

ACCESSING THE TOPOLOGICAL SUSCEPTIBILITY VIA THE GRIBOV HORIZON: AN UPDATE*

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We analyze the topological susceptibility in SU(3) and SU(2) gauge theories, using Padé approximation and Refined Gribov–Zwanziger gluon propagator.

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1. Introduction

From [1], we know that the Faddeev–Popov construction is not valid at the non-perturbative level. In this regime, there are Gribov copies, multiple intersections of the gauge orbits with the hypersurface corresponding to a given gauge condition $f(A) = 0$. To “fix” this, Gribov proposed the Gribov region Ω [1], which is free from infinitesimal Gribov copies. Taking into account the dimension two condensates, $\langle A_\mu^a A_\mu^a \rangle$ and $\langle \bar{\varphi}_\mu^{ab} \varphi_\mu^{ab} - \bar{\omega}_\mu^{ab} \omega_\mu^{ab} \rangle$, the Gribov region can be implemented by the action

$$S = S_{\text{YM}} + S_{\text{GF}} + S_{\text{RGZ}} + S_\tau, \quad (1)$$

where S_{YM} is the Yang–Mills action, S_{GF} is the Faddeev–Popov gauge-fixing in linear covariant gauges,

$$S_{\text{GF}} = \int d^4x \left(\frac{\alpha}{2} b^a b^a + i b^a \partial_\mu A_\mu^a + \bar{c}^a \partial_\mu D_\mu^{ab}(A) c^b \right), \quad (2)$$

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whereby α is the gauge parameter and $-\partial_\mu D_\mu^{ab} = \mathcal{M}^{ab}(A^h) = -\delta^{ab}\partial^2 + g f^{abc}(A^h)_\mu^c \partial_\mu$ is the Faddeev–Popov operator, S_{RGZ} is the Refined Gribov–Zwanziger (RGZ) action [2]

$$\begin{aligned} S_{\text{RGZ}} = & \int d^4x \left(-\bar{\varphi}_\nu^{ac} \mathcal{M}^{ab}(A^h) \varphi_\nu^{bc} + \bar{\omega}_\nu^{ac} \mathcal{M}^{ab}(A^h) \omega_\nu^{bc} \right. \\ & + \gamma^2 g f^{abc} (A^h)_\mu^a (\varphi_\mu^{bc} + \bar{\varphi}_\mu^{bc}) + \frac{m^2}{2} \int d^4x (A^h)_\mu^a (A^h)_\mu^a \\ & \left. + M^2 \int d^4x (\bar{\varphi}_\mu^{ab} \varphi_\mu^{ab} - \bar{\omega}_\mu^{ab} \omega_\mu^{ab}) \right), \end{aligned} \quad (3)$$

and where γ is the Gribov parameter fixed by the gap equation [3]

$$\left\langle f^{abc} A_\mu^{h,a} (\varphi_\mu^{bc} + \bar{\varphi}_\mu^{bc}) \right\rangle = 2d (N^2 - 1) \gamma^2 g^{-2}. \quad (4)$$

The field configuration A_μ^h is a gauge-invariant non-local power series in the gauge field. It can be obtained by minimizing the functional $f_A[u]$ along the gauge orbit of A_μ [4]

$$f_A[u] \equiv \min_{\{u\}} \text{Tr} \int d^4x A_\mu^u A_\mu^u, \quad A_\mu^u = u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u. \quad (5)$$

The result of this is a local minimum given by the transverse field configuration

$$\begin{aligned} A_\mu^h &= \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} \right) \phi_\nu, \quad \partial_\mu A_\mu^h = 0, \\ \phi_\nu &= A_\nu - ig \left[\frac{1}{\partial^2} \partial A, A_\nu \right] + \frac{ig}{2} \left[\frac{1}{\partial^2} \partial A, \partial_\nu \frac{1}{\partial^2} \partial A \right] + O(A^3). \end{aligned} \quad (6)$$

We lay [2, 5]

$$A_\mu^h = (A^h)_\mu^a T^a = h^\dagger A_\mu^a T^a h + \frac{i}{g} h^\dagger \partial_\mu h, \quad (7)$$

with $h = e^{ig\xi^a T^a}$. We get the local gauge invariance of A_μ^h under

$$h \rightarrow u^\dagger h, \quad h^\dagger \rightarrow h^\dagger u, \quad A_\mu \rightarrow u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u, \quad (8)$$

with $u \in \text{SU}(N)$. The term

$$S_\tau = \int d^4x \tau^a \partial_\mu (A^h)_\mu^a \quad (9)$$

implements the transversality of the composite operator $(A^h)_\mu^a$, $\partial_\mu(A^h)_\mu^a = 0$, where τ is the Lagrange multiplier. Furthermore, the action S is BRST invariant, $sS = 0$, see [2] for more details.

