PERSPECTIVES OF MODEL PREDICTIVE CONTROL IN HIGH-ENERGY PHYSICS EXPERIMENTS*

Maciej Ławryńczuk

Institute of Control and Computation Engineering
Faculty of Electronics and Information Technology
Warsaw University of Technology
Nowowiejska 15/19, 00-665 Warszawa, Poland
M.Lawrynczuk@ia.pw.edu.pl

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This work shortly discusses the general idea of Model Predictive Control (MPC) algorithms and emphasises their advantages over the classical PID controller. Some extensions of the rudimentary MPC techniques are briefly mentioned. Finally, the potential of MPC algorithms in high-energy physics experiments is discussed.

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1. Classical control

Let us define the basic control problem: it is necessary to calculate on-line the value of the manipulated variable of the process, u, (*i.e.* its input) in such a way that the process controlled variable, y, (*i.e.* its output) is close to the set-point, y^{sp}. In general, the set-point may be time-varying or constant, the process may be affected by some disturbances. In the case of the well-known classical Proportional-Integral-Derivative (PID) controller, the value of the manipulated variable is

$$u(t) = u_0 + K\left(e(t) + \frac{1}{T_{\mathrm{i}}}\int_0^t e(\tau)\mathrm{d}\tau + T_{\mathrm{d}}\frac{\mathrm{d}e(t)}{\mathrm{d}t}\right), \qquad (1)$$

where u_0 is the input offset, $e(t) = y^{\mathrm{sp}}(t) - y(t)$ is the current control error, and the controller's parameters are: the gain, K, the integral time, T_{i}, and

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the derivative time, T_{d}, respectively. Of course, in contemporary control systems, the discrete-time versions of the PID controller are used since on-line calculations are carried out using digital computers.

Although the PID algorithm is very frequently used in practice, it is necessary to mention its limitations:

1. In its basic version, the PID algorithm is used for controlling Single-Input Single-Output (SISO) processes. Since the majority of technological processes are Multiple-Input Multiple-Output (MIMO) ones, *i.e.* $u = [u_1 \ \ldots \ u_{n_{\mathrm{u}}}]^{\mathrm{T}}$ and $y = \left[y_1 \ \ldots \ y_{n_{\mathrm{y}}}\right]^{\mathrm{T}}$, a set of single-loop PID controllers are typically used. Unfortunately, simplicity is the only advantage of such an approach. It is because in the MIMO case, there are usually strong cross-couplings, *i.e.* the consecutive inputs affect all the inputs. As a result, when a set of independent single-loop PID controllers are applied to a MIMO process, the control quality may be below expectations.

2. In its rudimentary version, the PID controller does not take into account the technological constraints of process variables. Although it is easy to take into account limits of the manipulated variable, the process outputs cannot be constrained.

3. The PID algorithm does not work correctly when the process is characterised by significant delays or the inverse response.

4. Finally, the PID control law (1) is a linear function of the control error, its integral and derivative. Therefore, when applied to significantly nonlinear processes, the obtained control quality may be low.

2. Model Predictive Control

In MPC algorithms [1–3], there is no explicit formula used to calculate the value(s) of the manipulated variable(s). Instead, at each sampling instant, k, as many as N_{u} future increments of manipulated variable(s), $\triangle \boldsymbol{u}(k) = [\triangle u(k|k) \ \triangle u(k+N_{\mathrm{u}}-1|k) \ ]^{\mathrm{T}}$ (N_{u} is the control horizon), are calculated from an optimisation problem. Typically, the minimised cost-function is

$$J(k) = \sum_{p=1}^{N} \sum_{m=1}^{n_{\mathrm{y}}} \mu_{p,m} \left(y_m^{\mathrm{sp}}(k+p|k) - \hat{y}_m(k+p|k)\right)^2 + \sum_{p=0}^{N_{\mathrm{u}}-1} \sum_{n=1}^{n_{\mathrm{u}}} \lambda_{p,n} \left(\triangle u_n(k+p|k)\right)^2 \,. \tag{2}$$

