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DOUBLE-LOGARITHMIC CONTRIBUTION TO POMERON*

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Pomeron is a term introduced in the 1960s in the framework of the phenomenological Regge theory. It describes the behavior of the total cross sections of any hadronic reaction at extremely large values of the invariant energy s. In the QCD context, the best-known contributions to the Pomeron come from the BFKL equation. The BFKL equation accounts for total resummation of Leading Logarithmic (LL) contributions, *i.e.* the terms where single-logarithmic contributions are multiplied by the overall factor s. The high-energy asymptotics of such resummation is known as the BFKL Pomeron. It predicts the total cross section to be $\sim s^{\Delta}$, where the exponent Δ is called the intercept of the BFKL Pomeron. In contrast, the Double-Logarithmic (DL) contributions are not accompanied by the overall factor s, so resummation of them leads to the asymptotic form $\sim s^{(\Delta_{\rm DL}-1)}$ which looks negligibly small compared to the BFKL result. However, the intercept $\Delta_{\rm DL}$ proves to be so large that its value compensates for the lack of the extra factor of s and makes the DL Pomeron of importance comparable to the BFKL Pomeron. By this reason, contributions of the DL Pomeron should be taken into account whenever the BFKL Pomeron applies.

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1. Introduction

The classic/phenomenological theory of Regge poles does not involve any specific model strong interactions. On the contrary, it is based on such

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fundamental concepts as analyticity, causality and unitarity. One of the basic results of the Regge theory is the prediction that asymptotic of the total cross section $\sigma_{\rm tot}$ of any hadronic reaction is

$$\sigma_{\rm tot} \sim s^{\alpha(0)-1}\,, \tag{1}$$

with $\alpha(0)$ being the Regge trajectory $\alpha(t)$ at $t = 0$. The Froissart–Martin bound states that $\alpha(0) \leq 1$. Pomeranchuk [1] presumed that there could exist a Reggeon with maximally possible value of $\alpha(0) = 1$. Such Reggeon is called Pomeron. The Regge theory is not able to estimate values of $\alpha(t)$. Even more serious drawback of the Regge theory is that there are no means to estimate the energy scale, where the Regge asymptotics of Eq. (1) reliably describes the total cross section.

Several different Pomerons are known in the QCD framework but the BFKL Pomeron [2] is presently the most popular. The BFKL equation sums to all orders in $\alpha_{\rm s}$ the leading logarithmic (LL) contributions, *i.e.* scattering amplitudes $A_{\rm LL}$ in the LL approximation (LLA) are represented as the sum of single-logarithmic (SL) contributions multiplied by the overall factor s

$$A_{\rm LL} = s \sum c_n (\alpha_{\rm s} \ln s)^n\,. \tag{2}$$

The high-energy asymptotics of $A_{\rm LL}$ is called the BFKL Pomeron. Logarithmic contributions in Eq. (2) are called SL because each power of the coupling is multiplied by one log. In contrast, the double-logarithmic approximation (DLA) is also known, where the coupling is multiplied by two logs in each order of the perturbation expansion, see Ref. [3]. As a result, scattering amplitudes $A_{\rm DL}$ in DLA are represented through total resummation of DL contributions

$$A_{\rm DL} = \sum c'_n (\alpha_{\rm s} \ln^2 s)^n\,. \tag{3}$$

In Eqs. (2) and (3), c_n and c'_n are numerical factors. The high-energy asymptotics of $A_{\rm DL}$ is the DL Pomeron. Although each term in the series in Eq. (2) contains less logarithms than the one in Eq. (3) and, therefore, the $n^{\rm th}$ term is suppressed by the factor $\ln^n s$, the sum of such terms in Eq. (2) is multiplied by the overall factor s which seems to be so great that there exists the unanimous opinion among "logarithmic QCD society" stating that the LLA contribution (2) to any scattering amplitude involving a Pomeron exchange prevails a lot over the DLA contribution (3) and, therefore, the DL Pomeron can only be a small correction to the BFKL Pomeron. In Ref. [4], we proved that this opinion was a misconception. In fact, the DL Pomeron is not less important than the BFKL one. In the present paper, we demonstrate it calculating first the amplitude $A_{\gamma\gamma}$ of the elastic $\gamma^*\gamma^* \to \gamma^*\gamma^*$ scattering in DLA, then calculating the high-energy asymptotics of $A_{\gamma\gamma}$ *i.e.* the DL Pomeron and comparing it with the BFKL Pomeron.

