PSEUDORAPIDITY AND INITIAL-ENERGY DENSITIES IN p+p AND HEAVY-ION COLLISIONS AT RHIC AND LHC*

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A known exact and accelerating solution of relativistic hydrodynamics for perfect fluids is utilized to describe pseudorapidity densities of $\sqrt{s_{NN}} =$ 5.02 TeV Pb+Pb and $\sqrt{s} = 13$ TeV p+p collisions at the LHC. We evaluate a conjectured initial-energy densities $\epsilon_{\rm corr}$ in these collisions, and compare them to Bjorken's initial-energy density estimates, and to results for Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV and p+p collisions at $\sqrt{s} = 7$ and 8 TeV.

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1. Introduction

Relativistic hydrodynamics is an efficient theoretical framework to study the properties of strongly interacting Quark–Gluon Plasma (sQGP) produced in relativistic heavy-ion collisions [1, 2]. Both analytical and numerical results of hydrodynamics highlighted important details of the time evolution of sQGP [3–12] as reviewed in Ref. [13]. A brief review on the successful applications of exact analytic solutions of relativistic hydrodynamics to describe the evolution of longitudinal phase-space in high-energy collisions was recently given in Section 2 of Ref. [14].

In recent publications [14–16], the pseudorapidity distributions of various colliding systems were analyzed to study the longitudinal expansion dynamics at the RHIC and LHC energies. These works were based on an

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accelerating and explicit, but rather academic family of exact solutions of relativistic hydrodynamics, as found by Csörgő, Nagy, and Csanád (CNC) in Refs. [9, 10]. Given that the selected CNC solutions were 1+1 dimensional, the transverse momentum distributions were phenomenologically modeled utilizing also the 1+3 dimensional Buda–Lund hydro-model [7]. The initial thermodynamic quantities for $\sqrt{s_{NN}} = 200$ GeV Cu+Cu, $\sqrt{s_{NN}} = 130$ and 200 GeV Au+Au, $\sqrt{s_{NN}} = 2.76$ TeV Pb+Pb, and $\sqrt{s} = 7$ and 8 TeV p+p collisions at the RHIC and LHC energies were estimated and published recently in Refs. [15, 16], so we do not detail them here, due to space limitations. Instead, we present new results for the pseudorapidity distributions and for the initial-energy densities at top LHC energies, for Pb+Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV and p+p collisions at $\sqrt{s} = 13$ TeV.

2. Accelerating hydrodynamics and initial-energy densities

The dynamical equations of relativistic perfect fluid hydrodynamics correspond to the local conservation entropy and four-momentum

$$\partial_\mu(\sigma u^\mu) = 0\,, \tag{1}$$
$$\partial_\nu T^{\mu\nu} = 0\,, \tag{2}$$

where the entropy density is denoted by σ, the four velocity field by u^μ, the energy density by ϵ, the pressure by p and the energy-momentum four-tensor of perfect fluids is $T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu}$. The equation of state, $\epsilon = \kappa p$, closes the above set of dynamical equations. For the case of vanishing baryochemical potential $\mu_{\rm B} = 0$, the fundamental thermodynamical relation $\epsilon + p = T\sigma$ can also be utilized to solve these equations and, here, we also assume that $\kappa = 1/c_{\rm s}^2 \neq \kappa(T)$, so the speed of sound $c_{\rm s}$ is modeled with a temperature T-independent, average value. An accelerating but rather academic family of exact solutions was detailed in Refs. [9, 10]

$$u^\mu = (\cosh(\lambda\eta_x), \sinh(\lambda\eta_x))\,, \tag{3}$$
$$p = p_0 \left(\frac{\tau_0}{\tau}\right)^{\lambda\frac{\kappa+1}{\kappa}}\,, \tag{4}$$

where the longitudinal proper time is denoted by $\tau = \sqrt{t^2 - r_z^2}$, the space-time rapidity is denoted by $\eta_x = 0.5\log\left[(t + r_z)/(t - r_z)\right]$ and, here, we limit the discussion only to 1+1 dimensional solutions with $x^\mu = (t, r_z)$ and $u^\mu = (u^0, u^1)$ that correspond to one of the five different classes of solutions that were detailed in Refs. [9, 10]. In these solutions, the longitudinal acceleration parameter is a free fit parameter, denoted by λ and the initial values for the pressure and thermalization time are denoted by p_0 and τ_0, respectively. The price for the freedom in λ was a fixed value of super-hard equation of state,

