# THE LIGHT SCALAR $K_{0}^{*}(700)$ IN THE VACUUM AND AT NONZERO TEMPERATURE* 

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There is mounting evidence toward the existence of a light scalar kaon $\kappa \equiv K_{0}^{*}(700)$ with quantum numbers $I\left(J^{P}\right)=1 / 2\left(0^{+}\right)$. Here, we recall the results of an effective model with both derivative and non-derivative terms in which only one scalar kaonic field is present in the Lagrangian (the standard quark-antiquark "seed" state $K_{0}^{*}(1430)$ ): a second "companion" pole $K_{0}^{*}(700)$ emerges as a dynamically generated state. A related question is the role of $K_{0}^{*}(700)$ at nonzero $T$ : since it is the lightest scalar strange state, one would naively expect that it is relevant for $\pi$ and $K$ multiplicities. However, a repulsion in the $\pi K$ channel with $I=3 / 2$ cancels its effect.

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## 1. Introduction

The lightest scalar kaonic state listed in the PDG [1] is $K_{0}^{*}(700)$ (previously called $K_{0}^{*}$ (800), see PDG 2016 [2] and older versions). This state, sometimes called $\kappa$, still "needs confirmation", but many works do find a pole in that energy region, see Ref. [3] and references therein. The PDG reports at present the following result:

$$
\begin{equation*}
\text { pole } \kappa[\mathrm{PDG}]:(630-730)-i(260-340) \mathrm{MeV}, \tag{1}
\end{equation*}
$$

(hence, the pole width lies between $520-680 \mathrm{MeV}$ ), while the Breit-Wigner (BW) mass and widths are

$$
\begin{equation*}
\mathrm{BW}[\mathrm{PDG}]: m_{\kappa, \mathrm{BW}}=824 \pm 30 \mathrm{MeV}, \quad \Gamma_{\kappa, \mathrm{BW}}=478 \pm 50 \mathrm{MeV} \tag{2}
\end{equation*}
$$

The BW and the pole widths are compatible, but the BW mass is somewhat larger. There is, however, no friction, since BW and pole masses are different

[^0]quantities which coincide only when a resonance is narrow. This is definitely not the case for the $\kappa$, which is a very broad state with a width-to-mass ratio larger than 0.5.

In a certain sense, the light $\kappa$ can be regarded as the "brother" of the light $\sigma \equiv f_{0}(500)$ meson [1]. This state is also very broad and for a long time, it was not clear if there is a pole on the complex plane. Now, its existence is confirmed by many studies and the state is listed in the PDG, see also the review paper [4]. The destiny of the light $\kappa$ looks somewhat similar: its final confirmation is probably just a matter of time.

Yet, a different issue is the nature of the $\kappa \equiv K_{0}^{*}(700)$ and the $\sigma \equiv$ $f_{0}(500)$. According to mounting evidence, both states are not simple quarkantiquark states, but are rather four-quark objects, either in the form of a tetraquark nonet together with $a_{0}(980)$ and $f_{0}(980)$ [5] or as dynamically generated molecular-like states [6]. The $\kappa$ can be then interpreted as a diquark-antidiquark state $([u, d][\bar{d}, \bar{s}], \ldots)$ and/or as $K \pi$ state (mixing among these configurations is of course possible and rather probable to occur). If $\kappa$ is not $\bar{q} q$, where should be the scalar strange quarkonium? According to the quark model [7] and modern chiral approaches [8], the lightest $\bar{q} q$ kaonic state $(u \bar{s}, \ldots)$ is the well-established $K_{0}^{*}(1430)$ (similarly, the lightest scalar/isoscalar quarkonium is the state $f_{0}(1370)$ ). The question that we review in this work is the link between the standard state $K_{0}^{*}(1430)$ and the dynamically generated state $K_{0}^{*}(700)$. We find (see Sec. 2) that the $\pi K$ loops dressing $K_{0}^{*}(1430)$ generate $K_{0}^{*}(700)$ as a companion pole (a peculiar four-quark object) [9] (similarly, the $a_{0}(980)$ emerges as a companion pole of $a_{0}(1450)$ [10]).

There is, however, a related important question: if the light $\kappa$ is existent, should it be included into thermal hadronic models [11]? At a first sight, the answer is 'yes'. In fact, the light $\kappa$ is the second-lightest state with nonzero strangeness, thus potentially relevant. Yet, a detailed analysis of the problem [12] shows that one should better not include this state into a thermal model (see Sec. 3). Namely, also repulsive channels contribute to the thermodynamics [13-15]. Just as for the $f_{0}(500)$ whose contribution is cancelled by $\pi \pi$ scattering with $I=2$, the contribution of the $\kappa$ is cancelled by the repulsion in $\pi K$ channel with $I=3 / 2$. Thus, the easiest thing to do is to neglect both the $f_{0}(500)$ and the $K_{0}^{*}(700)$ when studying hadronic thermal models for the late stage of heavy-ion collisions.

