

DYNAMICS OF RELATIVISTIC SPIN-POLARIZED FLUIDS*

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We briefly review the foundations of a new relativistic fluid dynamics framework for polarized systems of particles with the spin one half. Using this approach, we numerically study the dynamics of the spin polarization of a rotating medium resembling the ones created in high-energy heavy-ion collisions.

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1. Introduction

Very recently, the STAR Collaboration [1] made an intriguing observation of non-zero global spin polarization of Λ hyperons emitted from the medium produced in high-energy heavy-ion collisions. This raised the questions concerning the relation between polarization and vorticity of matter created in these processes. The coupling between the two arises in the global equilibrium state of a rotating system [2, 3]. However, the most natural framework for such studies is provided by relativistic hydrodynamics, which forms the basis of our current understanding of heavy-ion collisions dynamics [4, 5]. Recently, a new framework for relativistic perfect fluid hydrodynamics of spin-polarized media was presented [6, 7] (see also Refs. [8, 9]), aiming at an extension of the work of Refs. [2, 3] to systems in local equilibrium. In this contribution, we review the framework proposed in Refs. [6–9] and use it for numerical studies of polarization dynamics in heavy-ion collisions.

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2. Evolution equations for spin-polarized fluids

In Refs. [6, 7], a fluid dynamical framework for polarized systems of spin-1/2 particles and antiparticles was proposed. It is based on conservation laws of baryon charge, energy, linear momentum and total angular momentum

$$\partial_\alpha N^\alpha = 0, \quad (1)$$

$$\partial_\alpha T^{\alpha\beta} = 0, \quad (2)$$

$$\partial_\alpha J^{\alpha,\beta\gamma} = 0. \quad (3)$$

Employing the kinetic-theory definitions of Ref. [10] together with the equilibrium spin density matrices proposed in Ref. [3], one finds that the baryon current N^α and the energy-momentum tensor $T^{\alpha\beta}$ in Eqs. (1)–(2) have the following perfect-fluid structure:

$$N^\alpha = nu^\alpha, \quad (4)$$

$$T^{\alpha\beta} = (\varepsilon + P)u^\alpha u^\beta - Pg^{\alpha\beta}, \quad (5)$$

respectively, where $g^{\alpha\beta} = \text{diag}(+1, -1, -1, -1)$ is the metric tensor and u^α denotes the four-velocity of the fluid. In addition, one finds that the entropy current, computed with the Boltzmann formula, takes the form of

$$S^\alpha = su^\alpha. \quad (6)$$

Rewriting the total angular momentum tensor $J^{\alpha,\beta\gamma}$ in Eq. (3) as a sum of the orbital $L^{\alpha,\beta\gamma} = x^\beta T^{\gamma\alpha} - x^\gamma T^{\beta\alpha}$ and spin $S^{\alpha,\beta\gamma}$ parts and employing Eqs. (2) and (5), one finds that the spin tensor is separately conserved, $\partial_\alpha S^{\alpha,\beta\gamma} = 0$. Moreover, assuming that the latter has the form proposed in Ref. [2], namely,

$$S^{\alpha,\beta\gamma} = \frac{wu^\alpha}{4\zeta} \omega^{\beta\gamma}, \quad (7)$$

Eq. (3) takes the form of

$$\dot{\bar{\omega}}^{\mu\nu} = 0. \quad (8)$$

Herein, $\omega^{\mu\nu}$ is the spin polarization tensor, which is antisymmetric and satisfies the relation $\epsilon_{\alpha\beta\gamma\delta}\omega^{\alpha\beta}\omega^{\gamma\delta} = 0$, $\bar{\omega}^{\mu\nu} \equiv \omega^{\mu\nu}/(2\zeta)$ is the normalized spin polarization tensor with $\zeta \equiv \frac{1}{2\sqrt{2}}\sqrt{\omega_{\mu\nu}\omega^{\mu\nu}}$ being real, and $(\dot{}) \equiv u \cdot \partial$ denotes the comoving derivative.

