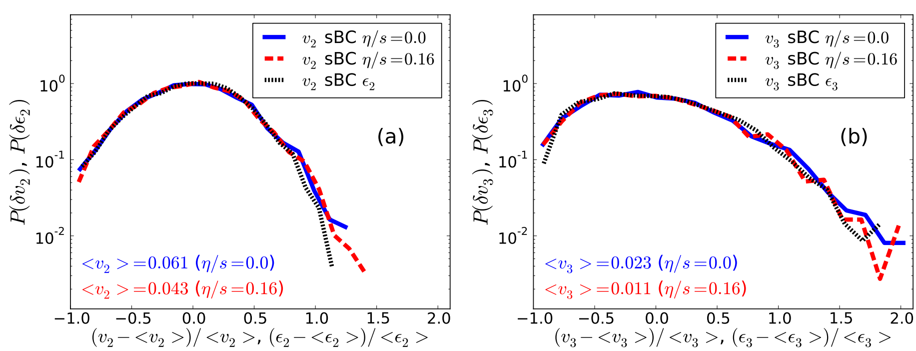

Fig. 3. Probability distributions: (a) $P(\delta v_2)$ and $P(\delta \epsilon_2)$, and (b) $P(\delta v_3)$ and $P(\delta \epsilon_3)$ in the 20–30% centrality class with the sBC Glauber model initialisation and two values of η/s, $\eta/s = 0$ and $\eta/s = 0.16$. $\delta v_n = (v_n - \langle v_n \rangle)/\langle v_n \rangle$ and $\epsilon_n = (\epsilon_n - \langle \epsilon_n \rangle)/\langle \epsilon_n \rangle$. Figures are from [12].

4. Temperature dependence of η/s

In the calculations discussed in the previous section, the η/s ratio was assumed to be constant. We know no fluid where the η/s ratio would be temperature-independent, and there are theoretical reasons to expect it to depend on temperature with a minimum around T_c [17]. Thus, the temperature-independent η/s is only an effective viscosity, and its connection to the physical, temperature-dependent, shear viscosity coefficient is unclear. What complicates the determination of the physical shear viscosity coefficient is that the sensitivity of the anisotropies to dissipation varies during the evolution of the system. As studied in [18], at RHIC ($\sqrt{s_{NN}} = 200$ GeV), v_2 is insensitive to the value of η/s above T_c, but very sensitive to its minimum value around T_c, and to its value in the hadronic phase below T_c. At the lower LHC energy, $\sqrt{s_{NN}} = 2.76$ TeV, η/s in the plasma phase does affect the final v_2, but not more than η/s in the hadronic phase.

Thus, constraining the temperature dependence of shear viscosity is difficult. As shown in figure 4, very different temperature dependencies (left panel) can lead to a good reproduction of v_2 at LHC [15], and once v_2 is reproduced, v_3 and v_4 do not provide further resolving power either. However, the situation changes once we calculate the anisotropies at RHIC using the same parametrisations for η/s (right panel of figure 4): Not all parametrisations work equally well, and some can be excluded. Unfortunately, some ambiguity remains and two quite different parametrisations work equally well. Thus, we cannot yet say how η/s depends on temperature, nor what its minimum value is, but we know that its minimum value near T_c is close to the postulated minimum of $\eta/s = 1/4\pi$.

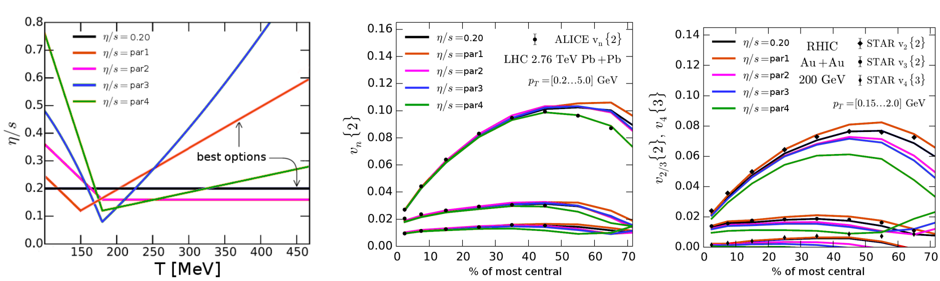

Fig. 4. Left: Parametrisations of the temperature dependence of η/s. Center: Centrality dependence of the flow coefficients $v_n\{2\}$ of charged hadrons in $\sqrt{s_{NN}} =$ 2.76 TeV Pb+Pb collisions at the LHC, and right: the coefficients $v_2\{2\}, v_3\{2\}$ and $v_4\{3\}$ of charged hadrons in 200 GeV Au+Au collisions at RHIC calculated using the five $(\eta/s)(T)$ parametrisations shown in the left panel [15]. The experimental data are from the ALICE [19] and STAR [20] collaborations. Results from [15], figures courtesy to Harri Niemi.

