FLUCTUATIONS IN SMALL SYSTEMS*

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We review the main features of event-by-event fluctuations of the content of the Fock states of onia (as models for dilute hadrons, or as bare hadronic components of virtual photons), as well as some of their observable consequences. We briefly address the total scattering cross section of a small onium off a nucleus, then of two small onia. Finally, we explain that the multiplicity in the final state of collisions of large onia with nuclei may directly be related to the gluon density in the former. We provide first predictions for the event-by-event fluctuations of the gluon density.

1. Dressed hadronic states from a branching process

Consider an asymptotic onium, *i.e.* a bare color-neutral quark–antiquark pair. To a nucleus with which it interacts during some finite time (of the order of a few Fermi), it appears, generally speaking, as a more complicated quantum state. When the onium has a large rapidity with respect to the nucleus, its typical Fock state as "seen" from the point of view of the nucleus is a dense set of gluons.

The probability of a given partonic configuration can, in principle, be calculated from the field theory. However, analytical expressions may be obtained only in appropriate asymptotic limits. For large rapidities, the color dipole model [1] provides a procedure to generate unweighted parton configurations and to derive evolution equations in the rapidity variable. It is formulated in the large-number-of-color (N_c) limit of QCD, in which a globally color-neutral set of gluons can be represented by color dipoles, and in transverse position space. In the framework of the dipole model with an increase of the rapidity of a color dipole by the small quantity dy there

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is associated a probability $\mathrm{d}P$ that a gluon be emitted. If r_0 is the size vector of the initial dipole, then the probability that a gluon be emitted at position r_1 with respect to the quark, up to $\mathrm{d}^2 r_1$, reads [1]

$$\mathrm{d}P = \bar{\alpha}\,\mathrm{d}y\,\frac{\mathrm{d}^2 r_1}{2\pi}\,\frac{r_0^2}{r_1^2(r_0 - r_1)^2}\,, \qquad \text{with} \qquad \bar{\alpha} \equiv \frac{\alpha_\mathrm{s} N_\mathrm{c}}{\pi}\,. \qquad (1)$$

In the large-N_c limit, the emitted gluon together with the quark part of the initial dipole, and the same gluon together with the antiquark part of the initial dipole, form two new dipoles, which replace the initial one. Furthermore, each dipole evolves independently of the other ones present in the Fock state[1] until the maximum rapidity is reached; see Fig. 1. Hence, in the color dipole model, the state of the interacting onium is built from a $1 \to 2$ *branching process* (for a review, see Ref. [2]).
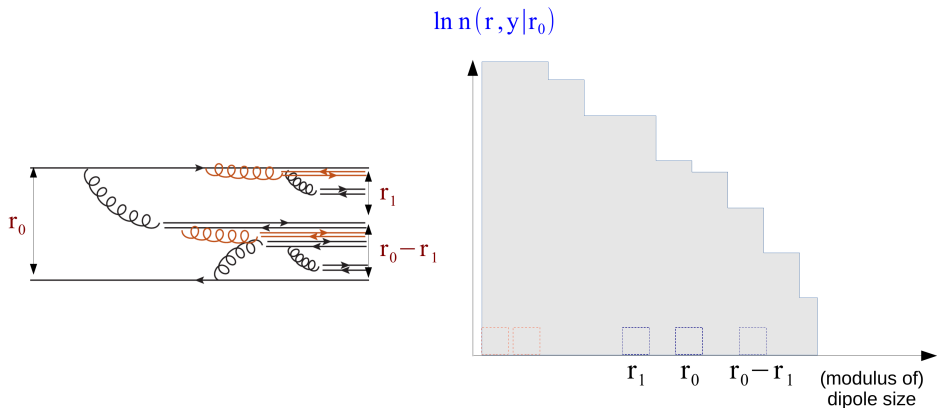

Fig. 1. Schematic representation of QCD evolution in rapidity. Left: An initial $q\bar{q}$ pair of size r_0 boosted to the rapidity Y gets dressed with (essentially) gluons. In the color dipole model, in which color-neutral sets of gluons are represented by sets of color dipoles, the Fock state is built through a $1 \to 2$ branching process. Right: Plot of the dipole number n at rapidity Y, in one particular realization of the QCD evolution of the onium, as a function of the dipole size. The mean shape is given by the solution to the deterministic BFKL equation, but fluctuations in the beginning of the evolution may shift the whole distribution randomly.