The thermodynamic quantities, namely, energy density, pressure, baryon density, and spin density

$$\varepsilon = 4 \cosh(\zeta) \cosh(\xi) \varepsilon_{(0)}(T), \quad P = 4 \cosh(\zeta) \cosh(\xi) P_{(0)}(T), \quad (9)$$

$$n = 4 \cosh(\zeta) \sinh(\xi) n_{(0)}(T), \quad w = 4 \sinh(\zeta) \cosh(\xi) n_{(0)}(T), \quad (10)$$

respectively, satisfy the fundamental thermodynamic relation $\varepsilon + P = sT + \mu n + \Omega w$ with $s = 4 \cosh(\zeta) \cosh(\xi) s_{(0)}(T)$ being the entropy density. The quantities $\xi \equiv \mu/T$ and $\zeta \equiv \Omega/T$ parametrize the baryon μ and the spin Ω chemical potentials, and are, together with the temperature T , treated as the independent thermodynamic variables entering the grand canonical potential. The thermodynamic quantities of (9)–(10) are expressed in terms of auxiliary ones describing a corresponding system of spin-0 particles

$$\begin{aligned} n_{(0)}(T) &= \frac{\kappa}{2\pi^2} T^3 \hat{m}^2 K_2(\hat{m}), \\ \varepsilon_{(0)}(T) &= \frac{\kappa}{2\pi^2} T^4 \hat{m}^2 \left[3K_2(\hat{m}) + \hat{m} K_1(\hat{m}) \right], \\ P_{(0)}(T) &= T n_{(0)}(T), \end{aligned}$$

where $s_{(0)}(T) = \frac{1}{T} [\varepsilon_{(0)}(T) + P_{(0)}(T)]$, $\hat{m} \equiv m/T$ and $\kappa \equiv g/(2\pi)^3$, with g denoting the number of internal degrees of freedom excluding spin.

It is instructive to study projections of Eq. (2). In particular, one finds that the projection of the energy-momentum conservation law onto directions orthogonal to the fluid four-velocity leads to the relativistic Euler equations

$$(\varepsilon + P)\dot{u}^\mu = \partial^\mu P - u^\mu \dot{P}, \quad (11)$$

where $\theta \equiv \partial \cdot u$ is the expansion scalar. On the other hand, projecting Eq. (2) onto the fluid flow and using the differentials of the pressure $P = P(T, \mu, \Omega)$ yields

$$T \partial_\mu (s u^\mu) + \mu \partial_\mu (n u^\mu) + \Omega \partial_\mu (w u^\mu) = 0. \quad (12)$$

Requiring entropy and baryon number conservation, which makes first and second term in (12) vanish, leads to

$$\partial_\mu (s u^\mu) = \dot{s} + s \theta = 0, \quad (13)$$

$$\partial_\mu (n u^\mu) = \dot{n} + n \theta = 0, \quad (14)$$

$$\partial_\mu (w u^\mu) = \dot{w} + w \theta = 0. \quad (15)$$

Equations (11), (13), (14) and (15) are six coupled partial differential equations which determine the dynamics of the space-time-dependent quantities

$\mu(x)$, $\Omega(x) \equiv \frac{T}{2\sqrt{2}} \sqrt{\omega_{\mu\nu} \omega^{\mu\nu}}$, $T(x)$ and $u^\mu(x)$. On top of their evolution, one has to solve Eq. (8) for the normalized components of the polarization tensor.

In Refs. [6, 7], it was shown that Eqs. (11), (13), (14) and (15) have the stationary vortex-like solution corresponding to a rotating global equilibrium state [2, 3] with

$$u^\mu = \gamma \left(1, -\tilde{\Omega} y, \tilde{\Omega} x, 0 \right), \quad (16)$$

$T = T_0 \gamma$, $\mu = \mu_0 \gamma$ and $\Omega = \Omega_0 \gamma$, where $\gamma = 1/\sqrt{1 - \tilde{\Omega}^2 r^2}$ is the Lorentz factor, $r = \sqrt{x^2 + y^2}$ and T_0 , μ_0 , and Ω_0 are arbitrary constants. The corresponding non-trivial form of the polarization tensor is $\omega_{ij} = -\omega_{ji} = \tilde{\Omega}/T_0 = 2 \Omega_0/T_0$ for $i = x$ and $j = y$, and $\omega_{ij} = 0$ otherwise. In the next section, we study Eqs. (11), (13), (14), (15) and (8) in the case where the equilibrium is achieved only locally.

3. Numerical results

In the following, we study numerically the solutions of the hydrodynamic equations presented above for a rotating spin-polarized medium, which resembles the systems created in the low-energy heavy-ion collisions. It is modelled by a Gaussian source $T_i = T_0 g(x, y, z)$, where $g(x, y, z) = \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{z^2}{2\sigma_z^2}\right)$. The initial flow is assumed to have the form of (16), where we replace $\tilde{\Omega}$ with $(1/r) \tanh(r/r_0)$. The parameter $r_0 = 1$ sets the rotation speed. The initial spin chemical potential is given by $\Omega_i = 0.03 T_i/2$, while the initial baryon chemical potential is given by $\mu_i = \mu_0 g(x, y, z)$, where $\mu_0 = 200$ MeV. With this setup, we let the system evolve in Minkowski time starting at $t_0 = 0.1$ fm using Eqs. (11), (13), (14) and (15). Once the evolution is finished, we initialize the normalized polarization tensor with

