EVENT-BY-EVENT FLUCTUATIONS OF THE SOURCE SHAPE: IMPLICATIONS FOR THE LÉVY SHAPE AND EVENT SHAPE SORTING*

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In the first part of this contribution, we show that the Lévy-stable shape of the correlation function can be caused by averaging of the measured correlation functions over a large number of events. In the second part, it is demonstrated how a sample of events sorted by the Event Shape Sorting technique exhibits different azimuthal dependence of correlation radii in each event class.

1. Motivation

It is widely understood that in the experiments with relativistic nuclear collisions, each event evolves differently from the others. This is true even if carefully selected centrality classes are studied. Event-by-event fluctuations of the hadron distributions and its anisotropies, parametrised through the coefficients v_n, demonstrate it clearly.

In spite of that, in femtoscopic studies, source sizes averaged over a very large number of events are always extracted and no attention is paid to their fluctuations. The impact of the fluctuations on the resulting event-averaged correlation function have recently been investigated in [1].

In this short contribution, we want to make two points : *(i)* The averaging over a large number of different sources influences the shape of the correlation function and may result in a shape given by the Lévy-stable distribution. *(ii)* The fluctuations of sizes and the space-time characteristics of the sources can be accessed through the Event Shape Sorting technique [2].

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2. Lévy-stable distribution from averaging over events

The measured correlation function is often non-Gaussian and fitted with the Lévy-stable distribution

$$C(q) - 1 = \lambda \exp\left(-|Rq|^\alpha\right),\tag{1}$$

where q is momentum difference of the pair and R, α, and λ are fit parameters. It has been pointed out already in [3] that such a shape may result from summing up production from sources with certain distribution of sizes. We recall that even on a single event, we have effectively many sources with different sizes, since each pair of bosons measures the size of the homogeneity region specified by its momentum. The averaging is on the level of the correlation function.

We demonstrate this with two examples. First, let us assume a simple toy-model source, which is given in transverse plane by two-dimensional Gaussian profile with sizes R_1 and R_2. Its axes are rotated with respect to the Cartesian x, y system by the angle θ_2. The correlation function from such a source is calculated the usual way

$$C(q) - 1 = \frac{\left|\int \mathrm{d}^4x\, S(x) e^{iqx}\right|^2}{\left(\int \mathrm{d}^4x\, S(x)\right)^2}.$$

Such a correlation function will be Gaussian. To represent the averaging over sources which have different sizes and are differently oriented, we then integrate over correlation functions which resulted from all the different Rs. They have been distributed like the two sizes of the overlapping region according to the optical Glauber model, and the rotation angle θ_2 is distributed uniformly. The resulting correlation function is no longer Gaussian. In Fig. 1 (left), we show its cut along the outward (or x) direction. We see that just from such a simple averaging, we obtain a correlation function which is best reproduced by the index parameter about $\alpha = 1.70$.

In our second example, we use a source given by the blast-wave model. This version of the blast-wave model is extended to describe transverse anisotropies to an arbitrary order [4], although here we will only go up to the second order. The important feature of the model is that it incorporates the anisotropies of the transverse size of the fireball $R(\varphi)$ and also the anisotropies of the transverse flow, with the help of the flow rapidity in the transverse direction $\rho(r, \varphi)$

$$R(\varphi) = R_0\left(1 - \sum_{n=2}^{\infty} a_n \cos\left(n(\varphi - \varphi_n)\right)\right),\tag{2}$$

$$\rho(r, \varphi) = \frac{r}{R(\varphi)}\rho_0\left(1 + \sum_{n=2}^{\infty} 2\rho_n \cos\left(n(\varphi - \varphi_n)\right)\right).\tag{3}$$
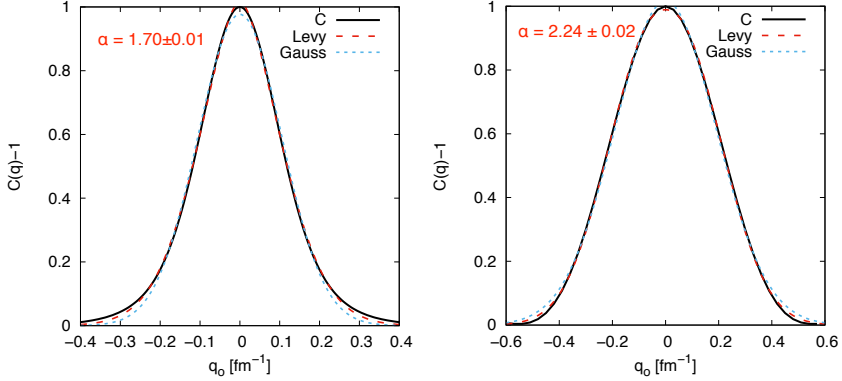
Fig. 1. The shapes of the correlation function as resulting from averaging over source sizes and orientations. Solid black line is the calculated correlation function, dashed red line is the best fit with distribution (1) with the parameter α indicated in the figure, and dotted blue line is the best fit with Gaussian distribution.

