# STRUCTURE AND PROPERTIES OF SUPER-HEAVY NUCLEI IN THE WORK OF ADAM SOBICZEWSKI AND HIS COLLABORATORS\*

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In honour of Professor Adam Sobiczewski

The history of the discovery of super-heavy nuclei is strongly related to the name of Professor Adam Sobiczewski who passed away two years ago. Already in 1966, he and his co-workers had predicted new proton and neutron magic numbers in very heavy nuclei that had not yet been observed at that time. During the last 50 years his and his group's theoretical estimates of properties of super-heavy nuclei such as spontaneous fission probabilities or alpha-decay half-lives have served to experimentalists all over the world as a guideline in exploring this *terra incognita* in the chart of atomic nuclei.

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## 1. Introduction

Professor Adam Sobiczewski, who passed away in 2017, was one of the leading Polish nuclear theoreticians. He made a seminal impact in the worldwide research of Super-Heavy Nuclei (SHN). He was an author of more than 150 scientific papers, which were cited around 6000 times in the world scientific literature.

Adam Sobiczewski has had many pupils. Apart from the two authors of the present paper, who were his first promoted doctor-students, he brought up eight other doctors of physics and got twelve scientific *grandchildren* and, till now, three *great grandchildren*. He inspired many scientists all over the world to investigate the existence probability of new elements, search for

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Fig. 1. Professor Adam Sobiczewski (1931–2017) during a lecture at the 7<sup>th</sup> Nuclear Physics Workshop in Kazimierz Dolny in 2000.

them experimentally and describe them theoretically. He brought up a large number of specialists in this field, who have been developing his ideas till now. He was, without any doubt, one of the founders and guiding figures of the Polish Nuclear Theory community.

The scientific research of Adam Sobiczewski has always been closely connected with the work of experimentalists. Theoretical estimates obtained by him and his group have served the experimentalists from the GSI-Darmstadt and the JINR-Dubna as well as other institutes all over the world as a guideline in their exploration of the *terra incognita* of super-heavy elements and *vice versa*, all experimental achievements and discoveries always served him to improve his models. His close collaboration with experimentalists, involved in the search for super-heavy nuclei, and his enthusiasm and engagements has strongly influenced the discovery of the new elements and their isotopes.

### 2. The begin of the dream on super-heavy elements

Already in 1966, Sobiczewski, Gareev and Kalinin [1] predicted the existence of SHN around the new magic numbers Z = 114 for protons and N = 184 for neutrons. In the abstract of this seminal paper, one can read how this prediction was made: The calculation is performed for the mass number values A = 209, 275, 299 and 355 with (in the case of protons) Z = 82+1, 106+1, 114+1, and 126+1, respectively. The radius of the potential is taken as  $R_0 = r_0 A^{1/3}$  with  $r_0 = 1.27$  fm, the parameter of the surface diffuseness a = 0.67 fm and the parameter characterizing the strength of the spin-orbit coupling  $\lambda = 32$ . The neutron potential depth is taken as  $V_{on} = 44$  MeV for all A values and the proton depth as  $V_{op} = 58$ , 60.8, 62 and 63 MeV for A = 209, 275, 299 and 355, respectively.

The level schemes for protons and neutrons are shown in Fig. 2.



Fig. 2. Proton (top) and neutron (bottom) single-particle energies obtained with the Woods–Saxon potential as a function of the mass number A. The plots taken from Ref. [1].

The new magic numbers Z = 114, N = 184 predicted in Ref. [1] were confirmed soon after by the Swedish–Polish group who had made similar calculations using the Nilsson, as well as the Woods–Saxon potential with parameters fixed by Rost [2]. The stability of nuclei with respect to fission and alpha-decay was then extensively studied by this Lund–Warsaw collaboration, where Adam Sobiczewski was an active member. The macroscopic– microscopic method developed by Myers and Świątecki [3] but with the Strutinsky shell correction [4] and a 6<sup>th</sup> order correction polynomial, instead of the 2<sup>nd</sup> order one originally proposed by Strutinsky, was used to evaluate the potential energy surfaces (PES) of transactinide and hypothetical SHN [5]. The detailed analysis of the PES helped to predict the groundstate masses and fission-barrier heights of these nuclei, what allowed then to make estimates of the alpha-decay and spontaneous fission probability. These first estimates were very optimistic, as one can see in Fig. 3 taken from Ref. [5]. The double magic nucleus (Z = 114, N = 184) was predicted



