ODDERON, HEGS MODEL AND LHC DATA*

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We show that the impact of the maximal odderon amplitude at $t = 0$ and $\sqrt{s} = 13$ TeV is small. We obtain a value of $\rho(t = 0)$ at $\sqrt{s} = 13$ TeV of the order of 0.12. The real part of the odderon amplitude grows like $\log(s/s_0)$ at high energies, and is calculated from the analytic properties of the amplitude. In the framework of the HEGS model, taking the same intercept for the odderon and the Pomeron leads to a good fit of the new LHC data at $\sqrt{s} = 13$ TeV. We also show that the main effect of the odderon can be seen in the region of the diffraction minimum of the differential elastic cross section. The form and energy dependence of the odderon amplitude determined in the HEGS model reproduce the characteristics of the diffraction minimum at $\sqrt{s} = 7, 8$ and 13 TeV.

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The fundamental measures of hadron interactions — the total cross section $\sigma_{\rm tot}(s)$ and the ratio of the real part to the imaginary part of the elastic hadron scattering amplitude $\rho(s,t)$ — are obtained from the analyses of the differential cross section of elastic scattering [1, 2]. The simplest phenomenological models parametrise the t dependence of the elastic scattering amplitude $\mathcal{A}(s,t)$ as a falling exponential. However, taking into account more realistic hadronic form factors for the interaction leads to a non-exponential behaviour in t. Furthermore, there can be other physical effects that can change the momentum-transfer dependence of the hadron cross sections [3].

The new measurements of $\sigma_{\rm tot}$ and ρ performed by the TOTEM Collaboration [4] at $\sqrt{s_{\rm LHC}} = 13$ TeV and $t = 0$ open the possibility to observe the "maximal odderon" (MO), *i.e.* a crossing-odd amplitude with an asymptotic energy dependence such that ${\rm Re}F_{\rm odd}(s, t = 0) \sim \log(s/s_0)^2$ and

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$\mathrm{Im}F_{\mathrm{odd}}(s,t=0) \sim \log(s/s_0)$, with $F(s,t)=\mathcal{A}(s,t)/s$. Martynov and Nicolescu [5] have proposed that the difference between the central prediction of the COMPETE Collaboration $\rho(s_{\mathrm{LHC}},t=0)=0.14$ and the recent measurement 0.10–0.09 results from the contribution of the MO.

To analyse the possibility of an odderon contribution to the differential cross sections at small $|t|$, we use the HEGS model [6, 7] which quantitatively describes, with only a few parameters, the differential cross section of pp and $p\bar{p}$ from $\sqrt{s}=9$ GeV up to 13 TeV, and includes the Coulomb-hadron interference region and the high-$|t|$ region up to $|t|=15$ GeV^2. However, to avoid possible problems connected with the low-energy region, we consider here only the data above $\sqrt{s}=100$ GeV.

The total elastic amplitude in general receives five helicity contributions, but at high energy, it is enough to write it as $F(s,t)=F^h(s,t)+F^{\mathrm{em}}(s,t)e^{\varphi(s,t)}$, where $F^h(s,t)$ comes from the strong interactions, $F^{\mathrm{em}}(s,t)$ from the electromagnetic interactions and $\varphi(s,t)$ is the interference phase factor between the electromagnetic and strong interactions [8, 9]. The Born term of the elastic hadron amplitude at large energy can be written as the sum of two Pomeron and two odderon contributions

$$F_{\mathbb{P}}(s,t) \;=\; \hat{s}^{\epsilon_0}\left(C_{\mathbb{P}}F_1^2(t)\;\hat{s}^{\alpha'\;t}+C'_{\mathbb{P}}A^2(t)\;\hat{s}^{\frac{\alpha' t}{4}}\right), \qquad (1)$$

$$F_{\mathbb{O}}(s,t) \;=\; \mathrm{i}\hat{s}^{\epsilon_0+\frac{\alpha' t}{4}}\left(C_{\mathbb{O}}+C'_{\mathbb{O}}/\left(1-r_{\mathbb{O}}^2 t\right)\right)A^2(t)\,. \qquad (2)$$