We have calculated the dependence of the correlation function on q_{out} and q_{side} from this model with $T = 120$ MeV, $R_0 = 7$ fm, $\rho_0 = 0.8$ and the anisotropy parameters are $a_2 = \rho_2 = 0.2$. The cut through the correlation function along the q_{out} axis is shown in Fig. 1 (right). While the best fit function belongs to family (1), it is no longer a Lévy-stable distribution, which is limited to $\alpha < 2$.

We summarise this section with the conclusion that the averaging over many different sources in the determination of the measurable correlation function may profoundly influence the actual shape of the correlation function.

3. Event Shape Sorting and femtoscopy

The recently proposed method of Event Shape Sorting (ESS) [2] may actually help to select events more selectively and avoid some of the averaging. The algorithm sorts the events in such a way that events with similar azimuthal distribution of hadrons end up close together. They may be assumed to have undergone similar fireball evolution. Except for femtoscopic studies, such an event selection may be interesting also for studies of jet quenching, since it may select events according to the actual azimuthal shape of the fireball which is relevant for the path length of the leading parton.

3.1. The difference to Event Shape Engineering

A treatment similar in its spirit is known under the name Event Shape Engineering (ESE) [5]. Let us explain the difference between ESE and ESS.

In Fig. 2, we show twelve shapes for which the radius follows the prescription:

$$R(\phi) = R_0 \left(1 + 2v_2 \cos(2\phi) + 2v_3 \cos(3(\phi - \Psi_{23}))\right) . \tag{4}$$

In ESE, one first selects the selection variable and then picks events with either highest or lowest values of that variable. Here, if *e.g.* that variable was v_2, then in one group, one would have shapes from the first and the third column and, in the other group, the remaining shapes. It is questionable if this is the natural way of grouping shapes. On the other hand, Fig. 3 shows average histograms of ESS-sorted events. We clearly see that the sorting respects the richer structure of the events, including both second and third order oscillations.
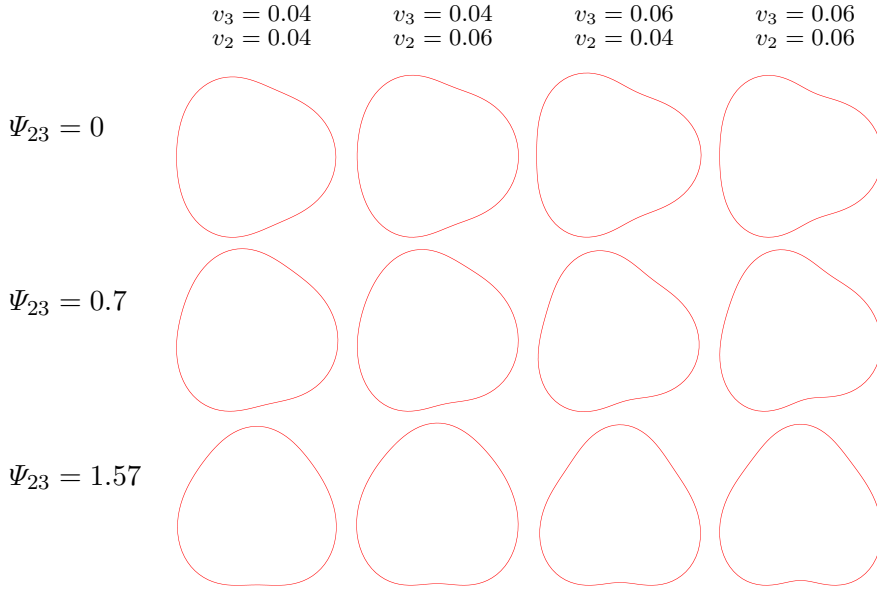

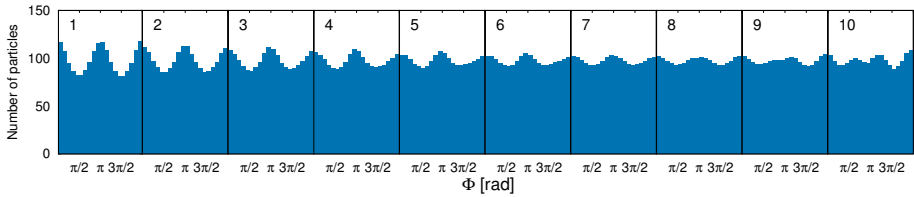Fig. 2. Shapes with different azimuthal anisotropies drawn according to Eq. (4).

