A NEW AND FINITE FAMILY OF SOLUTIONS OF HYDRODYNAMICS: PART III: ADVANCED ESTIMATE OF THE LIFE-TIME PARAMETER*

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We derive a new formula for the longitudinal HBT radii of the two particle Bose–Einstein correlation function from a new family of finite and exact, accelerating solution of relativistic perfect fluid hydrodynamics for a temperature-independent speed of sound. The new result generalizes the Makhlin–Sinyukov and Herrmann–Bertsch formulae and leads to an advanced life-time estimate of high-energy heavy-ion and proton–proton collisions.

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1. Introduction

This manuscript is the third part of a manuscript series. This series presents various applications of a new, accelerating, finite and exact family of solutions of perfect fluid hydrodynamics, the recently found Csörgő–Kasza–Csanád–Jiang (CKCJ) family of solution of Ref. [1]. The first part of this series [2] fixes the notation, summarizes this class of exact solutions and evaluates the rapidity and pseudorapidity density distributions. The second part [3] evaluates the initial-energy densities in high-energy collisions [1], and provides a fundamental correction to the renowned Bjorken estimate of initial-energy density [4].

In this manuscript, we evaluate the Bose–Einstein correlation functions in a Gaussian approximation from the CKCJ solutions [1]. Given that the considered dynamics is a 1+1 dimensional expansion, we evaluate $R_{\rm L}$, the Hanbury Brown–Twiss (HBT) radius in the longitudinal (beam) direction. This longitudinal HBT radius parameter is proportional to the mean freeze-out time of the fireball, thus the advanced evaluation of its transverse mass

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dependence and its constant of proportionality for finite, longitudinally non-boost-invariant fireballs may have important physics implications on lifetime determinations.

2. Bose–Einstein correlations and the longitudinal HBT radii

In high-energy heavy-ion collisions, the Bose–Einstein correlation functions (BECF) measure characteristic sizes of the particle emitting source, corresponding to lengths of homogeneity [5]. In high-energy heavy-ion collisions, the particle emitting source can be approximated as a locally thermalized fireball, surrounded by a halo of long-lived resonances, this is the so-called core–halo picture. The momentum-dependent intercept parameter λ_* of the two-particle Bose–Einstein correlation function can be interpreted in the core–halo picture of Ref. [6] as follows:

$$\lambda_* = \left(\frac{N_\mathrm{c}}{N}\right)^2 = \left(\frac{N_\mathrm{c}}{N_\mathrm{c}+N_\mathrm{h}}\right)^2 , \tag{1}$$

where $N = N_\mathrm{c} + N_\mathrm{h}$ is the total number of the emitted particles with a given momentum, adding the contributions from both the core N_c and the halo N_h. The fireball that undergoes a hydrodynamical evolution corresponds to core [7]. For locally thermalized sources, the lengths of homogeneity are expressible in terms of the derivatives of the fugacity, $\exp\left(\mu(x)/T(x)\right)$ and the locally thermalized momentum distribution, $\exp\left(-k^\mu u_\mu(x)/T(x)\right)$, corresponding to the so-called geometrical and thermal length scales [7]. Assuming an effective Gaussian source for the core particles, the BECF can be expressed in terms of the Bertsch–Pratt variables as follows:

$$C(\Delta k, K) = 1+\lambda_* \exp\left(-R^2_\mathrm{side}Q^2_\mathrm{side} - R^2_\mathrm{out}Q^2_\mathrm{out} - R^2_\mathrm{L}Q^2_\mathrm{L} - 2R^2_\mathrm{out,L}Q_\mathrm{out}Q_\mathrm{L}\right) . \tag{2}$$

All the fit parameters (λ_*, R_side, R_out, R_L and $R^2_\mathrm{out,L}$) depend on the mean momentum of the particle pair, $K^\mu = 0.5(k_1^\mu + k_2^\mu)$. The four-momentum of a given particle is denoted by $k = (E_k, \boldsymbol{k}) = (E_k, k_x, k_y, k_z)$. The three-components of the relative and mean momenta are denoted as

$$\begin{aligned} \Delta\boldsymbol{k} &= \boldsymbol{k}_1 - \boldsymbol{k}_2 , \\ \boldsymbol{K} &= 0.5\left(\boldsymbol{k}_1 + \boldsymbol{k}_2\right) . \end{aligned}$$

In the Bertsch–Pratt decomposition of the relative momentum [8, 9], the principal directions are defined as follows: The *out* direction is perpendicular to the beam axis and parallel to the mean transverse momentum of the boson pair; the longitudinal direction (indicated by subscript L) is parallel to the beam axis (r_z), and the *side* direction is orthogonal to the previous two

directions. This Bertsch–Pratt decomposition of the relative momentum is defined as follows:

$$Q_{\text{side}} = \frac{|\Delta \boldsymbol{k} \times \boldsymbol{K}|}{|\boldsymbol{K}|}\,, \tag{3}$$

$$Q_{\text{out}} = \frac{\Delta \boldsymbol{k} \cdot \boldsymbol{K}}{|\boldsymbol{K}|}\,, \tag{4}$$

