VORTEX MODEL OF THE QCD-VACUUM — SUCCESSES AND PROBLEMS*

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Describing the non-triviality of the QCD vacuum by closed and quantized color magnetic flux lines with some average thickness, the center vortex model can explain the non-perturbative phenomena of QCD, especially confinement and chiral symmetry breaking. There are various methods to identify vortices in lattice calculations, in particular direct and indirect maximal center gauge and adjoint Laplacian gauge. It turns out that the vortex identification suffers from a Gribov copy problem. The non-Abelian Stokes theorem may give a hint how to overcome this problem by investigating the color homogeneity of lattice configurations, especially its behavior for smooth field configurations. We discuss that the homogeneity of the color direction of the gauge field may facilitate detecting regions of center flux.

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1. Models of the QCD vacuum

The non-perturbative properties of QCD originate in the non-triviality of the QCD vacuum. Different models try to explain the corresponding non-vanishing gluon and quark condensates. In the *dual superconductor picture*, the color electric flux is collimated into a tube by Abelian monopoles, detected in lattice calculations via maximal Abelian gauge and Abelian projection, see [1] and references therein. In the *instanton–dyon liquid model*, kVBLL instantons, invented 1998 by Kraan, van Baal, Lee and Lu, are the relevant ingredients, see [2]. The *center vortex model* [3–5] is based on the center symmetry of the action in lattice quantum chromodynamics (LQCD). It describes the properties of the QCD vacuum by percolating vortices, which are closed and quantized magnetic flux lines of finite thickness, condensing in the vacuum. The deconfinement transition can be understood as a depercolation transition concerning the time direction.

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The vortex model combines the features of the above listed models and is capable of explaining:

— non-triviality of the vacuum → gluon condensate,

— area law of Wilson loops, see [5],

— area law for spatial Wilson loops at finite temperature,

— finite temperature phase transition → Polyakov loops,

— orders of phase transitions in SU(2) and SU(3),

— Casimir scaling of heavy-quark potential, see [6],

— spontaneous breaking of scale invariance, see [7],

— monopole picture of confinement → dual superconductor model,

— double winding Wilson loops,

— lumps of topological charge,

— color structure of vortices → instantons,

— chiral symmetry breaking, see [8, 9] → quark condensate.

An efficient method to detect vortices in LQCD is maximal center gauge and center projection, which is based on maximizing an appropriate gauge functional [5]. The projected and therefore thin vortices show the expected scaling behavior but suffer from a Gribov copy problem [10]. With numerical methods, only local maxima of the functional can be found. Some of these local maxima may not correspond to the configuration of thick vortices and, therefore, underestimate the string tension.

The aim of this work is to indicate a possible direction for the solution of this problem.

2. Non-Abelian Stokes law

Vortices form two-dimensional surfaces in four dimensions and pierce the minimal areas of Wilson loops, resulting in the area law if the piercings are uncorrelated. Each piercing contributes a non-trivial center element to a surrounding Wilson loop. We may, therefore, distinguish regions corresponding to trivial and non-trivial center elements, in SU(2) to ± 1, as schematically indicated in the left part of Fig. 1. Regions, fully enclosed by a Wilson loop, contribute with center elements, while regions at the boundary result in different group elements. Hence, the area of the Wilson loop can be split into two parts, as shown on the right-hand side of Fig. 1. The inner part evaluates to an Abelian factor commuting with the outer non-Abelian contribution. According to the non-Abelian Stokes theorem, the Abelian factor, actually only a sign depending on the number of piercings, can be extracted from the algebraic expression of the Wilson loop. This inner part accounts for an area law, while the outer part, proportional to the perimeter, leads to Coulombic behavior.

