RECENT DEVELOPMENTS IN SMALL-x RESUMMATION*

Marco Bonvini

INFN, Sezione di Roma 1, Piazzale Aldo Moro 5, 00185 Roma, Italy

(Received March 5, 2019)

There has been a revived interest in small-x resummation in recent times. The main motivation was its success in describing small-x HERA data without the inclusion of non-perturbative corrections. In this contribution, I will review the recent developments in the field.

DOI:10.5506/APhysPolBSupp.12.873

Let us consider an observable σ, *e.g.* a DIS structure function, within the context of the collinear QCD factorization theorem. It can be in general written as

$$\sigma\left(Q^2\right) = \sum_{i=g,q} \int_x^1 \frac{\mathrm{d}z}{z} C_i\left(z, \alpha_\mathrm{s}\left(Q^2\right)\right) f_i\left(\frac{x}{z}, Q^2\right)\,, \tag{1}$$

where C_i are perturbative coefficient functions and f_i are parton distribution functions (PDFs) satisfying the DGLAP evolution equation

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} f_i\left(x,\mu^2\right) = \sum_{j=g,q} \int_x^1 \frac{\mathrm{d}z}{z} P_{ij}\left(z, \alpha_\mathrm{s}\left(\mu^2\right)\right) f_j\left(\frac{x}{z}, \mu^2\right)\,, \tag{2}$$

where P_{ij} are splitting functions and the sums extend over all partons. It is well-known that perturbative quantities computed in QCD may contain logarithmic enhancements in some regions. This is, for instance, the case of $\alpha_\mathrm{s}^n \frac{1}{x} \log^k \frac{1}{x}$ terms, which appear in both splitting and coefficient functions in the singlet sector, and become large at small-x, spoiling the perturbativity of the α_s expansion. Resumming the small-x logarithms cures the instability

* Presented at the Diffraction and Low-x 2018 Workshop, August 26–September 1, 2018, Reggio Calabria, Italy.

of the fixed-order perturbative results. Small-x resummation is based on the interplay of the previous two equations with the k_T factorization theorem

$$\sigma\left(Q^2\right) = \sum_{i=g,q} \int_x^1 \frac{\mathrm{d}z}{z} \int_0^\infty \mathrm{d}k_\mathrm{T}^2\, \mathcal{C}_i\left(z, k_\mathrm{T}^2, \alpha_\mathrm{s}\right) \mathcal{F}_i\left(\frac{x}{z}, k_\mathrm{T}^2\right) , \tag{3}$$

where $\mathcal{C}_i$ are coefficient functions with off-shell initial-state partons (with off-shellness given by k_T^2), and $\mathcal{F}_i$ are unintegrated k_T-dependent PDFs. In the small-x limit, the unintegrated gluon PDF is related to the integrated integrated PDF by

$$\mathcal{F}_g\left(x, k_\mathrm{T}^2\right) = \mathcal{R}\, \frac{\mathrm{d}}{\mathrm{d}k_\mathrm{T}^2} x f_g\left(x, k_\mathrm{T}^2\right) , \tag{4}$$

where $\mathcal{R}$ is a scheme-dependent function. In the variant of the $\overline{\mathrm{MS}}$ scheme usually adopted in small-x resummation, denoted $Q_0\overline{\mathrm{MS}}$ scheme, $\mathcal{R} = 1$. The unintegrated gluon PDF satisfies the BFKL evolution equation

$$-x \frac{\mathrm{d}}{\mathrm{d}x} \mathcal{F}_g\left(x, k_\mathrm{T}^2\right) = \int_0^\infty \frac{\mathrm{d}q_\mathrm{T}^2}{k_\mathrm{T}^2} \mathcal{K}\left(\frac{k_\mathrm{T}^2}{q_\mathrm{T}^2}, \alpha_\mathrm{s}\right) \mathcal{F}_g\left(x, q_\mathrm{T}^2\right) , \tag{5}$$

where $\mathcal{K}$ is the BFKL kernel. Using Eq. (4) to translate the BFKL equation into an equation for the integrated PDF, it is then possible to require consistency between its solution and that of the DGLAP evolution Eq. (2) to find constraints between the splitting functions and the BFKL kernel, called duality relation, that allow to resum the small-x logarithms in splitting functions. Practically, the procedure is more complicated due to the perturbative instability of the BFKL kernel, that requires a number of operations to be performed before obtaining a perturbatively stable result. On top of this, the resummation of a class of subleading contributions originating from the running of the strong coupling turns out to be very important, as it changes the nature of the small-x behaviour. Resummation at next-to-leading logarithmic (NLL) level matched to fixed next-to-leading order (NLO) has been achieved by various groups (see *e.g.* [1–3]).