The first part of the cost-function (2) measures the predicted control errors, *i.e.* the deviations between the set-point trajectory, $y_m^{\mathrm{sp}}(k+p|k)$, and the predicted output trajectory, $\hat{y}_m(k+p|k)$, for all process outputs, *i.e.* $m = 1, \ldots, n_{\mathrm{y}}$. The predictions of process outputs are calculated over the prediction horizon, N, *i.e.* for $p = 1, \ldots, N$. The second part of the cost-function is a penalty term, it may be used for penalising big changes of the manipulated variables and to obtain good numerical properties of the MPC procedure. The tuning parameters are defined by $\mu_{p,m}$ and $\lambda_{p,n}$. The rudimentary MPC optimisation problem is

$$\begin{aligned}
&\min_{\triangle \boldsymbol{u}(k)} \Big\{ J(k) \Big\} \\
&\text{subject to} \\
&u_n^{\min} \leq u_n(k+p|k) \leq u_n^{\max}\,, \quad p = 0, \ldots, N_{\mathrm{u}}-1\,, \quad n = 1, \ldots, n_{\mathrm{u}}\,, \\
&-\triangle u_n^{\max} \leq \triangle u_n(k+p|k) \leq \triangle u_n^{\max}\,, \quad p = 0, \ldots, N_{\mathrm{u}}-1\,, \quad n = 1, \ldots, n_{\mathrm{u}}\,, \\
&y_m^{\min} \leq \hat{y}_m(k+p|k) \leq y_m^{\max}\,, \quad p = 1, \ldots, N\,, \quad m = 1, \ldots, n_{\mathrm{y}}\,.
\end{aligned} \tag{3}$$

Although at each sampling instant, k, the whole future optimal control policy, $\triangle \boldsymbol{u}(k)$, is determined, only its n_{u} first elements, *i.e.* the increments for the current sampling instant, are applied to the process. Next, the measurements of the process outputs are updated and the procedure is repeated. The predicted output trajectory, $\hat{y}_m(k+p|k)$, is calculated using a dynamic model of the controlled process, which must be explicitly known.

It is necessary to point out advantages of the MPC algorithms:

1. The optimisation procedure automatically calculates the best possible control policy, $\triangle \boldsymbol{u}(k)$, to minimise the predicted control errors. Hence, such an approach is straightforward in the case of MIMO processes since the predictions for all process outputs are taken into account, with all the existing cross-couplings. Furthermore, the number of process inputs and outputs may be different, which is a difficult problem for the single-loop PID controller.

2. The second great advantage of MPC is the ability of taking constraints into account. They are simply included in the optimisation problem. The constraints may be imposed on the values and rate of change of the manipulated variables and on the values of the predicted process outputs, which is impossible in the PID controller.

3. The formulation of MPC is very universal. It may be applied to processes which are difficult to control by the PID algorithms. When the process is characterised by a long delay, it is included in the model and taken into account during prediction and optimisation of the control

policy, $\triangle \boldsymbol{u}(k)$. The controlled process may be linear and nonlinear. In the latter case, a nonlinear model is necessary for prediction. Since the resulting MPC optimisation problem is a nonlinear task (the minimised cost-function and the output constraints are nonlinear), it is recommended to use computationally efficient nonlinear MPC with on-line model or trajectory linearisation [4]. In such approaches, easy to solve on-line quadratic optimisation problems are obtained rather than nonlinear ones.

The disadvantage of MPC is the fact that it is necessary to obtain a model of the process and it must be precise to calculate on-line the predictions.

3. Potential of Model Predictive Control in high-energy physics experiments

Although MPC algorithms have been used for years in numerous industrial applications, mainly in chemical and petrochemical engineering, paper industry and food industry, they may also give very good results in the area of high-energy physics experiments. In particular, MPC may be successfully applied to the so-called slow control systems, which are defined as typical process control (stabilisation of the flowrates, temperature, pressures, *etc.*) [5–7]. It is important to stress the fact that in such an application, the sampling time is relatively long, of the order of seconds or even minutes.

REFERENCES

[1] E.F. Camacho, C. Bordons, *Model Predictive Control*, Springer, London 1999.

[2] J.B. Rawlings, D.Q. Mayne, *Model Predictive Control: Theory and Design*, Nob Hill Publishing, Madison 2009.

[3] P. Tatjewski, *Advanced Control of Industrial Processes, Structures and Algorithms*, Springer, London 2007.

[4] M. Ławryńczuk, *Computationally Efficient Model Predictive Control Algorithms: a Neural Network Approach, Studies in Systems, Decision and Control*, Vol. 3, Springer, 2014.

[5] E. Blanco, C. de Prada, S. Cristea, J. Casas, *Control Eng. Pract.* **17**, 1136 (2009).

[6] F. Bonne, M. Alamir, P. Bonnay, *J. Process Contr.* **24**, 73 (2014).

[7] A.L. Edelen *et al.*, *IEEE Trans. Nucl. Sci.* **63**, 878 (2016).