Fig. 3. Theoretical estimates of the spontaneous fission (solid lines) and alphadecay (dashed lines) half-lives of the super-heavy nuclei obtained in 1969 by the Lund–Warsaw Group [5].

stable against fission  $(T_{1/2}^{\rm sf} > 10^{13} \text{ y})$  and to have an alpha-decay life-time around one year. Contrary to the case of the heaviest nuclei known at that time, the liquid-drop fission barrier is vanishing for these SHN and it is only shell effects that protect them against the fission instability, as shown in Fig. 4.



Fig. 4. Liquid drop (dashed lines) and macroscopic–microscopic (solid lines) fission barriers for selected heavy and super-heavy nuclei (figure taken from Ref. [5]).

Such very optimistic theoretical estimates have initiated a true run to the island of super-heavy nuclei. The experimentalists from the Lawrence Berkeley Laboratory (LBL) and the Joint Institute for Nuclear Research (JINR) in Dubna played a dominant role in that pioneering period. They have tried to synthesize the super-heavy nuclei by heavy-ion reactions or even to search them in nature. Already in the first decade after the paper of Sobiczewski et al. [1], three new elements: dubnium (Z = 105), seaborgium (Z = 106)and bohrium (Z = 107) were found. The discovery of the latest element made in 1976 by the JINR was confirmed in 1981 by the experimental group from the Geselschaft für Schwerionen-Forschung (GSI) in Darmstadt who had joint the hunting party for the SHN. Since that time the GSI and the JINR have played a dominant role in the discovery of SHN. The competition and collaboration between these two laboratories has led to the discovery of the next eleven elements up to oganesson (Z = 118). Adam Sobiczewski and his collaborators have always been in close contact with both these laboratories making the theoretical analysis of the probability for the production and decay properties of the new super-heavy isotopes.

Nowadays, in addition to the GSI and JINR, the experimental research in the super-heavy region of nuclei is also conducted at the Lawrence Livermore National Laboratory (LLNL), the Oak Ridge National Laboratory (ORNL), the Accelerator Laboratory of the University of Jyväskylä (JYU), the Nishina Center for Accelerator-Based Science (RIKEN), and the Grand Accelerateur National d'Ions Lourds (GANIL) in Caen.

Already in 60s and 70s, several theoreticians raised the question whether the possible elements would end at the island of SHN nuclei around Z =114, or whether even larger magic numbers for proton and neutron could possibly exist? Adam Sobiczewski, as a born optimist, together with his co-worker checked *e.g.* the stability of hypothetical nuclei around Z = 164 and N = 228. Their estimates of the spontaneous fission and alpha-decay life-times for these hyper-heavy nuclei are shown in Fig. 5. The very large predicted  $\alpha$ -decay probabilities (r.h.s. of Fig. 5) have unfortunately killed the dream about the existence of hyper-heavy nuclei in nature.



Fig. 5. Theoretical estimates of spontaneous fission (l.h.s.) and alpha-decay (r.h.s.) life-times of hyper-heavy nuclei around Z = 164 and N = 228. The figure taken from Ref. [6].

It was clear from the very beginning that a reliable theoretical model had to reproduce the data for existing nuclei as precisely as possible before one applies it to the unknown region of nuclei. An attempt to reproduce the spontaneous fission half-lives of actinide nuclei using the macroscopic– microscopic model based on the Nilsson single-potential, the Myers–Świątecki liquid-drop and the cranking inertia was made in Ref. [7]. In Fig. 6, the estimated  $T_{1/2}^{\text{sf}}$  of Ref. [7] are shown. The agreement with the data, one of the best on the market at that time, was rather not satisfactory. One had to decrease the value of the collective inertia by 20% and even then the theoretical estimates differ by around 2 orders of magnitude from the data. It was clear for Adam Sobiczewski and his co-workers that a more advanced model was necessary. The calculations should have been made in a multidimensional deformation space and the whole collective inertia tensor had to be taken into account.