$$Q_{\text{L}} = k_{1,z} - k_{2,z}\,. \tag{5}$$

If the Bose–Einstein correlation function is an approximately Gaussian in terms of the relative momenta, the Gaussian HBT radii $R^2_{i,j}$ can be introduced, with $\{i,j\}\,\epsilon\,\{\text{side},\text{out},\text{long}\}$. These Gaussian Bertsch–Pratt-radii can be related to the variances of the hydrodynamically evolving core, while the halo of the long-lived resonances is responsible for the effective reduction of the strength of the correlation function:

$$R^2_{i,j} = \langle \tilde{x}_i \tilde{x}_j \rangle_{\text{c}} - \langle \tilde{x}_i \rangle_{\text{c}} \langle \tilde{x}_j \rangle_{\text{c}}\,. \tag{6}$$

Here, the $\langle A \rangle_{\text{c}}$ stands for the average of quantity A in the core, i,j stand for directions (side, out or long) and

$$\tilde{x}_i = x_i - \beta_i t\,, \tag{7}$$

$$\beta_i = \frac{k_{i,1} + k_{i,2}}{E_1 + E_2}\,. \tag{8}$$

In this manuscript, we focus on the longitudinal radii, so the radii of the *side* and *out* direction are not discussed, see *e.g.* Ref. [7] for more details on this point. As discussed in [10] and illustrated in Fig. 1, for a 1+1 dimensional relativistic source, the longitudinal radius in an arbitrary frame reads as

$$R^2_{\text{L}} = \left(\beta_{\text{L}} \sinh\left(\eta^{\text{s}}_x\right) - \cosh\left(\eta^{\text{s}}_x\right)\right)^2 \tau_{\text{f}}^2 \Delta\eta_x^2 + \left(\beta_{\text{L}} \cosh\left(\eta^{\text{s}}_x\right) - \sinh\left(\eta^{\text{s}}_x\right)\right)^2 \Delta\tau^2\,, \tag{9}$$

where η_x is the space-time rapidity, and η^{s}_x is the main emission region of the source, which derived by the saddle-point calculation of the rapidity density, $\Delta\tau$ and $\Delta\eta_x$ are characteristic sizes around τ_{f} and η^{s}_x. This formula simplifies a lot in the LCMS (longitudinally co-moving system) frame of the boson pair, where $\beta_{\text{L}} = 0$:

$$R^2_{\text{L}} = \cosh^2\left(\eta^{\text{s}}_x\right) \tau_{\text{f}}^2 \Delta\eta_x^2 + \sinh^2\left(\eta^{\text{s}}_x\right) \Delta\tau^2\,. \tag{10}$$

Our new family of solutions are finite and limited to a narrow rapidity interval around midrapidity [1]. At midrapidity, if $\eta^{\text{s}}_x \approx 0$, the above equation can be simplified even further:

$$R_{\text{L}} = \tau_{\text{f}} \Delta\eta_x\,. \tag{11}$$

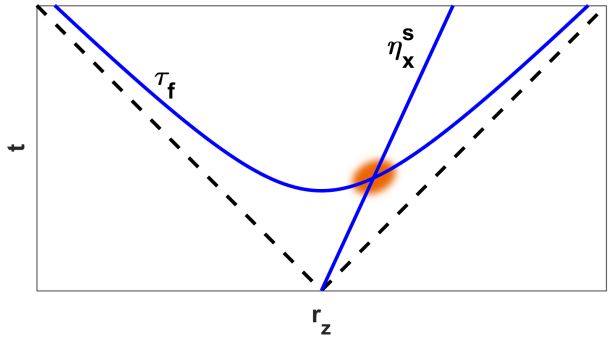


Fig. 1. Space-time picture of particle emission for longitudinally expanding fireballs.

3. Previous results on the longitudinal HBT radius

For a Hwa–Bjorken-type of accelerationless, longitudinal flow [4, 11], Makhlin and Sinyukov determined the longitudinal length of homogeneity in Ref. [5] as

$$R_{\mathrm{L}} = \tau_{\mathrm{Bj}} \sqrt{\frac{T_{\mathrm{f}}}{m_{\mathrm{T}}}} \,. \tag{12}$$

In this equation, T_{f} stands for the freeze-out temperature, m_{T} is the transverse mass of the particle pair and τ_{Bj} is the mean freeze-out time of the Hwa–Bjorken solution. This result makes it possible to determine the life-time, *i.e.* τ_{Bj} of the reaction from the measurement of the longitudinal HBT radius parameter, provided that $T_{\mathrm{f}} \approx m_{\pi} \approx 140$ MeV can be estimated from the analysis of the single particle spectra.