In recent Refs. [4–7], the formalism for small-x resummation, in the approach of Altarelli–Ball–Forte (ABF), has been extended in many respects. On top of (several) technical improvements, the main novelties that have been introduced are:

— matching the resummation to NNLO, to be able to construct DGLAP evolution at NNLO+NLL;

— making a prediction of the (yet unknown [8, 9]) N^{3}LO splitting functions at small x, and preparing all the ingredients to be able to match NLL resummation to N^{3}LO once available;

— providing an uncertainty on resummed results from subleading logarithmic contributions;

— releasing a public code, HELL [10], that implements the resummation and delivers resummed results for applications.

The first item of the list is particularly important, because the instability induced by small-x logarithms gets larger with increasing the order. This is seen in Fig. 1 (left), where the P_{gg} and P_{qg} splitting functions are shown at a low scale: the $\log\frac{1}{x}$ term at NNLO starts to grow for $x \lesssim 10^{-2}$ and invalidate the perturbative expansion. Once resummation is turned on, the behaviour changes substantially, and the NNLO+NLL result deviates significantly from the fixed-order result. A stronger effect is expected when matching the resummation to N^{3}LO, because at this order, extra powers of the log appear. A prediction (based on the expansion of the resummed result) is shown in Fig. 1 (right). The perturbative instability is apparent, especially when going to very small values of x. However, subleading logarithmic contributions which cannot be fixed by NLL resummation are potentially sizeable (difference between "N^{3}LO approx" and "N^{3}LO asympt" curves in the plot), so this prediction carries a huge uncertainty and it may only be useful combined with other information on the N^{3}LO result [8, 9].

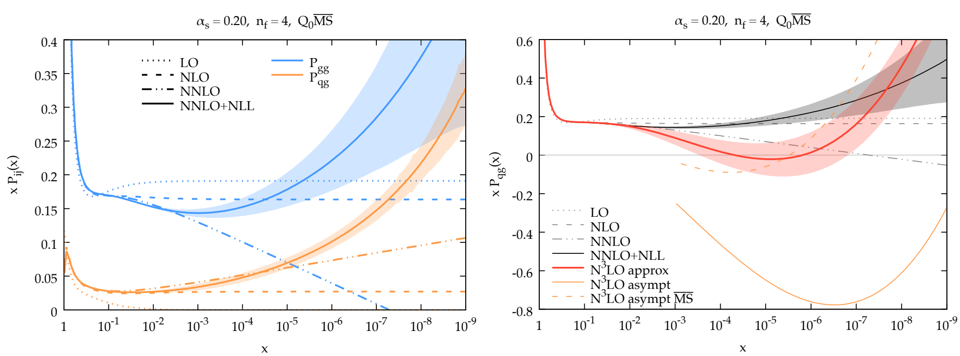

Fig. 1. Left: the P_{gg} and P_{qg} splitting functions at LO, NLO, NNLO and NNLO+NLL for $\alpha_s = 0.2$, $n_f = 4$. Right: N^{3}LO prediction for P_{gg}.

The resummation of coefficient functions is based on the direct comparison of the collinear and k_T-factorization formulae in Eqs. (1), (3), making use of a generalization of the relation Eq. (4). Moving to the Mellin N space, and introducing the DGLAP evolution factors $U_{ij}(N, \mu^2, \mu_0^2)$ from a scale μ_0 to a scale μ, we can rewrite Eq. (4) generalized to all flavours as

$$\mathcal{F}_k\left(N,k_{\rm T}^2\right) = \mathcal{R}_{kj}\,\frac{\rm d}{{\rm d}k_{\rm T}^2} U_{ji}\left(N,k_{\rm T}^2,Q^2\right) f_i\left(N,Q^2\right)\,, \tag{6}$$

so that by comparison between the two factorization formulae, we get

$$C_i\left(N,\alpha_{\rm s}\left(Q^2\right)\right) = \sum_{j=g,q} \int_0^\infty {\rm d}k_{\rm T}^2\,\mathcal{C}_k\left(N,k_{\rm T}^2,\alpha_{\rm s}\right)\,\mathcal{R}_{kj}\,\frac{\rm d}{{\rm d}k_{\rm T}^2} U_{ji}\left(N,k_{\rm T}^2,Q^2\right)\,, \tag{7}$$