Fig. 6. Theoretical estimates of spontaneous fission life-times of even-even isotopes and their shape-isomers (circles) compared with the experimental data (crosses). The figure taken from Ref. [7].

## 3. Dynamical effects

In the WKB approximation, the fission-barrier penetration probability is determined by the action integral (S) along the effective one-dimensional path to fission (L)

$$P = [1 + \exp S(L)]^{-1}.$$
 (1)

The magnitude of the action integral, given by the following formula:

$$S(L) = 2 \int_{s_1}^{s_2} \sqrt{\frac{2}{\hbar^2} [V(s) - E] B_s(s)} \,\mathrm{d}s$$
(2)

depends not only on the barrier height (V - E) but also on the magnitude of the collective inertia  $(B_s)$  along the path L. In most of the first estimates of the fission life-times, only a path which minimizes the fission barrier in the multidimensional deformation-parameter space, the so-called *static* path, was taken into account. Already in Ref. [8] it was shown, however, that in some cases, the static path does not correspond to the largest fissionbarrier penetrability and one has to find a *dynamical* path which takes also into account the variance with deformation of the inertia-tensor components. An effective way of finding the dynamical trajectory in the multidimensional space was proposed by Baran in Ref. [9]. The fission probability evaluated along the static path was typically a few orders of magnitude smaller than that along the dynamic path.

These dynamical effects in the two-dimensional space: elongation  $(\varepsilon)$ and neck  $(\varepsilon_4)$  were included in Ref. [10]. In addition, the reduction of the fission-barrier height due to the nonaxial  $(\gamma)$  and the left-right asymmetry  $(\varepsilon_3)$  degrees of freedom was taken into account. The results are presented in Fig. 7 taken from Ref. [10]. As seen in the figure, the fission barrier along the dynamic path is only slightly higher than that evaluated along the static path. The estimates of the spontaneous-fission life-times made in Ref. [10] are compared with the experimental data in the r.h.s. part of Fig. 7.



Fig. 7. Fission barrier (l.h.s. top) and collective inertia (l.h.s. bottom) of  $^{246}$ Cm along the static and dynamic paths to fission. Theoretical estimates of the spontaneous fission life-times obtained by inclusion of the dynamical and the left-right and nonaxial asymmetry degrees of freedom are compared with the experimental data for even-even isotopes (r.h.s.). Figures taken from Ref. [10].

The agreement found in Ref. [10] between the theoretical and experimental life-times was very satisfactory at that time. These are probably the best estimates obtained on the basis of the Nilsson potential. It was the hope of Adam Sobiczewski and his co-workers that taking into account a more realistic Woods–Saxon single-particle potential, one could obtain a better reproduction of the data for existing nuclei and thus give a better prediction of the life-times for the super-heavy nuclei.

### 4. Prediction of a new region of deformed super-heavy nuclei

New calculations [11] made with the Woods–Saxon potential and higher multipolarity deformations gave the results qualitatively presented in Fig. 8. A new island of deformed super-heavy nuclei centred around Z = 108 and N = 162 was predicted.



Fig. 8. Regions of relatively long-lived heavy nuclei as believed earlier (a), and after inclusion of higher multipolarity deformations (b). The figure taken from Ref. [11].

The potential energy was calculated by the macroscopic–microscopic method. The ground-state deformation parameters  $\beta_2^0$  up to  $\beta_8^0$  for heavy and super-heavy nuclei found in Ref. [12] are shown in Fig. 9. The quadrupole deformation  $\beta_2^0$  of the SHN around  $Z \simeq 104$  and  $N \simeq 150$  even exceeds 0.24, a value comparable to the ground-state deformations of actinide nuclei. This was a rather unexpected result at that time, nowadays completely confirmed by the experimental data.