For our analysis, the general form of the gluon propagator [5]

$$D_{\mu\nu}(p) = D(p)P_{\mu\nu}(p) + L(p)\frac{p_\mu p_\nu}{p^2} \quad (10)$$

is more interesting. We get that in the RGZ case,

$$D(p) = \frac{p^2 + M^2}{p^4 + (M^2 + m^2)p^2 + M^2m^2 + \lambda^4} \quad (11)$$

is the transverse form factor containing all non-trivial information and $L(p) = \alpha/p^2$ is the longitudinal one, with

$$P_{\mu\nu}(p) = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}, \quad L_{\mu\nu}(p) = \frac{p_\mu p_\nu}{p^2} \quad (12)$$

the transversal and longitudinal projectors, respectively. $L(p)$ is exactly known due to BRST invariance.

2. The topological susceptibility

Due to Witten and Veneziano's work [6], the topological susceptibility can be linked to the η' mass by

$$m_{\eta'}^2 = \frac{4N_f}{f_\pi^2} \chi_{\theta=0, N_f=0}^4 = \mathcal{O}(1/N), \quad (13)$$

where θ is the vacuum angle and f_π the pion decay constant. In [6], the authors analyzed the vacuum topology fluctuations by the Veneziano ghost,

$$p_\mu p_\nu \langle K_\mu K_\nu \rangle_{p=0} \neq 0, \quad (14)$$

whereby K_μ is the topological Chern–Simons current

$$K_\mu = \frac{g^2}{16\pi^2} \epsilon_{\mu\nu\rho\sigma} A_{\nu,a} \left(\partial_\rho A_\sigma^a + \frac{g}{3} f^{abc} A_\rho^b A_\sigma^c \right). \quad (15)$$

In Euclidean conventions [7], the topological susceptibility χ^4 is given by

$$\chi^4 = - \lim_{p^2 \rightarrow 0} p_\mu p_\nu \langle K_\mu K_\nu \rangle \geq 0. \quad (16)$$

The current correlator $\langle K_\mu K_\nu \rangle$ can be calculated via the Källén–Lehmann spectral density representation,

$$\begin{aligned} \langle K_\mu(p) K_\nu(-p) \rangle &= \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \mathcal{K}_\perp(p^2) + \frac{p_\mu p_\nu}{p^2} \mathcal{K}_\parallel(p^2) \\ &\equiv \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \int_0^\infty d\tau \frac{\rho_\perp(\tau)}{\tau + p^2} \\ &\quad + \frac{p_\mu p_\nu}{p^2} \int_0^\infty d\tau \frac{\rho_\parallel(\tau)}{\tau + p^2}, \end{aligned} \quad (17)$$

based on Euclidean invariance. Then (16) becomes

$$-\chi^4 = \lim_{p^2 \rightarrow 0} p^2 \mathcal{K}_\parallel(p^2) = \lim_{p^2 \rightarrow 0} p^2 \int_0^\infty d\tau \frac{\rho_\parallel(\tau)}{\tau + p^2}. \quad (18)$$

From dimensional analysis, we just need 2 subtractions ($\rho_\parallel(\tau) \sim \tau$ for $\tau \rightarrow \infty$) to remove divergences, then the topological susceptibility is only given by

$$-\chi^4 = \lim_{p^2 \rightarrow 0} p^6 \int_0^\infty d\tau \frac{\rho_\parallel(\tau)}{(\tau + p^2) \tau^2}. \quad (19)$$

The spectral density to 1-loop, associated with (17), is given by [8]

$$\rho_\parallel(\tau) = -2A_+ A_- \frac{g^4 (N^2 - 1)}{2^{2d+5} \pi^{7/2} \Gamma\left(\frac{d-1}{2}\right)} \frac{(\tau^2 - 4b^2 - 4a\tau)^{(d-1)/2}}{\tau^{d/2}} \quad (20)$$

for $\tau \geq \tau_c = 2(a + \sqrt{a^2 + b^2})$, where $a = M_2^2/2$ and $b = \sqrt{M_3^4 - M_2^4/4}$.

In the MOM scheme $D(p^2 = \mu^2) = 1/\mu^2$, the gluon propagator is given by

$$D(p^2) = Z \frac{p^2 + M_1^2}{p^4 + M_2^2 p^2 + M_3^4}, \quad Z = \frac{1}{\mu^2} \frac{\mu^4 + M_2^2 \mu^2 + M_3^4}{\mu^2 + M_1^2}. \quad (21)$$

For the dynamical mass parameters, rather than attempting to solve their gap equations (4), we resort to estimates obtained from fitting (21) to lattice propagator data [9].