All terms are supposed to have the same intercept $\alpha_0=1+\epsilon_0=1.11$, and the Pomeron slope is fixed at $\alpha'=0.24$ GeV^{-2}. The model takes into account the two hadron form factors $F_1(t)$ and $A(t)$ which correspond to the charge and matter distributions [10]. Both form factors are calculated as first and second moments of the same Generalised Parton Distributions (GPDs). It has four free parameters (the constants C) at high energy: two for the two Pomeron amplitudes and two for the odderon. The real part of the hadronic elastic scattering amplitude is determined through the complexification $\hat{s}=-\mathrm{i}s$ to satisfy the dispersion relations. We find that the extra factor $1/(1-r_{\mathbb{O}}^2 t)$ is needed in the odderon case to reproduce the data, and we also add the constant $C_{\mathbb{O}}$ to allow for a possible odderon contribution at $t=0$. The final elastic hadron scattering amplitude is obtained after unitarisation of the Born term through the standard one-channel eikonal representation.

We include a total of 882 experimental points for the energy region $\sqrt{s}>100$ GeV. In the fit, we take into account only the statistical errors. The systematic errors are accounted for through additional normalisations, one for each separate set of data. The description of the new data on the differential cross section from TOTEM at $\sqrt{s}=7$ and 13 TeV is shown in Fig. 1. The low-$|t|$ data are presented with an additional normalization coefficient $f=0.9$.

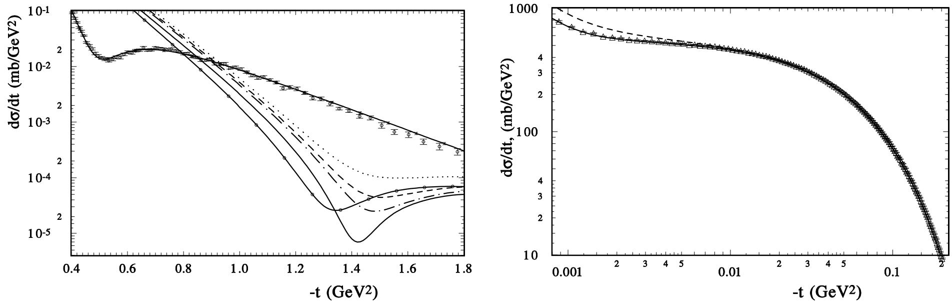

Fig. 1. $\mathrm{d}\sigma/\mathrm{d}t$ (left) in the region of the diffraction minimum (lines show the HEGS results and the experimental points are the data of the TOTEM Collaboration at 7 TeV; lines on the left correspond to $\sqrt{s} = 13.4, 16.8, 19.4, 30.4, 52.8, 7000$ GeV, respectively, dotted, short-dashed, dot-dashed, solid, solid+circles, solid+ants) and (right) at small $|t|$ at $\sqrt{s} = 13$ TeV (plain line is for pp and short-dashed line for $\bar{p}p$, triangles — the data of the TOTEM Collaboration at 13 TeV).

If we allow for the constant $C_{\mathbb{O}}$ in the odderon amplitude, we obtain $\chi^2 = 1132$ instead of $\chi^2 = 1143$ for $C_{\mathbb{O}} = 0$, and the best value is $C_{\mathbb{O}} = -0.07 \pm 0.03$. The second odderon constant is then $C'_{\mathbb{O}} = -0.29 \pm 0.01$. Hence, the odderon contributions at $t = 0$ is very small and cannot heavily impact the value of $\rho(t = 0)$. Finally, if we neglect the odderon contribution altogether, we have only 2 fitting parameters in the framework of the HEGS model. The value of χ^2 slightly increases to $\chi^2 = 1207$, and the resulting fit describes the new data of the TOTEM Collaboration at $\sqrt{s} = 13$ TeV.