which encodes the small-x resummation provided the DGLAP evolution factors are themselves computed with resummed splitting functions. This formulation of the resummation (introduced for the first time in Ref. [4]) is equivalent to previous approaches [2, 11, 12], but it is very convenient from a numerical point of view, and it allows for a simpler implementation of new processes in the resummation code HELL [10]. Thanks to this new formulation, there have been a number of developments also in the context of coefficient functions resummation [5, 7]:

- resummation of all neutral- and charged-current DIS structure functions F_2, F_L and F_3, both in the massless limit and including mass effects;
- implementation of a variable flavour number scheme at small x in $\overline{\rm MS}$-like schemes;
- resummation of heavy-quark matching conditions which give the initial conditions for the PDFs when transitioning from a scheme with $n_{\rm f}$ active flavours to a scheme with $n_{\rm f}+1$ active flavours;
- resummation of LHC observables (only Higgs production in gluon fusion so far, Drell–Yan is under investigation).

The third item turns out to be particularly interesting. Indeed, the transition from the $n_{\rm f}$ to the $n_{\rm f}+1$ scheme happens at a (unphysical) matching scale, that can be varied to assess the impact of unknown higher order contributions to the matching procedure. Once resummation is included in the matching and in DGLAP evolution, the matching scale uncertainty is drastically reduced at small x, thereby showing a stabilization of the perturbative expansion. This is shown for the charm PDF in Fig. 2. The gap between the various curves at large scale (*i.e.* in the $n_{\rm f}=4$ scheme) almost disappears once resummation is included.

Thanks to all these recent developments, and importantly to the availability of the public code HELL [10] that delivers resummed splitting and coefficient functions, it has been possible to perform two PDF fits including small-x resummation, one in the context of the NNPDF methodology [14] and the other one using the xFitter toolkit [13]. The striking effect of small-x

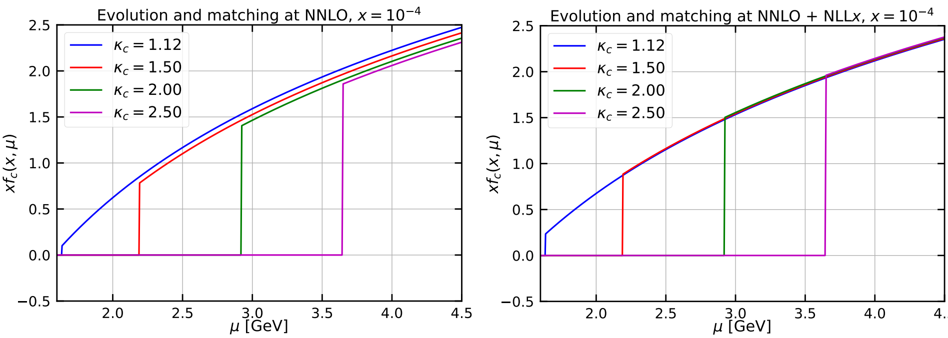

Fig. 2. Fixed NNLO (left) and resummed NNLO+NLL (right) charm PDF generated perturbatively at different charm matching scales $\mu_c \equiv \kappa_c m_c$, with m_c the charm mass and $\kappa_c = 1.12, 1.5, 2, 2.5$, at a small value of $x = 10^{-4}$ (figure taken from Ref. [13]).

resummation is a dramatic improvement in the description of the low-x low-Q^2 HERA data, leading to a significantly different gluon (and quark-singlet) PDF at NNLO+NLL with respect to the NNLO fit at small x.