It was found in Ref. [13] that the deformation energy  $(E_{\text{def}})$  defined as the difference between the binding energy of a spherical and ground-state deformation could reach 12 MeV, as can be seen on the l.h.s. of Fig. 10. The contribution of different multipolarities  $(\beta_{\lambda})$  to the energy of <sup>270</sup>Hs nucleus is analysed on the r.h.s. plot of Fig. 10. It can be seen that one has to take into account all deformations up to  $\lambda = 10$  when one would like to predict the ground-state energy with a reasonable accuracy [14].

The spontaneous fission life-time of the SHN from Rf (Z = 104) up to Fl (Z = 114) estimated in Ref. [15] using the macroscopic-microscopic model with the Woods-Saxon single-particle potential and taking into ac-



Fig. 9. Ground-state deformations  $\beta_2$  to  $\beta_8$  of heavy and super-heavy nuclei (the figure taken from Ref. [12]).



Fig. 10. Ground-state deformation energy of heavy and super-heavy nuclei (l.h.s.) and contribution of different multipolarities to the deformation energy of  $^{270}$ Hs (Z = 108) (r.h.s.). Figures taken from Refs. [13] and [14], respectively.

count higher multipolarity deformations is presented in Fig. 11. A good agreement of the estimates (open squares) made by Adam Sobiczewski and co-workers with the experimental data available at that time (black squares) can be observed in Fig. 11. Later experimental findings have shown how accurate these predictions were (confer *e.g.* Ref. [16]).



Fig. 11. Spontaneous fission and  $\alpha$ -decay life-times of even–even isotopes from Rf (Z = 104) to Fl (Z = 114). Theoretical estimates (open squares) are compared with the experimental data (full squares). The figure taken from Ref. [15].

### 5. $\alpha$ -decay half-lives

Apart from spontaneous fission, the  $\alpha$ -particle emission is one of the most important decay mode of SHN. That is why Sobiczewski and his group have studied this question with special care. Already in 1995 [15], they obtained very reasonable estimates of the  $\alpha$ -decay life-times of super-heavy nuclei as shown in Fig. 11. The probability of an  $\alpha$ -decay depends on the  $\alpha$ -particle energy which is determined through the mass difference of mother and daughter nuclei. That is why good estimates of the nuclear binding energies are so important when one would like to describe properly  $\alpha$ -decay life-times of unknown nuclei. This is a difficult task, and Sobiczewski, who was rather pragmatic, decided to find a liquid-drop-type mass formula which reproduced accurately the binding energy of the heaviest nuclei only. The difference between the experimental masses and the macroscopic–microscopic estimates made using this local liquid-drop mass formula for heavy nuclei (called HN) [18] is shown in Fig. 12.



Fig. 12. Difference between calculated ( $M_{th}$ ) and measured ( $M_{exp}$ ) masses of nuclei with proton number Z = 84-93 (l.h.s.) and Z = 94-108 (r.h.s.). The figure taken from Ref. [18].

A next step done by Sobiczewski and Parkhomenko was to modify the well-known Viola–Seaborg formula to better describe the  $\alpha$ -decay life-time of known heavy nuclei [17]. Their new formula (PS) reads

$$\log_{10} T_{\alpha}^{\rm ph}(Z,N) = aZ \left( Q_{\alpha} - \bar{E}_i \right)^{-1/2} + bZ + c \,, \tag{3}$$

where  $\bar{E}_i$  is the average excitation energy of the nucleus emitting the  $\alpha$ -particle and equals 0 for even-even nuclei and equals  $\bar{E}_p$  and  $\bar{E}_n$  for odd-proton and odd-neutron nuclei, respectively. In addition, one has assumed that  $\bar{E}_i = \bar{E}_p + \bar{E}_n$  for odd-odd system. The 5 free parameters  $(a, b, c, E_p, E_n)$  of Eq. (3) were fitted in Ref. [17] to the known life-times of nuclei heavier than <sup>208</sup>Pb. During the next years, Eq. (3) was successfully used to predict the  $\alpha$ -decay life-times in the super-heavy region of nuclei. The masses of unknown isotopes, which were necessary to obtain the energy  $Q_{\alpha}$ , were evaluated in the macroscopic-microscopic model using the local liquid-drop formula for the heavy nuclei [18]. The estimates of the  $T_{\alpha}$  life-time obtained in Ref. [15] for even-even SHN are shown in Fig. 11. The agreement of the estimates with the experimental data is obviously very good.