$$\bar{\omega}_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{\omega}_{xy} & \bar{\omega}_{xz} \\ 0 & -\bar{\omega}_{xy} & 0 & \bar{\omega}_{yz} \\ 0 & -\bar{\omega}_{xz} & -\bar{\omega}_{yz} & 0 \end{bmatrix}, \quad (17)$$

and evolve it on top of our hydrodynamic results, using Eq. (8) starting from $t = t_0$. The results of the latter calculations are presented in Fig. 1. Initially (top left panel), the $\bar{\omega}_{xy}$ (medium grey/red) component dominate $|y| < 6$ region, while the $\bar{\omega}_{yz}$ and $\bar{\omega}_{zz}$ components dominate the $y > 6$ and $y < -6$ regions, respectively. During the subsequent evolution, the spin polarization is transferred to different regions of space due to the non-trivial velocity field of the medium. At the very end of the evolution, the component $\bar{\omega}_{xy}$ dominates over the other components in almost the entire space.

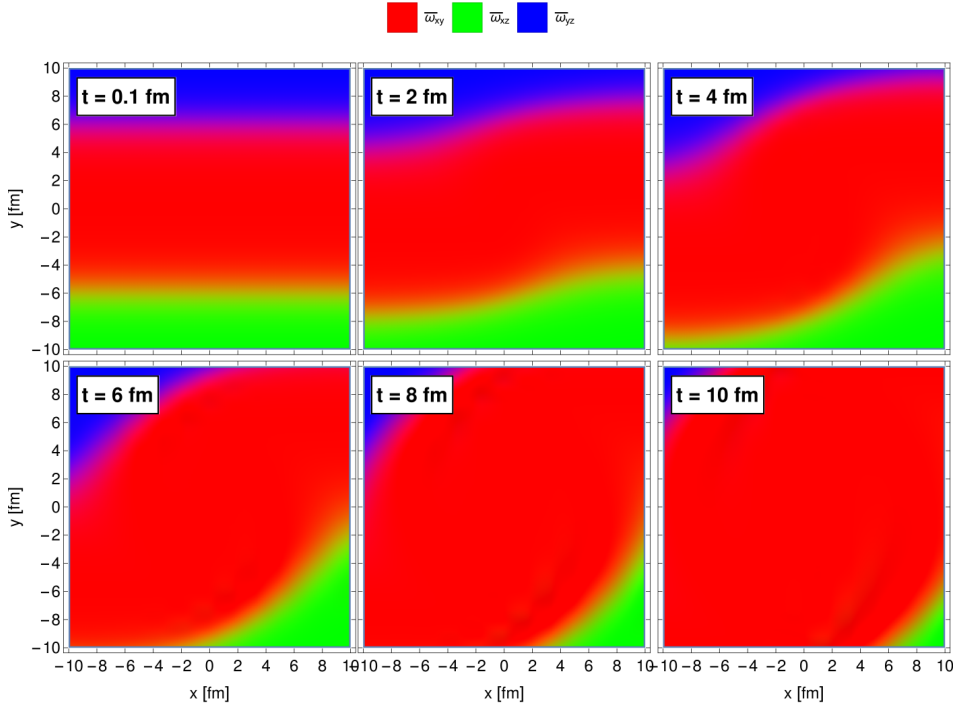


Fig. 1. (Colour on-line) The components of the spin polarization tensor: $\bar{\omega}_{xy}$ (medium grey/red), $\bar{\omega}_{xz}$ (light grey/green) and $\bar{\omega}_{yz}$ (dark grey/blue) in the reaction plane (x - y) at times: $t = 0.1, 2, 4, 6, 8, 10$ fm.

Our study shows that, if the dynamics of the spin polarization is included in the fluid modelling, the spin polarization of the medium may change significantly over its evolution. Such a fluid dynamic stage is missing in the models used so far for interpreting the data. Thus, it is of great importance to properly model the early-time dynamics of the spin polarization as this may significantly affect the final results.

4. Summary

Employing a novel formulation of relativistic perfect fluid hydrodynamics for polarized systems of particles with the spin $1/2$, we numerically studied the evolution of a rotating spin-polarized source resembling the ones created in high-energy heavy-ion collisions. We find that spin polarization tensor undergoes a non-trivial evolution in the hydrodynamic stage which may significantly affect the final results used for the interpretation of experimental data.

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