## 2. The light $\kappa$ in the vacuum

As a first step, we write down a Lagrangian that contains only one scalar state $K_{0}^{*}$, to be identified with $K_{0}^{*}(1430)$, coupled to $K \pi$ pairs

$$
\begin{equation*}
\mathcal{L}_{K_{0}^{*}}=a K_{0}^{*+} K^{-} \pi^{0}+b K_{0}^{*+} \partial_{\mu} K^{-} \partial^{\mu} \pi^{0}+\ldots \tag{3}
\end{equation*}
$$

where dots refer to other isospin channels. Note, there is no $\kappa \equiv K_{0}^{*}(700)$ in the model (yet). There are both derivative and non-derivative terms: the former naturally dominates in the context of chiral perturbation theory and also emerge from the extended Linear Sigma Model [8]. The decay width reads

$$
\begin{equation*}
\Gamma_{K_{0}^{*} \rightarrow K \pi}(m)=3 \frac{\left|\vec{k}_{1}\right|}{8 \pi m^{2}}\left[a-b \frac{m^{2}-M_{K}^{2}-M_{\pi}^{2}}{2}\right]^{2} F_{\Lambda}(m) \tag{4}
\end{equation*}
$$

with the vertex function $F_{\Lambda}(m)=\exp \left(-2 \vec{k}_{1}^{2} / \Lambda^{2}\right)$. Here, $\Lambda$ is an energy scale describing the nonlocal nature of mesons [16], $\vec{k}_{1}$ the three-momentum of one outgoing particle, $M_{K}$ the kaon mass, and $M_{\pi}$ the pion mass. (For details and phenomenology of the spectral function, see Ref. [17].)

The propagator of $K_{0}^{*}$ is given by $\Delta_{K_{0}^{*}}\left(m^{2}\right)=\left[m^{2}-M_{0}^{2}+\Pi\left(m^{2}\right)+i \varepsilon\right]^{-1}$, $M_{0}$ being the bare mass of $K_{0}^{*}(1430)$ and $\Pi\left(m^{2}\right)$ the one-loop contribution. The spectral function $d_{K_{0}^{*}}(m)=\frac{2 m}{\pi}\left|\operatorname{Im} \Delta_{K_{0}^{*}}\left(p^{2}=m^{2}\right)\right|$ is the mass probability density (its integral is normalized to unity). Typically, for the Breit-Wigner value $M_{\mathrm{BW}}$ determined as $M_{\mathrm{BW}}^{2}-M_{0}^{2}+\operatorname{Re} \Pi\left(M_{\mathrm{BW}}^{2}\right)=0$, the spectral function has a peak's width $\Gamma_{\mathrm{BW}}=\operatorname{Im} \Pi\left(M_{\mathrm{BW}}\right) / M_{\mathrm{BW}}$. A useful approximation, valid if the width is sufficiently small, is the relativistic Breit-Wigner expression

$$
\begin{equation*}
d_{K_{0}^{*}}(m) \approx d_{K_{0}^{*}}^{\mathrm{BW}}(m)=N\left[\left(m^{2}-M_{\mathrm{BW}}^{2}\right)^{2}+M_{\mathrm{BW}}^{2} \Gamma_{\mathrm{BW}}^{2}\right]^{-1} \tag{5}
\end{equation*}
$$

Under this approximation, there is only one pole in the complex plane at $m^{2} \simeq M_{\mathrm{BW}}^{2}-i M_{\mathrm{BW}} \Gamma_{\mathrm{BW}}$ (hence, $m \simeq M_{\mathrm{BW}}-i \Gamma_{\mathrm{BW}} / 2$ ). However, when a resonance is broad, these approximations are not valid anymore.

We now turn to $\pi K$ scattering. Within our framework, the pion-kaon phase shift is given by [9]

$$
\begin{equation*}
\delta_{\pi K, S-\mathrm{wave}}(m)=\delta_{(I=1 / 2, J=0)}(m)=\frac{1}{2} \arccos \left[1-\pi \Gamma_{K_{0}^{*}}(m) d_{K_{0}^{*}}(m)\right] \tag{6}
\end{equation*}
$$

where $\delta_{(I, J)}(m)$ is the general phase shift for a given isospin $I$ and total spin $J$. The amplitude of the process and the phase-shift are linked by $a_{(I, J)}=\left(e^{i \delta_{(I, J)}(m)}-1\right) /(2 i)$. The parameters $\left(a, b, M_{0,} \Lambda\right)$ entering in Eq. (3) were determined via a fit to $\pi K$ phase-shift data [18], see Ref. [9] for details. A very good description of data is achieved. A study of the complex plane shows an interesting fact: besides the pole corresponding to the well-known $K_{0}^{*}(1430)$ state $(1.413 \pm 0.057)-i(0.127 \pm 0.011) \mathrm{GeV}$, there is a second pole which correspond to $K_{0}^{*}(700)$

$$
\begin{equation*}
(0.745 \pm 0.029)-i(0.263 \pm 0.027) \mathrm{GeV} \tag{7}
\end{equation*}
$$

The numerical value is compatible with the PDG value of Eq. (1). A large- $N_{\mathrm{c}}$ study confirms that, while the first pole tends to the real axis (and hence is a $\bar{q} q$ state), the second one moves away from it, as it is expected for a dynamically generated state.

