TIME IN QUANTUM PROCESSES*

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We discuss the problem of time in quantum systems. We present experimental observations which are hard or impossible to explain on the grounds of conventional quantum mechanics. The need of introducing time as an observable and not just a numerical parameter is stressed. We show that this is possible in the projection evolution approach.

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1. Introduction

Time is a handy measure, commonly used in our everyday life. It allows to order events and quantify the age. The evolution of physical systems is in most cases described as a function of time.

Time appears in almost all physical models as a quantity which is common for every object in the system. We silently assume that we can always measure and compare time. We do not try to answer the question about the origin of this quantity, one exception being the relativity theory, in which time is treated as the fourth coordinate of space-time. In the relativistic approach, even though time changes between the coordinate systems, it is always present and can be transformed to the new coordinate system together with the spatial coordinates.

In this paper, we briefly point out that the parameter-like time in quantum mechanics is not sufficient to consistently describe some phenomena seen in experiments. We present few selected experimental observations which strongly suggest that time in the form of an observable is needed in

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our models. We finish the presentation by discussing the projection quantum mechanics in which time evolution of the system has the form of a set of projections of the density matrix onto appropriate spaces. This model can explain the presented experiments in a natural way.

2. Experiments on quantum systems

Below, we present a selection of experiments on quantum systems. The traditional quantum mechanics has problems with the description of the time evolution of the test objects. One basic reason is the fact that in quantum mechanics, there is no time operator and, consequently, time cannot be measured — it is not an observable.

2.1. Interference on time slits

Many different experiments indicated that the notion of time is not properly understood on the quantum level. One of the first temporal realisations of the Young double slit experiment was described by Hauser *et al.* in [1]. In this work, a weak single-photon source was used. A rotating disc with slits was inserted between the source and the detector. The photons could have either passed through a slit or be blocked by the space between two subsequent slits, depending on the adjustable speed of rotation of the disc. The authors were measuring the energy spectrum of the photons in the detector and they observed an interference pattern. Even though there was no proper theoretical explanation given, the authors tried to describe this finding using the Fourier transform of the photon's energy spectrum. As time is canonically conjugate to energy, this experiment seems to indicate that the photon which passed one of the slits was interfering with the same photon which passed another slit. These two events happened at different instances of time, so the interference must have occurred in the temporal regime, leading to an interference pattern in the energy spectrum. The faster the disc was rotating, the more fringes could have been found in the recorded spectrum, which is in agreement with the fact, that more slits appeared on the photon's path during its travel to the detector.

Another version of this experiment was performed by Lindner *et al.* [2], but instead of using a mechanical device, a special laser pulse played the role of the two slits (see Fig. 1). The laser pulse was absorbed by electrons, leading to photoionisation of the medium. The energy needed by the electrons is marked in Fig. 1 by the dashed lines. On the negative part of the pulse, there is one region (a single "slit"), while two such regions are on the positive part of the vector potential (two "slits"). The photoionisation process needed more time to complete than the time distance between the two maxima on the positive side of the pulse. Whenever the electron was

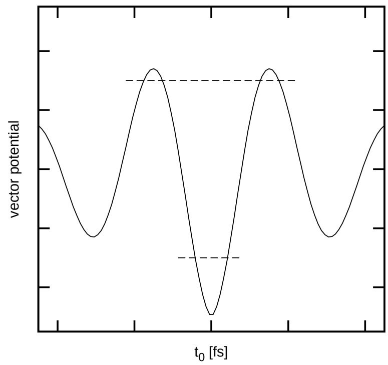

Fig. 1. The shape of the laser pulse in the Lindner *et al.* experiment [2]. The energy required for the photoionisation is marked by the dashed lines.

interacting with the positive part of the pulse, its proper description should have the form of a superposition of states in which it was ionised by the first or the second maximum. As a result, interference fringes were observed. As in the Hauser *et al.* experiment, they were representing the self-interference of the electron interacting with different maxima of the potential, *i.e.*, interactions in different instances of time. The measurements allowed to assess the time-width of the "slits" to be approximately 500 as.

In order to describe the interference of the wave functions (or density matrices) in time, time must be a coordinate. It follows that we need a time operator which projects the wave functions onto the time axis. The standard Schrödinger time evolution, in which time is introduced as a numerical parameter, does not allow for such description.

2.2. Delayed choice experiments

In the delayed choice experiment, alterations to the set-up are introduced when the test object has already passed a part of it. It has been observed that the test particle behaves in accordance with the changes made to the set-up, as if the modifications influenced the behaviour of the test object in the past.

In 1984 Wheeler proposed a Gedankenexperiment [3] based on an interferometer (see Fig. 2). If there is only one beamsplitter in the set-up (BS1, left panel), the photon goes either along path 1 or path 2. In the case of two BSs (middle panel), the photon goes along both paths and interferes with itself on the second BS2. Wheeler argued that in the delayed choice version, when the second BS2 is inserted in the set-up after the photon has passed BS1, the photon should behave as if the BS2 were in the set-up from the beginning.

