

HEAVY QUARKONIUM AND DYNAMICAL GLUON MASS AT NON-ZERO TEMPERATURE IN INSTANTON VACUUM MODEL*

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In the framework of the Instanton Liquid Model, we evaluate the heavy-quark $\bar{Q}Q$ potential at nonzero temperature T . The potential has two components: a contribution due to direct interaction with instantons, and a modification of the one-gluon exchange contribution via instanton-generated dynamical “electric” gluon mass $M_{\text{el}}(q, T)$. We conclude that the nonperturbative ILM contributions to the $\bar{Q}Q$ potential have pronounced temperature dependence, which might be tested in phenomenological analyses of charmonia production data.

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1. Motivation and introduction

Heavy quarkonium $\bar{Q}Q$ states created in the high-energy heavy-ion collisions can be used as a thermometer of the hot Quark–Gluon Plasma formed at later stages due to final-state interactions. For this reason, the analysis of heavy-quark dynamics occupies one of the central places in hot matter studies [1]. In the heavy-quark mass limit, the dynamics of $\bar{Q}Q$ pair is perturbative, however numerically, the mass of charmed quark, $m_c \approx 1.27$ GeV, is not very large, and for this reason, nonperturbative effects might be important, though to a lesser degree than for the light quarks. For this reason, heavy quarks might be used as a probe sensitive to the onset of the nonperturbative dynamics, and seek for one of the constituents of QCD vacuum, the instantons. The distribution of instantons in the QCD vacuum is described in the Instanton Liquid Model (ILM) framework. At nonzero temperature, $T \neq 0$, this framework predicts the temperature dependence of the mean instanton size $\bar{\rho}(T)$ and density $n(T)$, and thus provides an effective approach which might be considered as a model of Quark–Gluon Plasma.

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In these proceedings, we present results for the temperature-dependent $Q\bar{Q}$ potential due to interactions with instantons. We found that the ILM-induced potential differs significantly from the perturbative Coulomb-like shape, has a pronounced temperature dependence and thus affects the observed quarkonia yields.

1.1. Instanton Liquid Model

1.1.1. Instanton Liquid Model (ILM) at zero temperature

As was discussed in [2, 3], the ILM manages to describe all the nonperturbative physics using only two main parameters, the average instanton size $\bar{\rho}$ and density n . These parameters have been estimated independently in different approaches: the phenomenological [2] and variational [3] estimates of these parameters give $n^{-1/4} = R \approx 1$ fm, $\bar{\rho} \approx 0.33$ fm; the lattice studies [4] result in $R \approx 0.89$ fm, $\bar{\rho} \approx 0.36$ fm, and the phenomenological estimates with account of $1/N_c$ corrections [5] yield $R \approx 0.76$ fm, $\bar{\rho} \approx 0.32$ fm. As we can see, all these estimates agree with each other within 10% accuracy, and in what follows, for the sake of definiteness, we will use for numerical calculations the first set of parameters. The packing fraction parameter $\pi^2 \bar{\rho}^4 n \sim 0.1$, which corresponds to the fraction of the whole space occupied by instantons, is small, which justifies independent averaging over the collective coordinates of instantons. Due to interactions with instanton ensemble, the light quarks acquire the dynamical mass $M \sim (\text{packing parameter})^{1/2} \bar{\rho}^{-1} \approx 365$ MeV. The ILM approach manages to describe reasonably well a large number of low-energy observables involving light hadrons [5].

1.1.2. ILM instanton size vs. light hadron and heavy quarkonium $Q\bar{Q}$ sizes

The average instanton size, $\bar{\rho}$, determines average distance at which the instanton effects are pronounced. As could be seen from the above-given estimates, this size is comparable to the average size of the dynamic quark inside nucleon $r_N \sim 0.3\text{--}0.45$ fm [6], and to the size of the $Q\bar{Q}$ charmonia, as could be seen from Table I (see [7] for details). For this reason, we expect that the lowest quarkonia should be significantly affected by the contributions of instantons. The lowest charmonia due to their small size should

TABLE I

Sizes of charmonia evaluated in potential models. Data are from [7].

State	J/ψ	χ_c	ψ'	Υ	χ_b	Υ'	χ'_b	Υ''
Mass [Gev]	3.07	3.53	3.68	9.46	9.99	10.02	10.26	10.36
Size r [fm]	0.25	0.36	0.45	0.14	0.22	0.28	0.34	0.39

be insensitive to the large-distance confinement and, for this reason, we expect that ILM framework supplemented with perturbative gluons should be applicable for their description.

1.1.3. ILM at nonzero temperature

The temperature dependence is introduced in the model as (anti)periodical boundary condition for all fields, with period in time $\beta = 1/T$, and with additional contributions due to fluctuations of thermal gluon (and light quarks). In figure 1, we demonstrate the temperature dependence of the ILM parameters. We can see that the instanton gas density and average instanton size decrease as a function of temperature T .