## 6. Fission barrier heights of super-heavy nuclei

The barrier height is one of the most important factors determining the life-times of spontaneously fissioning nuclei (confer *e.g.* Ref. [19]). That is why it is so important to describe their values as accurately as possible when one would like to study the fission probability.

The detailed calculations of the potential energy surfaces in the multidimensional deformation-parameters space, based on the expansion of nuclear shapes in spherical harmonics were performed by the Sobiczewski group for actinide nuclei [20]. The macroscopic-microscopic model with the Woods-Saxon potential and the FRLD model [21] was used there. The obtained fission-barrier heights for actinides were rather close to the experimental estimates (see Fig. 13), what has given some hope that the predicted fission-barrier heights of the SHN were reliable.



Fig. 13. Difference between theoretical and experimental inner fission-barrier heights for even–even actinide nuclei as a function of the neutron number. The figure taken from Ref. [20].

One has to mention here the role of the nonaxial quadrupole and higher order deformations in determining the barrier height. An example of the PES shown in Fig. 14 illustrates well this effect. The barrier height for the  $^{302}120$  nucleus is reduced by 1.8 MeV when one takes the nonaxial degrees of freedom into account. To be sure that all important deformation modes were included, the authors have even studied for some selected SHN the effect of the nonaxial octupole-type deformation which leads to tetrahedrallike shapes [24], what indicates how accurate these investigations were.



Fig. 14. Contour map of the potential-energy surface of the nucleus  ${}^{302}120$ . Positions of ground state  $[E_I \text{ (circle)}]$  and higher  $[E_A^{\text{NAX(II)}} \text{ (cross)}]$  and lower  $[E_A^{\text{NAX(I)}} \text{ (star)}]$  nonaxial saddle points are indicated. The axial saddle point  $E_A^{\text{AX}}$  is marked by a filled square (the figure taken from Ref. [20]).

The resulting values of the fission-barrier heights of heavy and superheavy nuclei predicted in Ref. [20] are shown in Fig. 15. Apart from the deformed SHN region in which for some nuclei the estimated barrier heights exceed even 6 MeV, another island of high barriers is located around  $^{294}$ Lv.

As has been mentioned before, a precise prediction of the binding energies of unknown nuclei is absolutely crucial for reliable estimates of barrier heights and  $Q_{\alpha}$  values. Both quantities decide about the stability of nuclei to be possibly discovered. This is the main reason why Adam Sobiczewski always checked very carefully the nuclear mass predictions given by different theoretical models. In his last paper [25], written together with Litvinov and Palczewski, they compared the quality of the reproduction of experimental



Fig. 15. Contour map of calculated fission-barrier heights  $B_{\rm f}$  for even-even superheavy nuclei (the figure taken from Ref. [20]).

binding energies of nuclei from different mass regions. Ten theoretical models were compared. The main goal was to show the predictive power of different mass formulae. The root-mean square deviations of the estimates of each model from the data available in 2003 and in 2017 are compared. In Fig. 16, the discrepancies of the estimates made using the LSD [26] and the FRLD [21] from the experimental masses are shown. Such an analysis made for several models will certainly be very helpful in future theoretical calculations of properties of not yet discovered nuclei.



Fig. 16. R.m.s. deviation of LSD (l.h.s.) and FRDM (r.h.s.) estimates from the experimental data available in 2003 (solid line) and in 2017 (dahed line). The symbols: L, M-I, M-II, H and G denote the regions of light, medium-I, medium-II, heavy, and all considered nuclei, respectively (figures taken from Ref. [25]).