2. Amplitude of the elastic photon–photon scattering in DLA

We consider the elastic scattering of virtual photons in the forward kinematics $s \gg -t$

$$\gamma^*(q)\gamma^*(p) \to \gamma^*\left(q'\right)\gamma^*\left(p'\right)\,, \tag{4}$$

assuming all the photons to be unpolarized and keeping the standard notations

$$s = (p+q)^2\,, \qquad t = \left(p-p'\right)^2\,, \qquad \left|p^2\right| \approx \left|p'^2\right| = Q_1^2\,, \qquad \left|q^2\right| \approx \left|q'^2\right| = Q_2^2\,. \tag{5}$$

We introduce the infrared cut-off μ in order to regulate infrared divergences. Expressions for the scattering amplitude of process (4) are simpler when $Q_1^2, Q_2^2 \leq \mu^2$. In this case, the amplitude does not depend on $Q_{1,2}^2$ with DL accuracy. When $Q_{1,2}^2 \gg \mu^2$, the situation is more involved. As it is known (see *e.g.* Refs. [5, 6]), there are two kinematic regions, where $A_{\gamma\gamma}$ is described by different expressions. First, there is the region of moderately virtual photons where

$$s\mu^2 > Q_1^2 Q_2^2\,. \tag{6}$$

We denote $A_{\gamma\gamma}^{(M)}$ the scattering amplitude $A_{\gamma\gamma}$ in region (6). We call the region of deeply virtual photons the opposite region, where

$$s\mu^2 < Q_1^2 Q_2^2 \tag{7}$$

and denote $A_{\gamma\gamma}^{(D)}$ the scattering amplitude in region (7). It is convenient to use the Mellin transform for $A_{\gamma\gamma}$

$$A_{\gamma\gamma}\left(s, Q_1^2, Q_2^2\right) = \int\limits_{-\imath\infty}^{\imath\infty} \frac{\mathrm{d}\omega}{2\pi\imath} e^{\omega\rho} F_{\gamma\gamma}(\omega, y_1, y_2)\,, \tag{8}$$

where we have introduced the μ-dependent logarithmic variables ρ, y_1, y_2:

$$\rho = \ln\left(s/\mu^2\right)\,, \qquad y_1 = \ln\left(Q_1^2/\mu^2\right)\,, \qquad y_2 = \ln\left(Q_2^2/\mu^2\right)\,. \tag{9}$$

In order to calculate $A_{\gamma\gamma}^{(M,D)}$ we construct and solve the Infrared Evolution Equation. This method was suggested by Lipatov [7]. It is based on a possibility to isolate/factorize DL contributions of the softest partons *i.e.* the partons which have the minimal transverse momenta $k_\perp$. This property of the softest partons was first found in Ref. [8] in the QED context. The IR cut-off μ in the IREE method is introduced in the transverse momentum space. In DLA, all transverse momenta are ordered, so only integration over the softest momentum $k_\perp$ involves μ as the lowest integration limit. At the same time, μ plays the role of the mass scale. After factorization of the softest parton, its momentum $k_\perp$ plays the role of the IR cut-off/mass scale for all other virtual parton momenta. The IREE for $A_{\gamma\gamma}^{(M)}$ is illustrated by the graphs in Fig. 1. It involves the auxiliary amplitudes $A_{\gamma q}, A_{\gamma g}$ and their inverse.

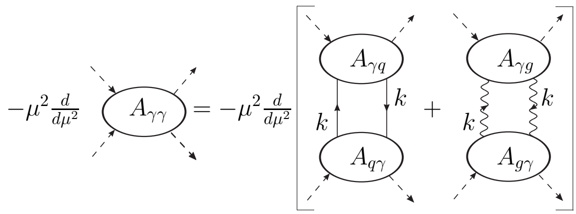

Fig. 1. IREE for $A_{\gamma\gamma}$. The dashed lines stand for the external photons, the straight (wavy) lines denote the quark (gluon) propagators of the softest partons with momenta k.