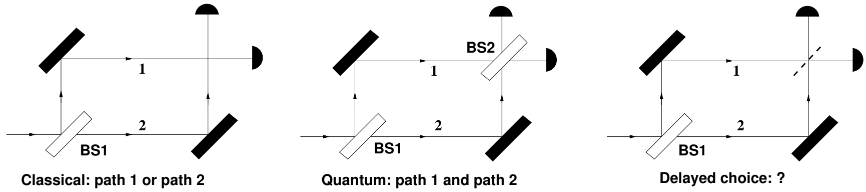

Fig. 2. Wheeler's Gedankenexperiment. Open rectangles represent beamsplitters, solid rectangles are perfect mirrors. In the right panel, the second beamsplitter is inserted after the photon has passed the first one.

Experimental realisation of the Wheeler's idea was first performed using an electro-optical set-up [4]. The results confirmed Wheeler's predictions. It may be argued that this particular result can be explained by the spatial width of the photon's wave function. As the length of the photon is comparable with the size of the experimental set-up, the interaction may have occurred through the spatial coordinate. Other experiments, especially those based on entangled particles, have investigated situations in which spatial interference was not enough to explain the results.

One may try to propose many different mechanisms which will explain the delayed choice behaviour. The most straightforward, however, is the one which assumes some time width of the test particle. It means that the particle occupies certain interval of time and exists in the "past" as well as in the "future", according to the laboratory clock. Being not sharply localised in time, the particle behaves in accordance with the state of the set-up even in the case, when the state changed "after" the particle should have been able to notice it (again, from the classical point of view). The particle remains not sharply localised in time as long as it is not measured, because the measurement can localise it in a space-time region. This picture requires again the time coordinate and the possibility to project the wave function (or density matrix) onto time intervals.

2.3. Experiments with entangled particles

Entangled quantum states of two or more particles take into account some conservation rule which works among them. These states are non-separable, which means that they cannot be written in the product form of the state of the first particle times the state of the second one. The conservation rule may describe, *e.g.*, the total spin of the pair of particles such that the change of the spin of one particle influences the other immediately, with no delay.

Entangled particles are vital for many quantum algorithms including quantum teleportation and quantum cryptography. The correlation between measurements performed on entangled particles, which have been previously separated, has been investigated many times in different conditions. In one of such experiments, the two laboratories were located on two different islands and separated by 144 km [5]. One of the entangled photons was entering the interferometer, while the other was transmitted to the second laboratory. A quantum eraser could have been used to erase the information which way the photon took in the interferometer. The two events: "second photon detected" and "quantum eraser used" were not in the same light cone, but despite this, when the data from both laboratories were compared, a perfect correlation between them was found. The authors conclude [5]: "*Our results demonstrate that the viewpoint that the system photon behaves either definitely as a wave or definitely as a particle would require faster-than-light communication. (...) we believe that such a viewpoint should be given up entirely.*"

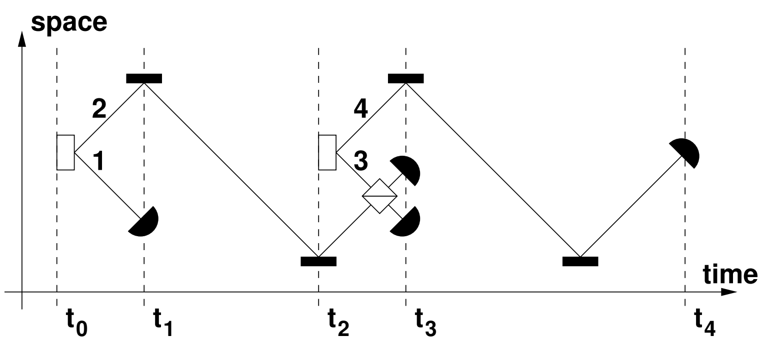

Fig. 3. Entanglement between particles that never coexisted in time (see [6]).

Another experiment showed [6] that one can entangle particles which never coexisted in time (see Fig. 3). A pair of entangled particles was created at $t = t_0$. Particle 1 was measured at $t = t_1$, while particle 2 was directed forwards. Later, a second pair of entangled particles was created from which particle 3 interacted with particle 2 by the projection on the Bell basis. In this way, these particles became entangled. Particle 4 was directed forwards and measured at $t = t_4$. The comparison of measurements of particles 1 and 4 showed a perfect correlation, *i.e.*, these particles were entangled even though particle 1 was caught by the detector well before particle 4 was created.