#### 7. Summary

It is difficult to make a selection of the most important steps in the more than 50 years long search of the late Professor Adam Sobiczewski for the region of super-heavy nuclei. The majority of his more than 150 scientific papers was devoted to this thematics. We have chosen here the papers which we consider to be most characteristic of his style of scientific research.

We miss Him very much and regret not having the opportunity to meet Him any more, discuss about life or science, walk in the mountains, sail or ski. He was our Teacher, Co-worker and Friend. It is a pity that our Master cannot continue further his inspiring work as a guideline for future scientists.

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#### REFERENCES

- [1] A. Sobiczewski, F.A. Gareev, B.N. Kalinkin, *Phys. Lett. B* 22, 500 (1966).
- [2] S.G. Nilsson et al., Nucl. Phys. A 115, 545 (1968).
- [3] W.D. Myers, W.J. Świątecki, *Nucl. Phys. A* 81, 1 (1966).
- [4] V.M. Strutinsky, Yad. Fiz. (USSR) 3, 614 (1966) [Sov. J. Nucl. Phys. 3, 449 (1966)]; Nucl. Phys. A 95, 420 (1967).
- [5] S.G. Nilsson *et al.*, *Nucl. Phys. A* **131**, 1 (1969).
- [6] A. Łukasiak, A. Sobiczewski, Acta Phys. Pol. B 6, 147 (1975).
- [7] J. Randrup *et al.*, *Phys. Rev. C* **13**, 229 (1975).
- [8] M. Brack et al., Rev. Mod. Phys. 44, 320 (1972).
- [9] A. Baran, *Phys. Lett. B* **76**, 8 (1978).
- [10] A. Baran, K. Pomorski, A. Łukasiak, A. Sobiczewski, *Nucl. Phys. A* 361, 83 (1981).
- [11] Z. Patyk, J. Skalski, A. Sobiczewski, S. Čwiok, Nucl. Phys. A 502, 591c (1989).
- [12] R. Smolańczuk, A. Sobiczewski, Proc. of XV Nuclear Physics Divisional Conference on Low Energy Nuclear Dynamics, St. Petersburg, Russia, 1995, World Scientific, Singapore, 1995, p. 313.
- [13] I. Muntian, A. Sobiczewski, Acta Phys. Pol. B **32**, 629 (2001).
- [14] Z. Patyk, A. Sobiczewski, Nucl. Phys. A 533, 132 (1991).
- [15] R. Smolańczuk, J. Skalski, A. Sobiczewski, Phys. Rev. C 52, 1871 (1995).
- [16] A. Sobiczewski, Acta Phys. Pol. B 29, 2191 (1998).

- [17] A. Parkhomenko, A. Sobiczewski, Acta Phys. Pol. B 36, 3095 (2005).
- [18] I. Muntian, Z. Patyk, A. Sobiczewski, Yad. Fiz. 66, 1051 (2003) [Phys. At. Nucl. 66, 1015 (2003)].
- [19] K. Pomorski, M. Warda, A. Zdeb, *Phys. Scr.* **90**, 114013 (2015).
- [20] M. Kowal, P. Jachimowicz, A. Sobiczewski, *Phys. Rev. C* 82, 014303 (2010).
- [21] P. Moller, J.R. Nix, W.D. Myers, W.J. Świątecki, At. Data Nucl. Data Tables 59, 185 (1995).
- [22] A. Sobiczewski, K. Pomorski, Prog. Part. Nucl. Phys. 58, 292 (2007).
- [23] Yu.Ts. Oganesian, A. Sobiczewski, G.M. Ter-Akopian, *Phys. Scr.* 92, 023003 (2017).
- [24] P. Jachimowicz et al., Int. J. Mod. Phys. E 20, 514 (2011).
- [25] A. Sobiczewski, Yu. Litvinov, M. Palczewski, At. Data Nucl. Data Tables 119, 1 (2018).
- [26] K. Pomorski, J. Dudek, *Phys. Rev. C* 67, 044316 (2003).