In conclusion, the simple model of Eq. (3) is able to describe $\pi K$ scattering data and naturally gives rise to the pole of $K_{0}^{*}(700)$ as a companion pole of the predominantly quark-antiquark resonance $K_{0}^{*}(1430)$.

## 3. The light $\kappa$ at nonzero temperature

The partition function of an hadronic gas can be expressed as the sum of the contributions of stable particles and their mutual interactions

$$
\begin{equation*}
\ln Z=\ln Z_{\text {pions }}+\ln Z_{\mathrm{kaons}}+\cdots+\ln Z^{\mathrm{int}}, \quad \ln Z^{\mathrm{int}}=\sum_{I, J} \ln Z_{(I, J)} \tag{8}
\end{equation*}
$$

The first term $\ln Z_{\text {pions }}=3 F_{1}\left(m_{\pi}\right)$ refers to pions and $\ln Z_{\text {kaons }}=4 F_{1}(\mathrm{~m})$ to kaons, where $F_{1}(m)=\int \frac{\mathrm{d}^{3} p}{(2 \pi)^{3}} \ln \left[1-e^{-\sqrt{\vec{p}^{2}+m_{\pi}^{2}} / T}\right]$ is the contribution of a free particle with mass $m$. The term $\ln Z_{I J}$ refers to the contribution of the interactions in the $(I, J)$ channel [13]

$$
\begin{equation*}
\ln Z_{(I, J)}=(2 I+1)(2 J+1) \int_{0}^{\infty} \frac{\mathrm{d} m}{\pi} \frac{\mathrm{~d} \delta_{(I, J)}(m)}{\mathrm{d} m} F_{1}(m) \tag{9}
\end{equation*}
$$

When in a certain channel a narrow resonance is present, one finds its standard contribution. For instance, for $I=J=1$, the $\rho$ meson is produced. In the nonrelativistic BW-limit, $\frac{1}{\pi} \frac{\mathrm{~d} \delta_{(1,1)}(m)}{\mathrm{d} m} \simeq \frac{\Gamma_{\rho}}{2 \pi}\left[\left(m-M_{\rho}\right)^{2}+\Gamma_{\rho}^{2} / 4\right]^{-1}$. (Moreover, for $\Gamma_{\rho} \rightarrow 0, \delta\left(m-M_{\rho}\right)$ emerges: the contribution of a stable $\rho$ is obtained.)

However, Eq. (9) is very general and can describe also broad resonances as well as non-resonant channels, such as repulsive ones. This is important for the $\kappa$. In the resonant $I=1 / 2, J=0$ channel in which the $\kappa$ is formed, one has (upon integrating up to 1 GeV$) \ln Z_{(1 / 2,0)}=\int_{0}^{1} \mathrm{GeV} \frac{2 \mathrm{~d} m}{\pi} \frac{\mathrm{~d} \delta_{(1 / 2,0)}(m)}{\mathrm{d} m}$ $F_{1}(m)$. This is sizable. However, one should also consider the repulsion in the $I=3 / 2, J=0$ channel. Remarkably, the sum
$\ln Z_{(1 / 2,0)}+\ln Z_{(3 / 2,0)}=\int_{0}^{1 \mathrm{GeV}} \mathrm{d} m\left(\frac{2}{\pi} \frac{\mathrm{~d} \delta_{(1 / 2,0)}(m)}{\mathrm{d} m}+\frac{4}{\pi} \frac{\mathrm{~d} \delta_{(3 / 2,0)}(m)}{\mathrm{d} m}\right) F_{1}(m)$
is small. Namely, while $\frac{\mathrm{d} \delta_{(1 / 2,0)}(m)}{\mathrm{d} m}>0$ (attraction), $\frac{4}{\pi} \frac{\mathrm{~d} \delta_{(3 / 2,0)}(m)}{\mathrm{d} m}<0$ (repulsion). Note: $\frac{1}{\pi} \frac{\mathrm{~d} \delta_{(1 / 2,0)}(m)}{\mathrm{d} m} \neq d_{K_{0}^{*}}(m)$. (This would be true only in the BW-limit). In conclusion, the light $\kappa$ can be safely neglected in the construction of thermal hadronic models.

## 4. Conclusions

We have described the emergence of the state $\kappa \equiv K_{0}^{*}(700)$ as a companion pole of $K_{0}^{*}(1430)$ by using an effective hadronic model [9]. The numerical value of the pole (7) is in agreement with the present PDG estimate of Eq. (1). On the other hand, contrary to the naive expectations, the light $\kappa$ is not relevant in a thermal hadronic gas. Namely, its influence on thermodynamical properties is cancelled by a repulsion in the $I=3 / 2$ channel. Either one includes both the light $\kappa$ and the repulsion, or - even easier - neglects both of them.

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