TWIST-2 TRANSVERSE MOMENTUM DISTRIBUTIONS AT NEXT-TO-NEXT-TO-LEADING ORDER IN QCD*

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The factorization theorem for the Drell–Yan process and semi-inclusive deep inelastic scattering holds for all leading-twist transverse momentum distributions. In this context, a QCD perturbative calculation shows several important characteristics of spin-dependent distributions. We consider all the different spin-dependent distributions which can be matched onto integrated twist-2 functions, focusing on the matching of the transversity and pretzelosity distributions up to next-to-next-to-leading-order. The pretzelosity case is specially relevant because, using a direct perturbative calculation, we obtain a null result up to two loops, which agrees with the experimental measurements.

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1. Introduction

The recent advances in the study of the transverse-momentum-dependent distributions (TMD) allow to unravel the structure of hadrons in great detail. Through the so-called factorization theorems, we construct expressions for cross sections of interesting processes as the Drell–Yan or semi-inclusive deep inelastic scattering (SIDIS) [1–3] in terms of TMD parton distribution functions (TMDPDF) and fragmentation functions (TMDFF). Higher order calculations in QCD for these TMD distributions are important to increase

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the predictive power of the framework [4, 5] in the description of the available experimental data. Increasing the perturbative order in the calculations of TMD distributions allows for better theoretical uncertainties.

The efforts to increase the perturbative order of the elements of TMD factorization theorems at next-to-next-to-leading-order (NNLO) have given us the evolution of the TMD distributions up to two and three loops [6–8]. The unpolarized TMD distribution matchings of TMDPDFs and TMDFFs have been studied up to two loops respectively in [9–11] and in [11].

The status of the polarized distributions is less advanced. We focus on the two transversely polarized distributions: transversity and pretzelosity TMDs. Their matching up to two loops is evaluated in [12]. These two distributions are very interesting because they have been recently subject of experimental, phenomenological and theoretical investigations. The relevant data for these extractions come mainly from HERMES [13] and COMPASS [14].

The TMD transversity distribution has been extracted using SIDIS data in *e.g.* [15] with Gaussian models without taking into account the TMD evolution. In these cases, the size of the theoretical errors is difficult to estimate. In order to provide this information, we need to introduce higher order perturbative information as the calculation of the matching coefficients we are going to recall in these proceedings. For the unpolarized TMD distribution, this analysis has been recently done in [5] decreasing the size of the theoretical uncertainties substantially. In principle, a similar analysis can also be done for the polarized distribution that we have studied.

For the pretzelosity distribution, we outline here the recent analysis made in [16, 17]. In these analyses, a practically null value for this distribution is obtained that agrees with our analysis done at up to two loops in [12, 18].

2. Transversely polarized distributions

The transversity and pretzelosity TMD distributions are derived from a general transversely polarized TMD distribution

$$\begin{aligned}\Phi_{q\leftarrow h}^{[i\sigma^{\alpha+}\gamma^5]}(x,\boldsymbol{b}) &= \frac{1}{2}\int\frac{\mathrm{d}\lambda}{2\pi}e^{-ixp^+\lambda} \\ &\times\langle P,S|\bar{T}\left\{\bar{q}\left(\lambda n+\boldsymbol{b}\right)\tilde{W}_n^{\mathrm{T}}\left(\lambda n+\boldsymbol{b}\right)\right\} i\sigma^{\alpha+}\gamma^5\, T\left\{\tilde{W}_n^{\mathrm{T}\dagger}(0)q(0)\right\}|P,S\rangle\,, \quad (1)\end{aligned}$$

where the index α is transverse and n is a light-like vector. The collinear Wilson lines $W_n^{\mathrm{T}}(x)$ introduce the rapidity divergences, unique feature of TMDs. They are renormalized by the proper rapidity renormalization factor R, which is built from the TMD soft factor [1, 3, 6, 19] in terms of soft Wilson lines

$$S(\boldsymbol{b}) = \frac{\mathrm{Tr}_{\mathrm{color}}}{N_{\mathrm{c}}} \langle 0| \left[S_n^{\mathrm{T}\dagger} \tilde{S}_{\bar{n}}^{\mathrm{T}} \right] (\boldsymbol{b}) \left[\tilde{S}_{\bar{n}}^{\mathrm{T}\dagger} S_n^{\mathrm{T}} \right] (0)|0\rangle \,. \tag{2}$$