$\kappa = 1$. Combining the exact solution of relativistic hydrodynamics with a Cooper–Frye flux term, and embedding this solution to 1+3 dimensions with $x^\mu = (t, r_x, r_y, r_z)$ and $p^\mu = (E, p_x, p_y, p_z)$, the rapidity distribution $\mathrm{d}n/\mathrm{d}y$ was obtained in a saddle-point approximation as [9, 10]

$$\frac{\mathrm{d}n}{\mathrm{d}y} = \left.\frac{\mathrm{d}n}{\mathrm{d}y}\right|_{y=0} \cosh^{-\frac{\alpha}{2}-1}\left(\frac{y}{\alpha}\right) \exp\left\{-\frac{m}{T_\mathrm{f}}\left[\cosh^\alpha\left(\frac{y}{\alpha}\right) - 1\right]\right\}, \tag{5}$$

where $\alpha = \frac{2\lambda-1}{\lambda-1}$, the freeze-out temperature is denoted by T_f, the mass of particles is m and the rapidity of the observed particles is denoted by $y = 0.5\log((E+p_z)/(E-p_z))$. The constant of normalization $\left.\frac{\mathrm{d}n}{\mathrm{d}y}\right|_{y=0}$ is proportional to $S_\perp$ that stands for the transverse cross section of the fluid.

The pseudorapidity density distribution $\frac{\mathrm{d}n}{\mathrm{d}\eta}$, with the help of an advanced saddle-point integration is given [7, 9] as a parametric curve $(\eta(y), \frac{\mathrm{d}n}{\mathrm{d}\eta}(y))$, where the parameter is the rapidity y

$$\left(\eta(y)\,, \frac{\mathrm{d}n}{\mathrm{d}\eta}(y)\right) = \left(\frac{1}{2}\log\left[\frac{\bar{p}(y) + \bar{p}_z(y)}{\bar{p}(y) - \bar{p}_z(y)}\right], \frac{\bar{p}(y)}{\bar{E}(y)}\frac{\mathrm{d}n}{\mathrm{d}y}\right), \tag{6}$$

where $\bar{A}(y)$ denotes the rapidity-dependent average value of the variable A including the various components of the four-momentum, and the Jacobian connecting the double differential $(y,\ m_\mathrm{T})$ and $(\eta,\ m_\mathrm{T})$ distributions has been utilized at the average value of the transverse momentum [9]. Based on the Buda–Lund hydrodynamic model [7], in the region of $p_\mathrm{T} < 2$ GeV, the relation between mean transverse momentum $\bar{p}_\mathrm{T}$ and the effective temperature T_eff at a given rapidity y can be written as

$$\bar{p}_\mathrm{T}(y) \; = \; \frac{T_\mathrm{eff}}{1 + \frac{\sigma_\mathrm{T}^2}{2}(y - y_\mathrm{mid})^2}, \tag{7}$$

where σ_T parameterizes the rapidity dependence of the average transverse momentum. In our case, σ_T and T_eff are free fit parameters. Their values can be determined either from fits to data on the rapidity-dependent transverse momentum spectra, or phenomenologically as in Ref. [7] or dynamically as in Ref. [14]. Midrapidity is denoted by y_mid. In our case, it is at $y_\mathrm{mid} = 0$.

Our fit results to pseudorapidity densities allow for advanced estimates of the initial-energy densities. The Bjorken-estimate [5] at midrapidity is

$$\epsilon_\mathrm{Bj} = \frac{1}{S_\perp \tau_0}\frac{\mathrm{d}E_\mathrm{T}}{\mathrm{d}\eta} = \frac{\langle E_\mathrm{T}\rangle}{S_\perp \tau_0}\frac{\mathrm{d}n}{\mathrm{d}y}\,. \tag{8}$$

In the case of a longitudinally accelerating flow, the acceleration effects modify Bjorken's estimate. A conjectured initial-energy density $\epsilon_{\rm corr}$ [15, 16] that corrects Bjorken's estimate for acceleration effects reads as

$$\epsilon_{\rm corr} = (2\lambda - 1)\left(\frac{\tau_{\rm f}}{\tau_0}\right)^{\lambda-1}\left(\frac{\tau_{\rm f}}{\tau_0}\right)^{(\lambda-1)\left(1-\frac{1}{\kappa}\right)}\epsilon_{\rm Bj}\,. \tag{9}$$

This estimate explicitely takes into account the bending of the fluid worldlines due to acceleration. However, it is based on results that are obtained exactly in the $\kappa = 1$ case only. Until most recently, the dependence of the initial-energy density on the speed of sound $c_{\rm s} = 1/\sqrt{\kappa}$ had not been derived exactly, only a conjecture was known so far. Given that the speed of sound is an important physical property of the sQGP, it is crucial to cross-check and derive exact results for realistic values of the speed of sound, corresponding to $c_{\rm s}^2 \approx 0.1$. However, let us emphasize that this conjecture, Eq. (9), is based on the determination of the acceleration parameter λ from fits to the measured pseudorapidity density distributions. The dependence of the initial-energy density on the initial and freeze-out proper-times, τ_0 and $\tau_{\rm f}$, is a topic of ongoing research, with first results presented in Refs. [19, 20].