5. Further reading

My talk was an updated version of the talks I gave in the Exited QCD 2015 [21] and the 2$^{\rm nd}$ International Conference on Particle Physics and Astrophysics [22]. A reader interested in the theory of hydrodynamics in ultra-relativistic heavy-ion collisions can find a good introduction in [23]. General reviews about hydrodynamics and flow can be found in [24–26].

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REFERENCES

[1] K.H. Ackermann *et al.* [STAR Collaboration], *Phys. Rev. Lett.* **86**, 402 (2001).

[2] P.F. Kolb *et al.*, *Phys. Lett. B* **500**, 232 (2001).

[3] D. Teaney, *Phys. Rev. C* **68**, 034913 (2003).

[4] M. Luzum, P. Romatschke, *Phys. Rev. C* **78**, 034915 (2008) [*Erratum ibid.* **79**, 039903 (2009)].

[5] M.L. Miller *et al.*, *Annu. Rev. Nucl. Part. Sci.* **57**, 205 (2007).

[6] D. Kharzeev, E. Levin, M. Nardi, *Phys. Rev. C* **71**, 054903 (2005); H.-J. Drescher, Y. Nara, *Phys. Rev. C* **75**, 034905 (2007); **76**, 041903 (2007).

[7] H. Song *et al.*, *Phys. Rev. Lett.* **106**, 192301 (2011) [*Erratum ibid.* **109**, 139904 (2012)].

[8] B. Alver, G. Roland, *Phys. Rev. C* **81**, 054905 (2010) [*Erratum ibid.* **82**, 039903 (2010)].

[9] B. Schenke, S. Jeon, C. Gale, *Phys. Rev. C* **85**, 024901 (2012).

[10] H. Holopainen, Research report 4/2011, Department of Physics, University of Jyväskylä, `http://urn.fi/URN:ISBN:978-951-39-4371-4`

[11] P. Huovinen, *PoS* **ConfinementX**, 165 (2012).

[12] H. Niemi *et al.*, *Phys. Rev. C* **87**, 054901 (2013).

[13] J. Jia [ATLAS Collaboration], *Nucl. Phys. A* **904–905**, 421c (2013).

[14] B. Schenke, P. Tribedy, R. Venugopalan, *Phys. Rev. Lett.* **108**, 252301 (2012); *Phys. Rev. C* **86**, 034908 (2012).

[15] H. Niemi, K.J. Eskola, R. Paatelainen, *Phys. Rev. C* **93**, 024907 (2016).

[16] C. Gale *et al.*, *Phys. Rev. Lett.* **110**, 012302 (2013); *Nucl. Phys. A* **904–905**, 409c (2013).

[17] L.P. Csernai *et al.*, *Phys. Rev. Lett.* **97**, 152303 (2006).

[18] H. Niemi *et al.*, *Phys. Rev. C* **86**, 014909 (2012).

[19] K. Aamodt *et al.* [ALICE Collaboration], *Phys. Rev. Lett.* **107**, 032301 (2011).

[20] J. Adams *et al.* [STAR Collaboration], *Phys. Rev. Lett.* **92**, 062301 (2004); *Phys. Rev. C* **72**, 014904 (2005); L. Adamczyk *et al.* [STAR Collaboration], *Phys. Rev. C* **88**, 014904 (2013).

[21] P. Huovinen, *Acta Phys. Pol. B Proc. Suppl.* **8**, 327 (2015).

[22] P. Huovinen, *J. Phys.: Conf. Ser.* **798**, 012063 (2017).

[23] S. Jeon, U. Heinz, *Int. J. Mod. Phys. E* **24**, 1530010 (2015).

[24] U. Heinz, R. Snellings, *Annu. Rev. Nucl. Part. Sci.* **63**, 123 (2013).

[25] C. Gale, S. Jeon, B. Schenke, *Int. J. Mod. Phys. A* **28**, 1340011 (2013).

[26] P. Huovinen, *Int. J. Mod. Phys. E* **22**, 1330029 (2013).