The R factor depends on the rapidity regularization scheme used. In our case, it is the modified δ-regularization scheme, which is based in a regularization of the rapidity divergences directly at the operator level, by using modified Wilson lines. *E.g.* for soft ones

$$S_n = P \exp\left(-ig \int_0^\infty \mathrm{d}\sigma (n \cdot A_s)(n\sigma) e^{-\delta\sigma} \right) \tag{3}$$

in the limit $\delta \to 0$. In this scheme, the rapidity renormalization factor can be written simply as $R = 1/\sqrt{S}$. This fact makes possible to write a renormalized TMD distribution in a simple way

$$\Phi^{\mathrm{ren}}(x, \boldsymbol{b}; \mu, \zeta) = Z(\mu, \zeta|\epsilon) R(\boldsymbol{b}, \mu, \zeta|\epsilon, \delta) \Phi^{\mathrm{unsub}}(x, \boldsymbol{b}|\epsilon, \delta) \,, \tag{4}$$

where Z renormalizes the UV divergences and R the rapidity ones.

To study the leading-twist matching of the distributions we are interested in, we use the operator product expansion (OPE) that allows for the expansion of the TMD operator in powers of $\boldsymbol{b}$. The evaluation of the matrix elements of the small-b OPE for our TMD operator results in the following expression:

$$\Phi_{q\leftarrow h}^{[i\sigma^{\alpha+}\gamma^5]}(x, \boldsymbol{b}) = \sum_f \left[C_{q\leftarrow f;\mathrm{tw}-2}^{\alpha\beta}(\boldsymbol{b}) \otimes h_{f\leftarrow h}^{\beta;\mathrm{tw}-2} \right](x) + \ldots \,, \tag{5}$$

where h are collinear distributions, C are matching coefficients functions and $\otimes$ symbol stands for the Mellin convolution in the momentum fractions. The points include distributions that can be produced only at twist-3 [20].

The twist-2 coefficient functions have structures $\sim g_{\mathrm{T}}^{\alpha\beta}$ and $\sim b^\alpha b^\beta/\boldsymbol{b}^2$. Thus, the natural decomposition of this function is

$$C_{q\leftarrow f;\mathrm{tw}-2}^{\alpha\beta}(x, \boldsymbol{b}) = g_{\mathrm{T}}^{\alpha\beta} \delta C_{q\leftarrow f}(x, \boldsymbol{L}_\mu) + \left(\frac{g_{\mathrm{T}}^{\alpha\beta}}{2(1-\epsilon)} + \frac{b^\alpha b^\beta}{\boldsymbol{b}^2} \right) \delta^\perp C_{q\leftarrow f}(x, \boldsymbol{L}_\mu) \,, \tag{6}$$

where ϵ is the parameter of dimensional regularization ($d = 4 - 2\epsilon$) and $\boldsymbol{L}_\mu = \ln(\frac{\mu^2 \boldsymbol{b}^2}{4e^{-2\gamma_E}})$ is the only way in which the matching coefficients depend on the transverse position. As the pieces of the decomposition do not mix with each other, we find individual matching for each TMD distributions. We show here the one for transversity TMD distribution, having an analogous expression for pretzelosity

$$h_1^q\left(x,\boldsymbol{b}\right)=\int\limits_x^1\frac{\mathrm{d}y}{y}\sum_{f=q,\bar{q}}\delta C_{q\leftarrow f}\left(\frac{x}{y},\boldsymbol{L}_\mu\right)h_1^f(y)+\mathcal{O}\left(\boldsymbol{b}^2\right)\,. \tag{7}$$

3. Matching of the transversity distribution at NNLO

The calculation of the matching coefficients for the transversity distribution is straightforward from the unpolarized one done in [11]. The main difference is the non-mixing with gluons. This calculation stands as an explicit evidence that the factorization theorems of spin-dependent TMD distributions work properly. The evolution equations for spin-dependent TMD distributions and for the coefficients that result from its matching onto integrated PDFs have the same form as the ones for the unpolarized TMD, see *e.g.* [12].

In perturbation theory, the matching coefficients can be written as

$$\delta C_{f\leftarrow f'}\left(x,\boldsymbol{L}_\mu,\boldsymbol{l}_\zeta\right)=\sum_{n=0}^{\infty}\alpha_{\mathrm{s}}^n\sum_{k=0}^{n+1}\sum_{l=0}^{n}\boldsymbol{L}_\mu^k\,\boldsymbol{l}_\zeta^l\,\delta C_{f\leftarrow f'}^{(n;k,l)}(x)\,, \tag{8}$$

where $\alpha_{\mathrm{s}}=g^2/(4\pi)^2$ and $\boldsymbol{l}_\zeta=\ln(\mu^2/\zeta)$ are the rapidity logarithms. The coefficients $\delta C^{(n;k,l)}$ with $k+l>0$ are fixed order-by-order with the help of the renormalization group above-named. Thus, the only part that cannot be calculated in this way are the $\delta C^{(n;0,0)}$ coefficients.