Both of these examples show clearly that in the quantum system, the events ordering parameter is not the same as the time measured in the laboratory. In fact, the time instances as we perceive them are set by classical measurements, while the evolution of the quantum system between the measurements must be driven by something which does not need to be in a one-to-one correspondence with classical time.

3. Time in the projection quantum mechanics

To consistently explain the above-mentioned experiments, time in the quantum theory must be an observable, not a parameter. It was quite early realised by Pauli, that it is impossible to define a Hermitean operator which will represent the time observable canonically conjugate to the energy [7]. Recent research showed that the experimentally available observables can, in general, be represented by positive operator-valued measures (POVM) rather than Hermitean operators. Taking into account this weaker assumption, the Pauli theorem does not apply and it has been shown that constructing a time POVM should be possible [8].

What is time? In the relativity theory, time is introduced as the fourth dimension of our space-time. In contrast with the spatial distance, it does not seem to be an internal feature of the space, but rather a measure that allows us to order events. Each event, in turn, is related to some change of the state of the system. As a consequence, we postulate that change is the fundamental property of the universe and that changes give us the possibility to define and measure both spatial and temporal distances.

As time behaves like a coordinate, being timeless is unphysical in the same way as occupying zero volume is unphysical. Therefore, one has to remember that every physical object and process occupies a non-zero time interval. Since we introduce time as a quantity being canonically conjugate to the zeroth component of the four-momentum, a Heisenberg-like relation exists for them. The quantum states between measurements do not follow the classical chronology, describing the system's "past", "present", and "future" simultaneously. For our convenience, we introduce an ordering parameter τ which will number the events in the quantum system. The τ ordering has nothing to do with the time ordering; in fact, τ is not a measurable observable and should play no role after a measurement is performed.

The state of a quantum system is described by the density matrix $\rho(\tau, \nu)$, where τ is the ordering parameter and ν represents the set of quantum numbers. In general, in each step of the evolution, the system will evolve to one out of many different final states

$$\rho(\tau_0, \nu_0) \to \begin{cases} \rho\left(\tau_1, \nu_1^{(1)}\right) \\ \rho\left(\tau_1, \nu_1^{(2)}\right) \\ \quad\cdots \\ \rho\left(\tau_1, \nu_1^{(n)}\right) \end{cases} . \tag{1}$$

The choice of one particular state is random and obeys the probability distribution given by the evolution operators $\mathcal{E}$. We write, therefore,

$$\rho(\tau_n, \nu_n) = \frac{\mathcal{E}_n \rho(\tau_{n-1}, \nu_{n-1}) \mathcal{E}_n^\dagger}{\mathrm{Tr}\left(\mathcal{E}_n \rho(\tau_{n-1}, \nu_{n-1}) \mathcal{E}_n^\dagger\right)}\,, \tag{2}$$

where $\mathcal{E}_n$ project the state $\rho(\tau_{n-1}, \nu_{n-1})$ onto the space of states labelled τ_n, and the denominator gives the proper normalisation. The evolution $\tau_0 \to \tau_1 \to \cdots \to \tau_n$ takes the form of subsequent projections

$$\rho(\tau_n, \nu_n) = \frac{\mathcal{E}_n \mathcal{E}_{n-1} \dots \mathcal{E}_1 \rho(\tau_0, \nu_0) \mathcal{E}_1^\dagger \dots \mathcal{E}_n^\dagger}{\mathrm{Tr}\left(\mathcal{E}_n \mathcal{E}_{n-1} \dots \mathcal{E}_1 \rho(\tau_0, \nu_0) \mathcal{E}_1^\dagger \dots \mathcal{E}_n^\dagger\right)}\,. \tag{3}$$

To describe the evolution of a quantum system is to define the initial state $\rho(\tau_0)$ and the set of evolution operators $\mathcal{E}_{1,2,\dots}$. At the end, a connection to the classical world has to be established via a final measurement. The classical chronology comes in our model from the order of irreversible projection operators on the interface between the quantum and the classical part of the system. For more details about the problem of time in the quantum theory, see Refs. [9–17].

Going back to the previously mentioned experiments, the projection evolution model allows to naturally explain all of them. In the case of the interference on time slits, as ρ is not projected on any particular t by the quantum operations, it contains the information about the state of the system between the measurements. As a result, the interference between different τ components of the density matrices is possible and corresponds to the time width of the described system.

A similar situation is encountered in the delayed choice experiments. The measurement is done at the end of the experiment. At the same time, the particle's state has time width comparable with the duration of the experiment. This means that the particle has the possibility of interacting with the changed part of the set-up, even though it appears as a retrocausality as far as the laboratory clock is concerned.

In the case of entangled particles, one has again to remember that the whole process is ordered by τ, not t, and therefore is not bound by the speed of light measured according to the external clock. Moreover, the time width of the particle covers the time interval between the measurements. It is also important to mention that this interaction cannot transfer information, so the upper limit on the speed of information transfer is preserved.

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