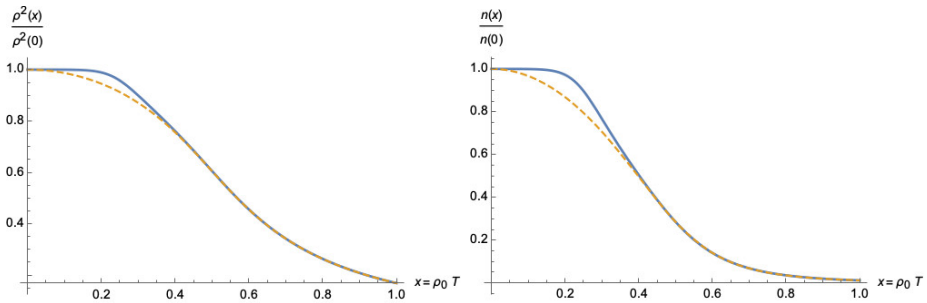


Fig. 1. Left: The temperature dependence of the average instantons size, normalized to unity at $T = 0$ ($\bar{\rho}^2(x = \bar{\rho}T)/\bar{\rho}^2(0)^2$). Right: The temperature dependence of the instanton density normalized to unity at $T = 0$ ($n(x)/n(0)$). In both plots, the dashed lines correspond to the full suppression of instantons due to the Debye screening [8], whereas the full line corresponds to an interpolation between no suppression below critical temperature $T_c \approx 150$ MeV and full suppression above T_c , with a width $T = 0.3 T_c$ [2].

2. Gluons in ILM

2.1. $T = 0$ case [9]

The interaction of gluons with instantons leads to generation of the dynamical momentum-dependent gluon mass. In order to find it, we have to solve the zero-mode problem, to average the total gluon propagator in ILM using the method of [10] extended to gluon sector and, finally, to find gauge-invariant dynamical gluon mass $M_g(q)$ as a function of Euclidean momentum q . It was found that the dynamical mass M_g has a form of $M_g(q) = M_g F(q)$, where the form factor $F(q) = q\bar{\rho}K_1(q\bar{\rho})$ is shown in the left panel of figure 2, and $M_g \sim M \sim (\text{packing parameter})^{1/2} \bar{\rho}^{-1} \approx 362$ MeV. The value of M_g also determines the strength of gluon–instanton interaction.

2.2. Nonzero temperature [11]

In this case, again, we have to impose the time periodicity condition for the fields, which breaks relativistic covariance of the gluon propagator, as well as take into account the temperature-induced modifications of ILM parameters. For our studies, the most important observable is the temperature dependence of the so-called “electric” dynamical gluon mass $M_{\text{el}}(q, T)$ shown in the right panel of figure 2. We can see that this dependence is mild in the region of $T < T_c$.

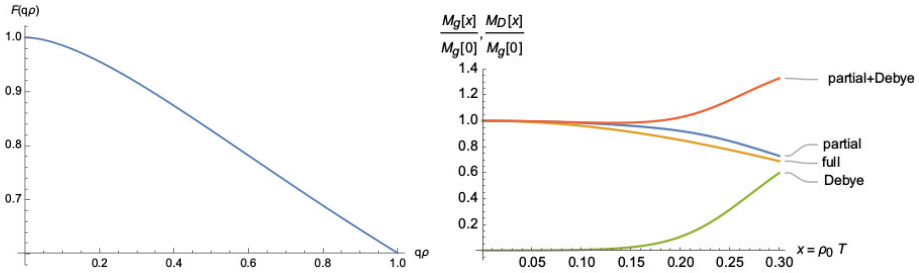


Fig. 2. Left: Momentum q dependence of the gluon electric mass at $T = 0$ normalized to unity at $q = 0$: $M_{\text{el}}(q, 0) = M_g F(q)$. Right: Temperature dependence of the gluon effective mass M_g and the Debye screening M_D , normalized to unity at $T = 0$. We use notations $x = T\bar{\rho}$; the critical temperature corresponds to $x_c = 0.25$. For $x > x_c$, the contribution of thermal gluon (and quark) fluctuations to the gluon propagator leads to the so-called gluon Debye screening mass $M_D(x)$, and we use for it the lattice parametrization $M_D(x) = 1.52 x / \bar{\rho} \Theta(x - x_c)$ [12]. The other notations are the same as in Fig. 1.

3. Singlet $Q\bar{Q}$ potential in ILM at $T \neq 0$. Direct instanton contribution to the singlet $Q\bar{Q}$ potential at $T \neq 0$

In figure 3, we present our results for the potential $V_c(r, T)$ evaluated from the Wilson loop in ILM, using the method introduced in [10]. At zero temperature, the potential has a finite limit, $V_c(r \rightarrow \infty, T = 0) = 2 \Delta m_Q \approx 140$ MeV, where Δm_Q is the ILM contribution to the heavy-quark mass, and is due to the heavy-quark-instanton interaction. As could be seen from Table II, the direct ILM effects are not small, are of the order of $\sim 30\%$ in comparison with the experimental data and strongly depend on the choice of ILM parameters.