Fig. 3. Histograms in azimuthal angle of ESS-sorted events generated by DRAGON.

3.2. Femtoscopy studies with Event Shape Sorting

We have sorted 150 000 events generated by DRAGON [6], which is an MC blast-wave generator with included resonances. It has been augmented to include azimuthal anisotropy according to Eqs. (2) and (3). The scatter plots in Fig. 4 show the v_2 and v_3 as they depend on the sorting variable μ (see [2]). The sorting goes dominantly according to v_2, but in classes 8–10, we observe an admixture of the third order anisotropy, as well.
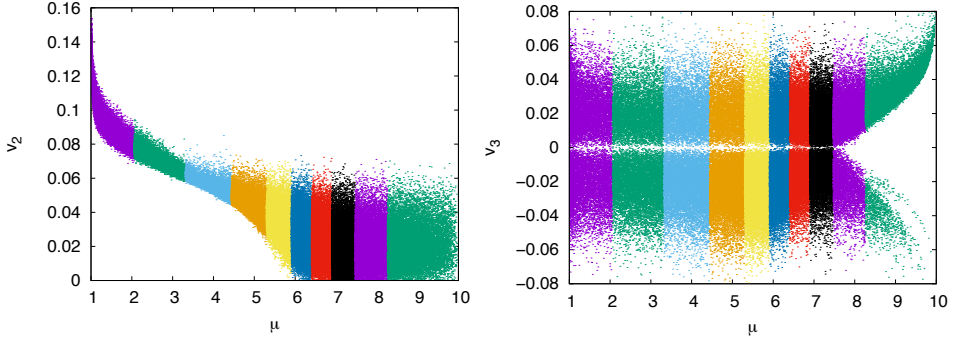

Fig. 4. (Colours on-line) Coefficients v_2 (left) and v_3 (right) of individual events as depending on the sorting variable μ. Different shades of grey/colours represent different event classes.

This picture is complemented by the azimuthal dependence of the correlation radii, which has been constructed for each event class separately. It is shown in Fig. 5. The third order component in the azimuthal dependence of the correlation radii is best seen in event classes 6, 9, and 10.
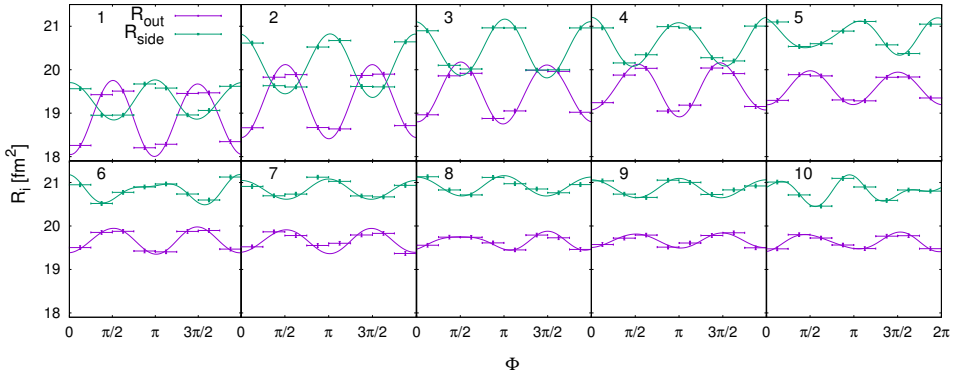

Fig. 5. The azimuthal dependence of R_{out} and R_{side} from all event classes.

Note that in experimental analyses so far, one could only look at each oder of the oscillation separately. If all events were azimuthally rotated with respect to n^{th} order event plane ($n = 2$ or 3), then only the nth order oscillation can be seen and the other order is averaged out. The ESS technique thus offers the unique opportunity to observe all orders together at once.

4. Conclusions

Averaging over a large number of events has an impact on the shape of the correlation function. We showed that it can cast the correlation function into a shape of a Lévy-stable distribution. It remains to be seen to what extent the present observations [7, 8] can be described by such averaging.

The analysis could be made more exclusive and less influenced by averaging with the help of the Event Shape Sorting technique. We have demonstrated here that the sorted classes differ also by the azimuthal dependence of the correlation radii.

Note finally that we have made the sorting algorithm [2] available [9].

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