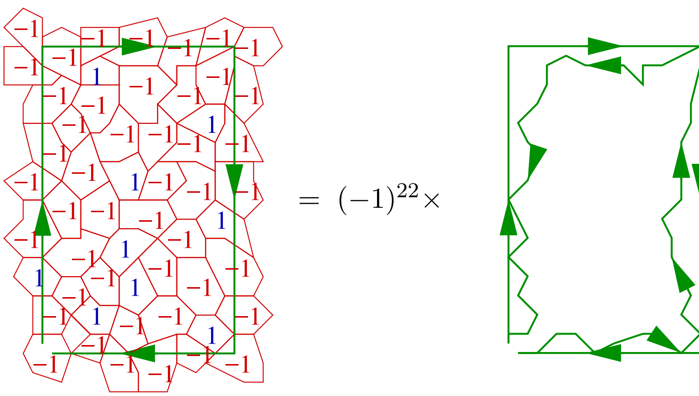

Fig. 1. Gluon field configurations can be split into regions with center flux. According to the non-Abelian Stokes theorem, the parallel transport around such full center fluxes leads to an Abelian factor counting the number of piercings which can be extracted from the algebraic expression for the Wilson loop.

Reversing the above argument about the Abelian factor, we conclude that the parallel transport around the boundary of a connected area, resulting in a center element, teaches whether the number of piercings is even or odd. We intend to use this knowledge as a constraint for the gauge fixing procedure and to improve the vortex detection. A possible method to detect the smallest center regions could be based upon the homogeneity of neighboring plaquettes.

3. Color homogeneity

To define the homogeneity of a 2×2-Wilson loop in SU(2)-LQCD, we decompose the matrix W of this 2×2-Wilson loop according to the non-Abelian Stokes theorem in four plaquette contributions W_j, building up the loop as depicted in Fig. 2.

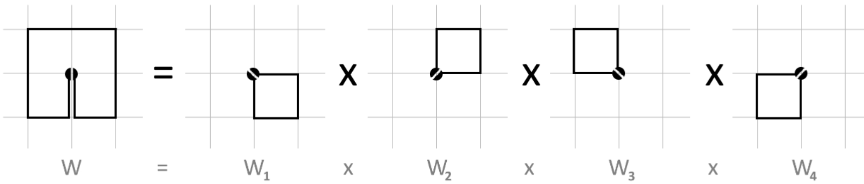

Fig. 2. Decomposition of the matrix W of a 2×2-Wilson loop.

$W_j \in \mathrm{SU}(2)$ can be formulated with Pauli matrices $\boldsymbol{\sigma}_k$, $\boldsymbol{n}_j \in S^2$ and $\boldsymbol{u}_j \in S^3$

$$W_j = \cos\alpha_j\, \boldsymbol{\sigma}_0 + i \sum_{k=1}^{3} \sin\alpha_j\, (\boldsymbol{n}_j)_k\, \boldsymbol{\sigma}_k = \sum_{\alpha=0}^{3} (\boldsymbol{u}_j)_\alpha\, \boldsymbol{\sigma}_\alpha\,. \tag{1}$$

S2-homogeneity h_{S2} and S3-homogeneity h_{S3} are defined as

$$h_{\mathrm{S2}} := \frac{1}{4}\Big|\sum_{j=1}^{4} \boldsymbol{n}_j\Big| \;\in\; [0,1]\,, \qquad\qquad h_{\mathrm{S3}} := \frac{1}{4}\Big|\sum_{j=1}^{4} \boldsymbol{u}_j\Big| \;\in\; [0,1]\,. \tag{2}$$

As shown in Fig. 3, the S2-homogeneity preserves a broad distribution during cooling, whereas the S3-homogeneity approaches a δ-distribution.

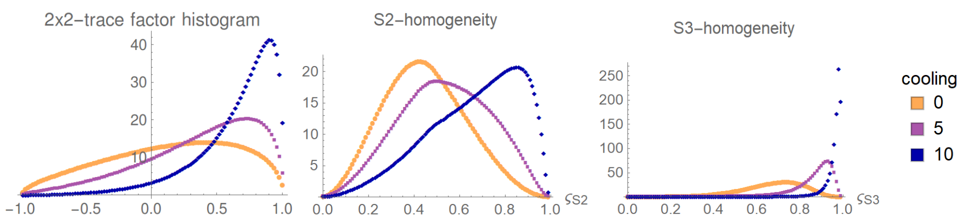

Fig. 3. Histograms of 2×2-trace factor $\frac{1}{2}\mathrm{Tr}\, W$, h_{S2} and h_{S3} for 400 Wilson action configurations on a 18^4 lattice at $\beta = 2.2$ for 3 different numbers of cooling steps.