In the MOM scheme, the strong coupling is effectively small. Therefore, we can consider the perturbative treatment, but first, we use the Padé approximation to estimate the infrared region, where the perturbative result

is not trustworthy. Before taking the limit in (19), we did an $[\mathcal{M} + 2, \mathcal{M}]$ Padé rational function in variable p^2 . The Padé approximation was around $p^2 = \mu^2$. Note that the Padé approximation should not be too small, then the (perturbatively) computed r.h.s. of (19) is still valid, and not too large, so the non-perturbative effects from the presence of the RGZ mass scales in (11) are taken into account, and execute a sensible extrapolation of the approximant to zero momentum to get an estimate for $\chi^4(\mu^2)$. The results for SU(3) are shown in Fig. 1 for $\mathcal{M} = 1, 2, 3$. They are compared with the lattice ball-park value of $\chi \sim 200$ MeV [10]. The error estimation from the uncertainty on $\vec{x} \equiv (M_1^2, M_2^2, M_3^4)$ was computed by $\sigma_\chi(\mu^2) = \sqrt{\sum_i (\partial\chi/\partial x_i)^2 \sigma_{x_i}^2}$, that is showed in Fig. 1 (right).

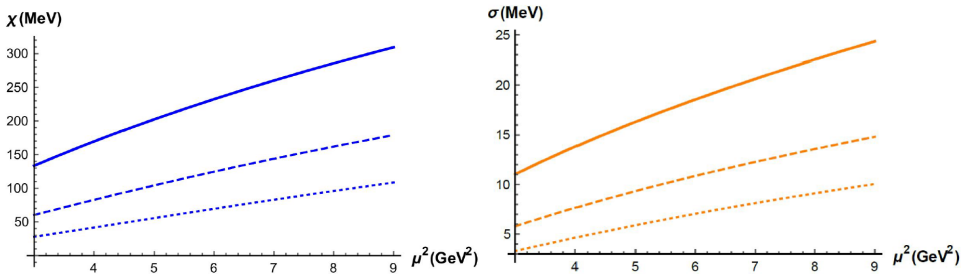


Fig. 1. The SU(3) topological susceptibility χ (left) and its respectively estimated error due to the uncertainty on the fitting parameters (right) for variable μ^2 for $\mathcal{M} = 1, 2, 3$ (solid, dashed, dotted) [8].

For the $N = 2$ case, we followed the same procedure as for $N = 3$, getting the graphs of Fig. 2.

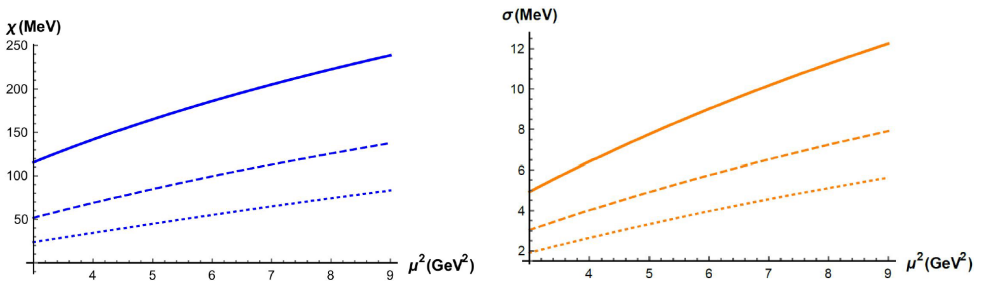


Fig. 2. The SU(2) topological susceptibility χ for variable μ^2 for $\mathcal{M} = 1, 2, 3$ (solid, dashed, dotted) [8].

In SU(2), the lattice prediction for the topological susceptibility is $\chi = 200\text{--}230$ MeV, see [8]. For more details about the calculus and references, see [8].

3. Conclusion

We have studied the topological susceptibility, χ^4 , in SU(2) and SU(3) Euclidean Yang–Mills theory in a generic linear covariant gauge starting from the RGZ action. We have checked that the topological susceptibility is gauge-invariant in the non-perturbative framework as expected. To get estimates for χ^4 , we have done a particular Padé rational function approximation based on the Källén–Lehmann spectral integral representation of the topological current correlation function $\langle KK \rangle$. An improvement of the result is to include the next order correction. However, this will be a challenge because of the enlarged set of vertices in the now considered BRST-invariant RGZ action for the linear covariant gauge.

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