The mean number density of gluons, namely averaged over events, is given by the well-understood solution to the BFKL equation [3]. Event-by-event fluctuations are important in the initial stages of the evolution,

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[1] This is because the leading order in $1/N_\mathrm{c}$ is given by planar graphs. Interference graphs for gluon emissions off different dipoles would be nonplanar, and hence can be discarded.

when the system is very dilute, and at all rapidities only in the evolution of the dipoles which have sizes close to that of the largest dipole in the event, for which the number density is low. A complete description of these fluctuations is currently out of reach, but we were able to model them and draw phenomenological consequences that we shall now describe [4].

2. Scattering cross sections

Let us consider an onium of size r_0, small compared to the typical hadronic size $1/\Lambda_{\text{QCD}}$, scattering off a large nucleus at total rapidity Y. We will discuss the total cross section. From the point of view of the nucleus, at high total rapidity, the latter typically appears dressed with many gluons, represented by a set of color dipoles. The S-matrix element for the elastic scattering of this particular state is the product of the S-matrix elements for the scattering of the individual dipoles present in the onium at the time of its interaction with the nucleus. If $\{r_i\}$ is the set of their size vectors, then

$$S(r_0) = \prod_i S_{\text{MV}}(r_i),$$

$$\text{with} \quad S_{\text{MV}}(r) = \exp\left[-\frac{r^2 Q_{\text{MV}}^2}{4} \ln\left(e + 4/r^2 \Lambda_{\text{QCD}}^2\right)\right], \quad (2)$$

where $Q_{\text{MV}} \sim 1$ GeV is the nuclear saturation momentum. The expression for the S-matrix element associated to the scattering of an elementary dipole at low rapidity, S_{MV}, stems from the McLerran–Venugopalan model [5].

$S(r_0)$ is close to one if *no dipole* in the state has a size much larger than $1/Q_{\text{MV}}$. It is close to zero if *at least one* dipole has a size larger than $1/Q_{\text{MV}}$. In other words, in a first approximation, $S(r_0)$ can be thought of as the probability that there be at least one dipole larger than the inverse saturation momentum of the nucleus in the Fock state of the photon at rapidity Y [4]. The amplitude for the scattering is just $T(r_0, Y) = 1 - \langle S(r_0)\rangle_Y$, where $\langle\cdot\rangle_Y$ represents the average over all possible Fock states, namely dipole configurations.

$T(r_0, Y)$ also obeys the Balitsky–Kovchegov equation [6, 7], whose solution exhibits the following feature:

$$T(r_0, Y) \sim \ln \frac{1}{r_0^2 Q_s^2(Y)} \left[r_0^2 Q_s^2(Y)\right]^{\gamma_0}$$

$$\text{for} \quad 1 \ll \left|\ln r_0^2 Q_s^2(Y)\right| \ll \sqrt{\chi''(\gamma_0)\bar{\alpha} Y}, \quad (3)$$

where the rapidity-dependent saturation scale $Q_s(Y)$ reads

$$Q_s^2(Y) \sim Q_{MV}^2 \frac{e^{\bar{\alpha}\chi'(\gamma_0)Y}}{(\bar{\alpha}Y)^{3/(2\gamma_0)}} \, . \tag{4}$$

Here, $\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$ and γ_0 solves $\chi'(\gamma_0) = \chi(\gamma_0)/\gamma_0$. The analytic form of the amplitude can be related to the distribution of the two kinds of fluctuations in the evolution of the onium, the ones in the beginning of the evolution, and the ones at the large-size tip of the distribution of the dipole sizes; see Ref. [4] for details.