The difference in the homogeneity of average 2×2-Wilson loops and those pierced by vortices is shown in the left diagram of Fig. 4. It has opposite signs for h_{S2} and h_{S3}. While h_{S2} grows during the first cooling steps, h_{S3} tends to zero. Further, h_{S2} is sensitive to the phase transition, see the left diagram of Fig. 4. Figure 5 reveals a correlation between h_{S2} and the trace factor $\frac{1}{2}\mathrm{Tr}\, W$, indicating that h_{S2} is sensitive to non-trivial center regions.

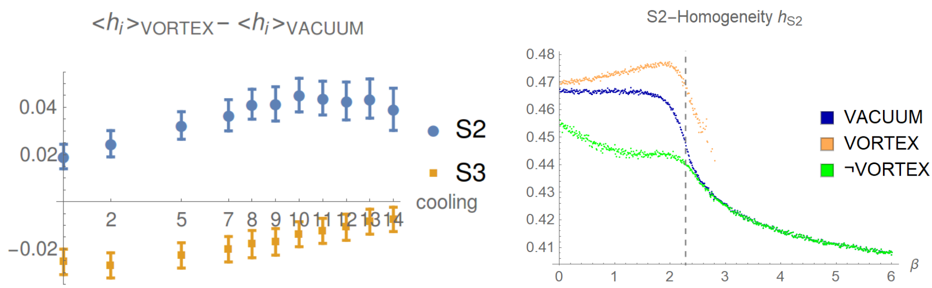

Fig. 4. Left: 12^4-lattice, 20 simulations per data point, Wilson action, $\beta = 2.2$. Right: $12^3 \times 4$-lattice, 20 simulations per data point. The dashed line marks $\beta_{\rm cr}$.

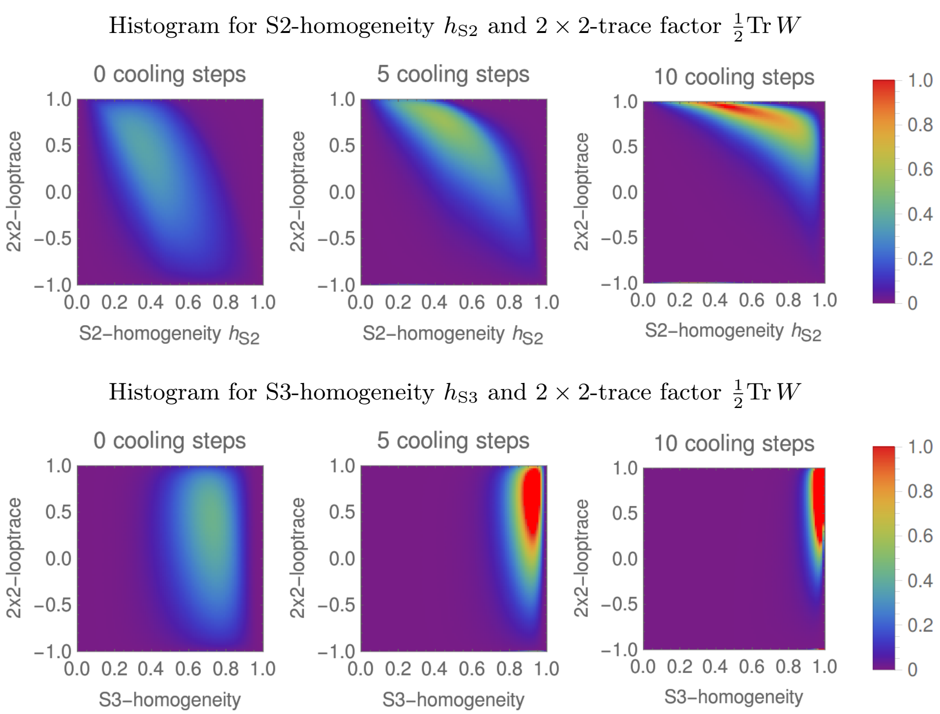

Fig. 5. Histograms for three different numbers of cooling steps, for 400 configurations on a 18^4-lattice with Wilson action at $\beta = 2.2$.

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