In Fig. 2, the results of the HEGS model are presented at 13 TeV, near the diffraction minimum and for a wide region of momentum transfer. It is already known that the model describes the diffraction minimum and its en-

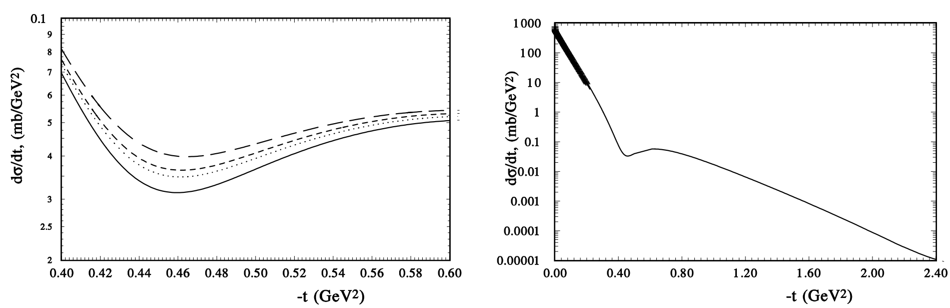

Fig. 2. $\mathrm{d}\sigma/\mathrm{d}t$ for pp-scattering at 13 TeV, in the region of the diffraction minimum (left) and for a wide region of $|t|$ (right). The experimental points are the data of TOTEM. On the left, we show the result of the HEGS calculation with odderon (plain lines and and long-dashed lines respectively for pp and $p\bar{p}$) and without odderon (short-dashed line and dotted line respectively for pp and $p\bar{p}$).

ergy dependence at lower energies [11]. As seen from the left-hand figure, for $\sqrt{s} = 13$ TeV, HEGS predicts a diffraction minimum at $-t_{\rm min} = 0.46$ GeV^2 and a maximum as $-t_{\rm max} = 0.62$ GeV^2. The maximum differential cross section is $R = 1.58$ larger than the minimum one. This seems to agree with the latest LHC data [12]: $-t_{\rm min} = 0.47$ GeV^2, $-t_{\rm max} = 0.638$ GeV^2 and $R = 1.78$.

The same figure also shows the differential cross section for $\bar{p}p$. The difference between pp and $\bar{p}p$ scattering comes from the odderon contribution. If we neglect the odderon amplitude, the difference between pp and $p\bar{p}$ scattering is entirely determined by the contribution from the Coulomb hadron interference and is quite small.

We can now turn to the value of ρ within the HEGS model. Figure 3 (left) shows $\rho(t)$ at $\sqrt{s} = 13$ TeV for pp and $p\bar{p}$ scattering. At $t = 0$, the best fit gives $\rho_{pp} = 0.12$, *i.e.* only slightly less than the COMPETE central value [13]. For $\bar{p}p$ scattering, we obtain $\rho_{\bar{p}p} = 0.13$. The difference is very small, about a fourth of that obtained in [5]. Near $-t = 0.1$, the difference $\rho_{pp}(t) - \rho_{p\bar{p}}(t)$ changes the sign, as required by the dispersion relations [14, 15]. At larger $|t|$, this difference is positive.