We now turn to the scattering of two small onia of respective sizes r_0 and $r_0' > r_0$. At moderate rapidities, the amplitude solves the BFKL equation, but to date, there is no sound formulation of the latter in the limit of large rapidities. However, the picture of QCD evolution as a branching process enabled us to find the asymptotic form of the amplitude [4]

$$T(r_0, Y) \sim \ln^2 \frac{1}{r_0^2 Q_s^2(Y)} \left[r_0^2 Q_s^2(Y) \right]^{\gamma_0} \, , \tag{5}$$

where now $Q_s^2(Y) \simeq e^{\chi'(\gamma_0)\bar{\alpha}Y}/r_0'^2$.

Note that in this picture, the cross section is essentially determined by the size distribution of the *largest* dipole in the Fock state.

3. Fluctuations of the number of particles produced in onium–nucleus collisions

Now, consider the scattering of an onium of size *larger* than $1/Q_{MV}$ off a large nucleus. When the impact parameter is within the radius of the nucleus, then the S-matrix element is clearly close to zero. One interesting quantity to study is the number of particles produced in a given rapidity slice in the fragmentation region of the proton *in a given event*. In Ref. [8], we suggested that it may be directly related to the gluon density in the onium integrated up to the saturation scale of the nucleus with which it interacts at a value of the momentum fraction x in correspondence with the rapidity of the final-state hadrons that are measured. Hence, measurements of the multiplicity of particles may provide an insight into the *event-by-event fluctuations* of the gluon density in the onium.

The dipole model, supplemented with a model for confinement, is an appropriate framework to compute these fluctuations: Indeed, it provides equations for the probability P_n to have n dipoles of size larger than a given threshold, namely n gluons of momenta smaller than the saturation scale, in a given realization. However, the equations for P_n are too complicated to be solved analytically. We were, nevertheless, able to obtain closed expressions

for P_n in asymptotical limits. For n large with respect to its expected value $\bar{n}$, we have found [8]

$$P_n \propto r_0^2 \Lambda_{\text{QCD}}^2 \times \frac{1}{n_1} e^{-n/n_1} \quad \text{for} \quad n \gg \bar{n}. \tag{6}$$

The mechanism for generating high-multiplicity events is the following: In the very beginning of the evolution, the onium of size r_0 promptly splits to the largest dipoles compatible with confinement of size $1/\Lambda_{\text{QCD}}$. This step has the probability $r_0^2 \Lambda_{\text{QCD}}^2$ in the limit of $r_0 \ll 1/\Lambda_{\text{QCD}}$. In a second step, the system decays into smaller dipoles over the remaining rapidity range: The number of dipoles resulting from this process is distributed as a decreasing exponential in the same way as the number of objects generated in a $1 \to 2$ simple branching process. The parameter n_1 of the exponential is the expected number of dipoles, namely the ordinary gluon density in a hadron evaluated at the saturation scale of the nucleus.

In the small-n limit instead, we have found the following expression for P_n [9]:

$$P_n \propto \exp\left(\frac{1}{2} \ln^2 n - \frac{1}{4} \frac{\ln^4 n}{\ln^2 \bar{n}} - \frac{1}{4} \ln^2 \bar{n} \right) \quad \text{for} \quad 1 \ll n \ll \bar{n}. \tag{7}$$

In order to check our understanding of the mechanism leading to dipole number fluctuations, we have written a Monte Carlo code implementing the color dipole model (supplemented with an infrared cutoff modeling confinement) [9]. The numerical results we have obtained are in very good agreement with the analytical formulas (6) and (7).

4. Outlook

From our recent theoretical results on the multiplicity distribution in the final state of onium–nucleus scattering processes, one may derive predictions for proton–nucleus collisions at the LHC, or for deep-inelastic scattering at a future electron–ion collider. The former is quite demanding, since a proton is not a quark–antiquark pair, and it is not clear whether a quark–diquark dipole would be a good model in this context. The latter is clearly more straightforward [10] since onium–nucleus interaction is a subprocess of electron–nucleus scattering, factorizable in the limit of high energies.

Next-to-leading order effects are known to be important practically; one eventually needs to implement them to arrive at a quantitative description of the data. A kinematical constraint [11], or a modified kernel incorporating resummations of collinear logarithms [12] may be good ways to effectively account for subleading effects.

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