The results are quite lengthly and we omit them in these proceedings. To see their explicit expressions see Eqs. (4.9), (4.10) (and Eqs. (6.9), (6.10) for the fragmentation case) of [12]. One interesting feature of the NNLO transversity matching coefficients is its relation with the unpolarized ones. Both coefficients can be written as

$$C^{(2;0,0)}(x)=P^{[1]}(x)F_1(x)+F_2(x)+\delta(1-x)F_3\,, \tag{9}$$

where $P^{[1]}(x)$ is LO DGLAP kernel of the corresponding PDF. The function $F_1(x)$ and the constant F_3 are exactly the same for unpolarized and transversity cases. This behavior is expected because these parts are the ones proportional to $1/(1-x)$ and $\delta(1-x)$ contributions, respectively. They come from diagrams where the quarks interact with the Wilson lines and are insensitive to the polarization structure of the operator. The only different part is the non-singular (at $x\to 1$) function $F_2(x)$.

4. Matching of the pretzelosity distribution at NNLO

The calculation of the matching of the TMD pretzelosity distribution over the integrated transversity PDF is similar to the one of the transversity TMD distribution. The one-loop result was given in [18]

$$\delta^{\perp}C^{[1]}_{q\leftarrow q}(x,\boldsymbol{b}) = -4C_F\boldsymbol{B}^{\epsilon}\Gamma(-\epsilon)\bar{x}\epsilon^2 = 0+\mathcal{O}(\epsilon)\,, \tag{10}$$

where $\bar{x}=1-x$ and $\boldsymbol{B}=\boldsymbol{b}^2/4$. We see that the result is ϵ-suppressed, as we anticipated before.

To calculate the matching coefficient at NNLO, it is interesting to organize the result of the sum of the diagrams by different color factors

$$\delta^{\perp}\Phi^{[2]}_{f\leftarrow f'} = C_F^2 A_F + C_F\left(C_F-\frac{C_A}{2}\right)A_{FA} + \frac{C_F C_A}{2}A_A + C_F N_{\mathrm{f}} A_N\,, \tag{11}$$

because we can see major cancellations between them, taking into account that $A_{FA}=A_A+\mathcal{O}(\epsilon), A_N=\mathcal{O}(\epsilon)$. So, the renormalized pretzelosity TMDPDF depends only on C_F^2 color factor. This remaining term is canceled in the matching equation for pretzelosity

$$\delta^{\perp}C^{[2]}_{q\leftarrow q}(x,\boldsymbol{b}) = h^{[2]}_{1T,q\leftarrow q}(x,\boldsymbol{b}) - \left[\delta^{\perp}C^{[1]}_{q\leftarrow q}(\boldsymbol{b})\otimes\delta f^{[1]}_{q\leftarrow q}\right](x) \tag{12}$$

because the convolution term is the same as for the result of the renormalized pretzelosity. Thus,

$$\delta^{\perp}C^{[2]}_{q\leftarrow f}(x,\boldsymbol{b}) = 0+\mathcal{O}(\epsilon)\,, \tag{13}$$

where $f=q,\bar{q}$.

5. Conclusions

In this article, the matching of the two transversely polarized TMD distributions at leading twist is shown. These results give us the first calculation of a polarized TMD (transversity) at the same level of accuracy of the unpolarized TMD distribution. As is discussed in [4], increasing the perturbative precision in the matching coefficients allows to decrease the theoretical errors and gives us more accurate information of the non-perturbative contributions. The improvement in the theory for polarized distributions shown in this article opens the path to phenomenological analyses with the same level of precision that the obtained for unpolarized TMD.

For another part, this calculation checks explicitly the spin independence of the TMD factorization theorems up to NNLO. Further on, this calculation stands also for a check of the spin independence of the double-scale evolution of the TMD distributions.

For the pretzelosity, an unexpected null result is found up to two-loop level. Some signs to stand that the twist-2 matching of this distribution is zero at all orders in perturbation theory are encountered. However, these statements are not a complete demonstration for the nullity of this matching at all orders. We conjecture that the twist-2 matching of the pretzelosity function is zero at all orders and only the first non-zero matching appears at the twist-4 level.

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