Evaluating the HBT radii from the same Hwa–Bjorken solution [4, 11], Herrmann and Bertsch obtained a more accurate result in Ref. [12], using a Gaussian approximation for the longitudinal HBT radius at midrapidity, in terms of Bessel functions $K_1(z)$and $K_2(z)$, as follows:

$$R_{\mathrm{L}} = \tau_{\mathrm{f}} \sqrt{\frac{T_{\mathrm{f}}}{m_{\mathrm{T}}}} \sqrt{\frac{2K_2(m_{\mathrm{T}}/T_{\mathrm{f}})}{K_1(m_{\mathrm{T}}/T_{\mathrm{f}})}} \,. \tag{13}$$

This formula improves the Sinyukov–Makhlin formula (12) for lower $m_{\mathrm{T}}/T_{\mathrm{f}}$ values, and approaches it in the large $m_{\mathrm{T}}/T_{\mathrm{f}}$ limit.

If the flow is accelerating, the estimated origin of the trajectories is shifted back in proper-time, thus τ_{Bj} is underestimating the life-time of the reaction. The correction was estimated, based on the modification of the flow-profile, from the Csörgő–Nagy–Csanád (CNC) solution [13, 14] as follows:

$$R_{\mathrm{L}} = \frac{\tau_{\mathrm{f}}}{\lambda} \sqrt{\frac{T_{\mathrm{f}}}{m_{\mathrm{T}}}} \,, \tag{14}$$

where $\tau_{\rm f}$ stands for the freeze-out time. In the $\lambda \to 1$ boost-invariant limit, this formula also reproduces the Makhlin–Sinyukov formula, but for the realistic $\lambda > 1$ parameter values, it yields larger life-times as compared to the Makhlin–Sinyukov formula.

4. The longitudinal HBT radius parameter of the CKCJ solution

Let us evaluate the emission function for the CKCJ solution of Refs. [1–3]. The integration of the Cooper–Frye formula is performed by the saddle-point approximation. Near to midrapidity, the fluid rapidity is well-approximated by a linear function of the space-time rapidity: $\Omega \approx \lambda\eta_x$. Using a saddle-point integration in η_x, we obtain the rapidity distribution

$$\frac{{\rm d}N}{{\rm d}y} \approx \frac{\left(2\pi\Delta\eta_x^2\right)^{1/2}}{2\pi\hbar}\left[k_\mu u^\mu \frac{\tau(\eta_x)}{\cosh(\Omega-\eta_x)} \exp\left(-\frac{k_\mu u^\mu}{T_{\rm f}(\eta_x)}\right)\right]_{\eta_x=\eta_x^{\rm s}}. \tag{15}$$

Here, $\eta_x^{\rm s}$ stands for the saddle-point, which is found to be proportional to the rapidity y: $\eta_x^s \approx \frac{y}{2\lambda-1}$. At midrapidity, the saddle-point vanishes and the emission function can be well-approximated by a Gaussian centered on zero. The width of this Gaussian is given by $\Delta\eta_x$ as

$$\Delta\eta_x \approx \sqrt{\frac{T_{\rm f}}{m_{\rm T}}}\frac{1}{\sqrt{\lambda(2\lambda-1)}}. \tag{16}$$

At midrapidity, these considerations lead to the following longitudinal HBT radius parameter:

$$R_{\rm L} = \tau_{\rm f}\Delta\eta_x \approx \frac{\tau_{\rm f}}{\sqrt{\lambda\,(2\lambda-1)}}\sqrt{\frac{T_{\rm f}}{m_{\rm T}}}. \tag{17}$$

Surprisingly, this result is independent of the equation of state, and it is formally different from the CNC estimate.

Our result thus presents an important step forward: once the parameter λ of the acceleration is determined from the fits to the (pseudo)rapidity distributions [2], this parameter combined with the longitudinal HBT radius measurement can be used to provide an advanced estimate of the life-time of the reaction, solving Eq. (17) for the life-time $\tau_{\rm f}$. The significance of our advanced formula is illustrated in figure 2.

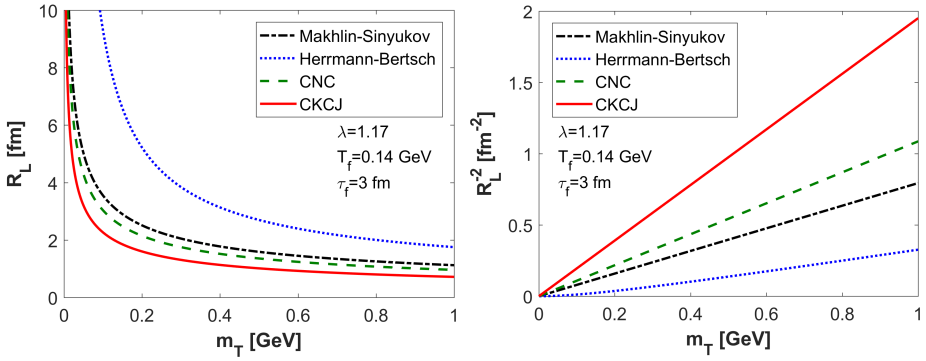


Fig. 2. (Color online) The HBT radius $R_{\mathrm{L}}(m_{\mathrm{T}})$ (left) and $1/R^2_{\mathrm{L}}(m_{\mathrm{T}})$ (right) of the CKCJ solution are shown with solid red lines and compared to earlier estimations. The parameters correspond to the fit results of the CKCJ solution to p+p collisions at $\sqrt{s} = 7$ TeV [2].

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