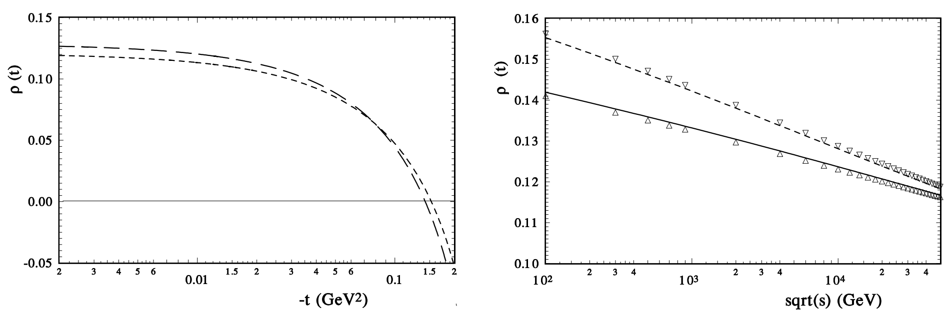

Fig. 3. Left: $\rho(t)$ at $\sqrt{s} = 13$ TeV for pp and $p\bar{p}$ scattering. Right: energy dependence of $\rho(s, t = 0)$ for pp and $p\bar{p}$ (hard and dashed lines); up and down triangles show the contributions from the constant odderon term.

In Fig. 3 (right), the energy dependence of $\rho(s, t = 0)_{pp}$ and $\rho(s, t = 0)_{p\bar{p}}$ is shown. This exponential behaviour is due to the fact that we neglected the non-asymptotic terms in the scattering amplitudes. The difference between $\rho(t)_{pp}$ and $\rho(t)_{p\bar{p}}$ tends to zero at asymptotic s, if we set $C_{\mathbb{O}} = 0$. A small value of $C_{\mathbb{O}}$ leads to a small additional contribution at asymptotic energies.

The energy dependence of the scattering amplitude in the HEGS model is the same for values of t on the first diffraction cone: the real part grows slightly faster than $\log(s)$ but more slowly than $s \log^2(s)$, and the imaginary part grows slightly slower than $s \log^2(s)$.

The corresponding overlap function in the impact parameters representation, calculated in the HEGS model does not reach the Black disc limit (BDL) at the LHC. The same result was obtained in the full HEGS model [7]. Hence, the hadron interactions are not in their asymptotic regime at 13 TeV.

Note that a recent paper [16] announced that the BDL is exceeded from the data of the TOTEM Collaboration at $\sqrt{s} = 13$ TeV. In this paper, the precision of the calculated $\sigma_{\rm tot}$ and $\rho(s, t = 0)$ is smaller than the precision of the experimental data by one order of magnitude. Their Eq. (12) shows that they used the modulus of the imaginary part of the scattering amplitude to calculate the profile function. However, the existence of a sharp diffraction minimum in the differential cross section at $-t = 0.45$ GeV^2 means that the imaginary part changes the sign in this domain of momentum transfer. The real part, which according the dispersion relations changes its sign in the region $-t \approx 0.1$, is larger than the imaginary part in the domain of the diffraction dip. Neglecting this phenomenon may be the reason for an overestimate of the profile function.

The analysis of the new TOTEM data at small momentum transfer at $\sqrt{s} = 13$ TeV, together with other experimental data at lower energies, allows to examine the energy dependence and the form of the odderon part of the hadron scattering amplitude. It has been shown that the impact of the maximal odderon amplitude at $t = 0$ and $\sqrt{s} = 13$ GeV is small and cannot lead to $\rho(\sqrt{s} = 13 \;\; \text{GeV}, t = 0) = 0.09$. The obtained value of $\rho(t = 0)$ at $\sqrt{s} = 13$ TeV is approximately equal to 0.12. The energy dependence of the odderon in the framework of the HEGS model with the same intercept as the Pomeron amplitude seems to agree with the new LHC data at $\sqrt{s} = 13$ TeV, if one allows for an additional normalisation coefficient which reflects the systematic errors.

The form and energy dependence of the odderon amplitude determined in the HEGS model are also in good agreement with the features of the diffraction minimum at $\sqrt{s} = 7$, 8 and 13 TeV.

Hence, it is possible that the Born terms of the Pomeron and odderon amplitudes have the same intercept. The real parts of the final scattering amplitudes both grow like $\log(s)$, as required by the analytical properties of the amplitude [17]. A very different approach, using the Good–Walker formalism, leads to very similar conclusions [18].

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