At very short distances, the dynamics of $\bar{Q}Q$ is described by the pQCD, so we add to ILM potential $V_c(r)$ the one-gluon exchange contribution

$$V_g(r, T) = \lambda \bar{\lambda} g^2 \int \frac{d^3 k}{(2\pi)^3} \exp(i\vec{k}\vec{r}) \left(\vec{k}^2 + M_{\text{el}}^2(\vec{k}, T) \right)^{-1},$$

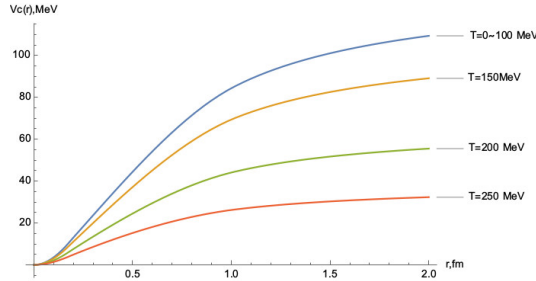


Fig. 3. The color singlet $\bar{Q}Q$ potential $V_c(r)$ at different temperatures T evaluated in the instanton liquid model.

TABLE II

ILM direct instanton contribution to the charmonium states at $T = 0$ [13]. $\Delta M_{c\bar{c}} = M_{c\bar{c}} - 2m_c$ in [MeV], $m_c = 1275$ MeV. Set I corresponds to the instanton vacuum parameters $\bar{\rho} = 0.33$ fm and $R = 1$ fm, set II uses values of $\bar{\rho} = 0.36$ fm, $R = 0.89$ fm.

$\Delta M_{c\bar{c}} (J^P)$	Set I	Set II	Exp.
$\Delta M_{\eta_c}(0^-)$	118.81	203.64	433.6 ± 0.6
$\Delta M_{J/\psi}(1^-)$	119.57	205.36	546.91 ± 0.11
$\Delta M_{\chi_{c0}}(0^+)$	142.43	250.86	864.75 ± 0.31

where $\lambda \bar{\lambda} = -4/3$ and $+1/6$ for $Q\bar{Q}$ in color singlet and color octet states, respectively, and we use gluon effective mass $M_g(q, T)$ evaluated earlier. Finally, in figure 4, we compare the full ILM potential with the Cornell potential, which consists of a sum of Coulomb-like and linearly growing confinement contributions [14]. We can observe that the potentials differ at large distances, which implies that the $\bar{Q}Q$ phenomenology should be reanalyzed.

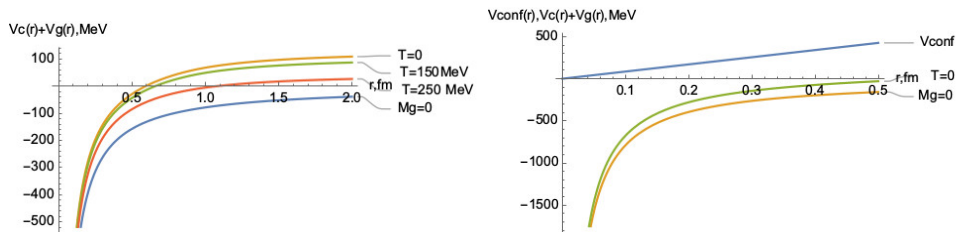


Fig. 4. Left: Comparison of the full ILM potential at different temperatures T with the Coulomb piece of Cornell potential ($M_g = 0$ line). Right: Comparison of ILM potential at zero temperatures with Coulomb and confinement pieces of the Cornell potential.

4. Discussion

The dynamics of the heavy $Q\bar{Q}$ pair is significantly affected by the non-perturbative QCD vacuum properties. We used the ILM as the model of QCD vacuum, both at zero temperature and in hot matter. At $T = 0$, heavy-quark–instanton interaction generates nonperturbative correction to the heavy-quark mass, $\Delta m_Q \sim 70$ MeV, whereas in the case of the light-quark–instanton and gluon–instanton interactions, the same interaction is more pronounced and gives the respective dynamical masses $M \sim M_g \sim 360$ MeV. The instanton-induced nonperturbative effects homogeneously decrease as a function of temperature T .

We also evaluated explicitly the $Q\bar{Q}$ potential, which receives sizable modifications due to direct interactions with instantons, as well as generation of the dynamical gluon mass in one-gluon exchange contribution. The evaluated ILM potential significantly differs from the phenomenological parametrizations available from the literature. At zero temperature, our findings might be tested through studies of the charmonia spectroscopy. At nonzero temperatures ($T \neq 0$), the average instanton size $\bar{\rho}(T)$ and density $n(T)$ are gradually decreasing functions of temperature, which leads to pronounced temperature dependence of the $Q\bar{Q}$ potential. The predicted T dependence might be tested by heavy-quarks production processes in hadron–hadron collisions at high energies.

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