To appreciate the importance of such effect, we show in Fig. 3 the comparison of the fixed-order and resummed predictions for the production of Higgs in gluon fusion at hadron colliders as a function of the collider energy [7, 16]. The effect of resummation (mostly coming from the use of resummed PDFs) is small and compatible within PDF uncertainty with the fixed-order result up to approximately the current LHC energy. For higher energies, the effect of resummation is a significant increase of the

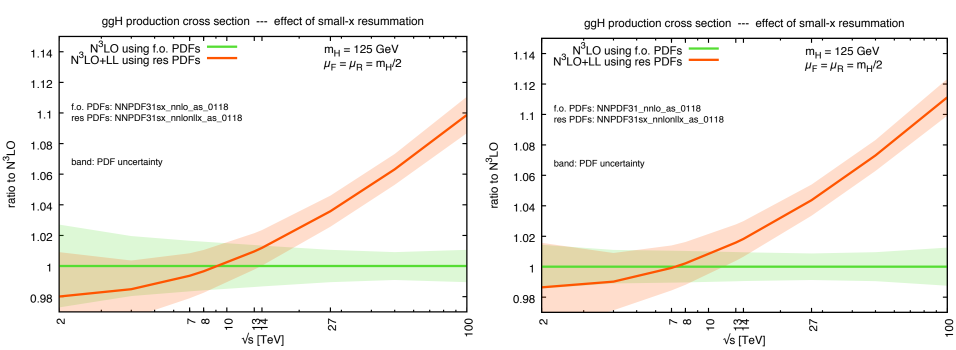

Fig. 3. Ratio of the resummed prediction of the Higgs cross section at N^{3}LO+LL to the fixed N^{3}LO result. In both plots, the resummed PDF set is the one from Ref. [14], while the fixed order (NNLO) set used for the fixed-order prediction is either the baseline of the resummed fit [14] (left plot), or the state-of-the-art NNPDF3.1 set of Ref. [15] (right plot).

cross section, rising with the energy, and reaching up to $\sim +10\%$ at a future circular collider of 100 TeV. This conclusion holds unchanged if using a different NNLO PDF set for the comparison, for instance the state-of-the-art NNPDF3.1 set of Ref. [15] (right plot), which has been fitted using a larger dataset. Subleading logarithmic contributions may have sizeable effects [7] and reduce (or enhance) the overall effect of resummation, but the significance of the effect is likely independent of them.

This work is supported by the Marie Skłodowska-Curie grant HiPPiE@LHC, number 746159.

REFERENCES

[1] M. Ciafaloni, D. Colferai, G. Salam, A. Stasto, *J. High Energy Phys.* **0708**, 046 (2007) [arXiv:0707.1453 [hep-ph]].

[2] G. Altarelli, R.D. Ball, S. Forte, *Nucl. Phys. B* **799**, 199 (2008) [arXiv:0802.0032 [hep-ph]].

[3] C.D. White, R.S. Thorne, *Phys. Rev. D* **75**, 034005 (2007) [arXiv:hep-ph/0611204].

[4] M. Bonvini, S. Marzani, T. Peraro, *Eur. Phys. J. C* **76**, 597 (2016) [arXiv:1607.02153 [hep-ph]].

[5] M. Bonvini, S. Marzani, C. Muselli, *J. High Energy Phys.* **1712**, 117 (2017) [arXiv:1708.07510 [hep-ph]].

[6] M. Bonvini, S. Marzani, *J. High Energy Phys.* **1806**, 145 (2018) [arXiv:1805.06460 [hep-ph]].

[7] M. Bonvini, *Eur. Phys. J. C* **78**, 834 (2018) [arXiv:1805.08785 [hep-ph]].

[8] J. Davies *et al.*, *Nucl. Phys. B* **915**, 335 (2017) [arXiv:1610.07477 [hep-ph]].

[9] S. Moch *et al.*, *J. High Energy Phys.* **1710**, 041 (2017) [arXiv:1707.08315 [hep-ph]].

[10] https://www.ge.infn.it/~bonvini/hell/

[11] S. Catani, F. Hautmann, *Nucl. Phys. B* **427**, 475 (1994) [arXiv:hep-ph/9405388].

[12] R.D. Ball, *Nucl. Phys. B* **796**, 137 (2008) [arXiv:0708.1277 [hep-ph]].

[13] H. Abdolmaleki *et al.* [xFitter Developers' Team], *Eur. Phys. J. C* **78**, 621 (2018) [arXiv:1802.00064 [hep-ph]].

[14] R.D. Ball *et al.*, *Eur. Phys. J. C* **78**, 321 (2018) [arXiv:1710.05935 [hep-ph]].

[15] R.D. Ball *et al.* [NNPDF Collaboration], arXiv:1706.00428 [hep-ph].

[16] M. Bonvini, S. Marzani, *Phys. Rev. Lett.* **120**, 202003 (2018) [arXiv:1802.07758 [hep-ph]].