3. Results

Measurements of the charged particle pseudorapidity distribution $\mathrm{d}n/\mathrm{d}\eta$ for $\sqrt{s_{NN}} = 5.02$ TeV Pb+Pb collisions and $\sqrt{s} = 13$ TeV p+p collisions were presented by the ALICE [17] and CMS collaborations [18]. Here, we extract the acceleration parameter λ of these collisions and apply it to calculate the energy density correction ratio $\epsilon_{\rm corr}/\epsilon_{\rm Bj}$ as a function of $\tau_{\rm f}/\tau_0$. Fit results to the ALICE and CMS data are shown in Figs. 1 and 2. Our advanced estimates of the initial-energy densities $\epsilon_{\rm corr}$ are given in Tables I and II for the squared speed of sound $c_{\rm s}^2 = 0.1$ and $\tau_{\rm f}/\tau_0 = 6 \pm 2$.

TABLE I

Acceleration parameters and initial-energy density estimations for 2.76 [16] and 5.02 TeV 0–5% centrality Pb+Pb data [17]. Auxiliary values of $T_{\rm f} = 90$ MeV, $T_{\rm eff} = 0.27 \pm 0.03$ GeV, $\bar{m} = 0.24$ GeV, $\sigma_{\rm T} = 0.9 \pm 0.1$ have been used based on Refs. [7, 15, 16].

$\sqrt{s}$	$\left.\frac{\mathrm{d}n}{\mathrm{d}\eta}\right\vert_{\eta=\eta_0}$	λ	$\epsilon_{\rm Bj}$ [GeV/fm^3]	$\epsilon_{\rm corr}$ [GeV/fm^3]
2.76 TeV	1615 ± 39	1.050 ± 0.005	12.50 ± 0.44	15.07 ± 0.81
5.02 TeV	1929 ± 46	1.046 ± 0.013	14.85 ± 0.53	17.40 ± 0.61

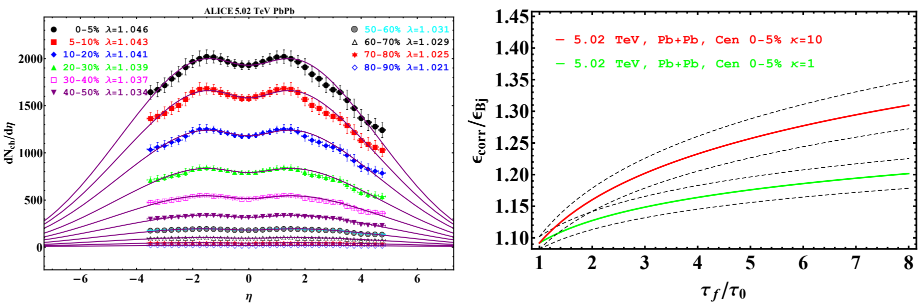

Fig. 1. (Color online) The left panel shows hydrodynamical fits using Eqs. (5)–(7) to $\mathrm{d}n_{\mathrm{ch}}/\mathrm{d}\eta$ data as measured by the ALICE Collaboration in $\sqrt{s_{NN}} = 5.02$ TeV Pb+Pb collisions. The right panel indicates with solid curves the $\epsilon_{\mathrm{corr}}/\epsilon_{\mathrm{Bj}}$ correction factor, as a function of the ratio of freeze-out time and thermalization time τ_{f}/τ_0, for the centrality class of 0–5%, both for the exact solution with $\kappa = 1$ superhard equation of state, and for the conjectured energy density values for the realistic $\kappa = 10$ soft equation of state, while the dashed lines represent the uncertainty of these estimates as determined from the errors of the fit parameters.

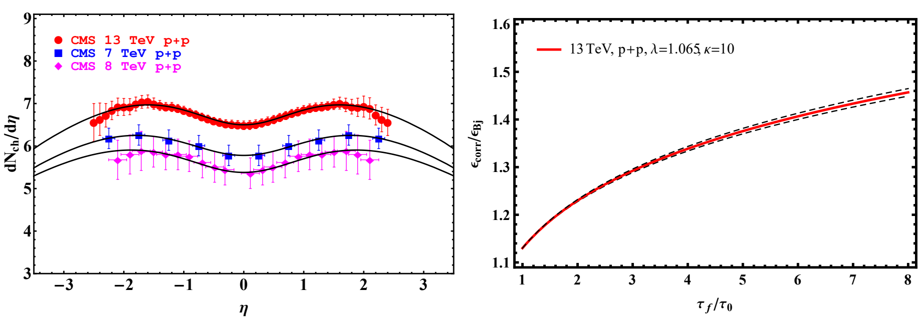

Fig. 2. The same as Fig. 1, but for $p+p$ collisions at $\sqrt{s} = 13$ TeV.

TABLE II

Acceleration parameters and initial-energy density estimations for $\sqrt{s} = 7$, 8 [15] and 13 TeV $p+p$ data [18]. Auxiliary values of $T_{\mathrm{f}} = T_{\mathrm{eff}} = 0.17 \pm 0.01$ GeV, $\bar{m} = 0.14$ GeV, $\sigma_{\mathrm{T}} = 0.81 \pm 0.04$ have been utilized, based on Refs. [7, 15, 16].

$\sqrt{s}$	$\left.\frac{\mathrm{d}n}{\mathrm{d}\eta}\right\|_{\eta=\eta_0}$	λ	ϵ_{Bj} [GeV/fm^3]	ϵ_{corr} [GeV/fm^3]
7 TeV	5.78 ± 0.01	1.073 ± 0.001	0.51 ± 0.01	0.64 ± 0.01
8 TeV	5.36 ± 0.02	1.067 ± 0.001	0.52 ± 0.01	0.64 ± 0.01
13 TeV	6.50 ± 0.02	1.065 ± 0.013	0.56 ± 0.02	0.69 ± 0.02

4. Summary and conclusions

We have evaluated the conjectured initial-energy densities $\epsilon_{\rm corr}$ in p+p and in Pb+Pb collisions at the currently available highest LHC energies, and compared them to Bjorken's initial-energy density estimates as well as to earlier results for Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV and p+p collisions at $\sqrt{s} = 7$ and 8 TeV. Our new results are similar to our recent results published in Ref. [14]. Our results were found to be not inconsistent — neither in proton–proton nor in heavy-ion reactions — with longitudinal expansion dynamics of hydrodynamical origin.

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REFERENCES

[1] S.A. Bass, M. Gyulassy, H. Stöecker, W. Greiner, *J. Phys. G* **25**, R1 (1999).
[2] M. Gyulassy, L. McLerran, *Nucl. Phys. A* **750**, 30 (2004).
[3] L.D. Landau, *Izv. Akad. Nauk Ser. Fiz.* **17**, 51 (1953).
[4] R.C. Hwa, *Phys. Rev. D* **10**, 2260 (1974).
[5] J.D. Bjorken, *Phys. Rev. D* **27**, 140 (1983).
[6] T.S. Biró, *Phys. Lett. B* **487**, 133 (2000).
[7] T. Csörgő, B. Lörstad, *Phys. Rev. C* **54**, 1390 (1996).
[8] T. Csörgő, F. Grassi, Y. Hama, T. Kodama, *Phys. Lett. B* **565**, 107 (2003).
[9] T. Csörgő, M.I. Nagy, M. Csanád, *Phys. Lett. B* **663**, 306 (2008).
[10] M.I. Nagy, T. Csörgő, M. Csanád, *Phys. Rev. C* **77**, 024908 (2008).
[11] M.I. Nagy, *Phys. Rev. C* **83**, 054901 (2011).
[12] S.S. Gubser, *Phys. Rev. D* **82**, 085027 (2010).

[13] R. Derradi de Souza, T. Koide, T. Kodama, *Prog. Part. Nucl. Phys.* **86**, 35 (2016).

[14] T. Csörgő, G. Kasza, M. Csanád, Z.F. Jiang, *Universe* **4**, 69 (2018).

[15] M. Csanád, T. Csörgő, Z.F. Jiang, C.B. Yang, *Universe* **3**, 9 (2017).

[16] Z.F. Jiang, C.B. Yang, M. Csanád, T. Csörgő, *Phys. Rev. C* **97**, 064906 (2018).

[17] J. Adam *et al.* [ALICE Collaboration], *Phys. Lett. B* **772**, 567 (2016).

[18] A.M. Sirunyan *et al.* [CMS, TOTEM Collaboration], *Phys. Rev. D* **96**, 112003 (2017).

[19] T. Csörgő, G. Kasza, M. Csanád, Z.-F. Jiang, *Acta Phys. Pol. B* **50**, 27 (2019).

[20] G. Kasza, T. Csörgő, *Acta Phys. Pol. B Proc. Suppl.* **12**, 175 (2019), this issue; T. Csörgő, G. Kasza, *Acta Phys. Pol. B Proc. Suppl.* **12**, 217 (2019), this issue.