Applying to this equation the standard Feynman rules and using transform (8), we can write down this IREE in the analytical form[1]. The IREE for $F_{\gamma\gamma}^{(M)}$ and $F_{\gamma\gamma}^{(D)}$ are different because $A_{\gamma\gamma}^{(D)}$ does not depend on μ

$$\begin{aligned} \omega F_{\gamma\gamma}^{(M)} + \frac{\partial F_{\gamma\gamma}^{(M)}}{\partial y_1} + \frac{\partial F_{\gamma\gamma}^{(M)}}{\partial y_2} &= \frac{1}{8\pi^2}\left[F_{\gamma q}F_{q\gamma} + F_{\gamma g}F_{g\gamma}\right] , \\ \frac{\partial A_{\gamma\gamma}^{(D)}}{\partial \rho} + \frac{\partial A_{\gamma\gamma}^{(D)}}{\partial y_1} + \frac{\partial A_{\gamma\gamma}^{(D)}}{\partial y_2} &= 0 . \end{aligned} \tag{10}$$

Solving these equations, we represent $A_{\gamma\gamma}^{(M,D)}$ in terms of the auxiliary amplitudes. All auxiliary amplitudes can also be found with constructing and solving IREEs for them. The explicit expressions for $A_{\gamma\gamma}^{(M,D)}$ can be found in Ref. [4]. They are rather complicated, so we do not reproduce them here. Instead, in this paper, we will focus on high-energy asymptotics of $A_{\gamma\gamma}^{(D)}$. As the asymptotics for $A_{\gamma\gamma}^{(M)}$ and $A_{\gamma\gamma}^{(D)}$ are the same, we will drop the superscripts M, D in what follows.

3. Asymptotics of $A_{\gamma\gamma}$

The asymptotics of $A_{\gamma\gamma}$ at $s \to \infty$ can be found with the standard mathematical means: the saddle/stationary point method. It states that

[1] See details in Ref. [4].

when $A_{\gamma\gamma}$ of Eq. (8) is represented as

$$A_{\gamma\gamma} = \int\limits_{-\imath\infty}^{\imath\infty} \frac{\mathrm{d}\omega}{2\pi\imath} e^{\omega\rho+\Psi}\,, \tag{11}$$

its asymptotics looks as follows:

$$A_{\gamma\gamma} \sim A^{\mathrm{as}}_{\gamma\gamma} = \frac{e^{\Psi(\omega_0)}}{\sqrt{2\pi\Psi''(\omega_0,\rho)}} \left(\frac{s}{\mu^2}\right)^{\omega_0}\,, \tag{12}$$

with the stationary point ω_0 being the rightmost solution of the equation $\rho + \Psi' = 0$. Applying this technique to the explicit expressions for $A_{\gamma\gamma}$, we obtain its asymptotics $A^{\mathrm{as}}_{\gamma\gamma}$:

$$A^{\mathrm{as}}_{\gamma\gamma} = \frac{\lambda(\omega_0)}{\sqrt{\rho^3}} \left(\frac{s}{\sqrt{Q_1^2 Q_2^2}}\right)^{\omega_0}\,, \tag{13}$$

where the λ stands for the impact-factors. The Reggeon in Eq. (13) is the DL contribution to Pomeron. It is altogether non-BFKL contribution. Estimating ω_0 was done in Ref. [4] for several different situations:

A. Approximation of fixed α_{s}; gluon contribution is accounted for, while quark contribution is neglected. In this case, the intercept $\omega_0 = 1.35$.

B. Approximation of fixed α_{s}; both gluon and quark contribution are accounted for. Impact of quark contribution decreases the intercept down to $\omega_0 = 1.35$.

C. QCD coupling α_{s} runs; quark contribution are neglected. Running coupling effects decrease of the intercept down to $\omega_0 = 1.25$.

D. QCD coupling α_{s} runs; both gluon and quark contribution are accounted for. Further decrease of the intercept down to $\omega_0 = 1.07$.

We see that, similarly to the BFKL Pomeron, the DL Pomeron is supercritical though its value decreases with the increase of the accuracy of calculations.

Asymptotics $A^{\mathrm{as}}_{\gamma\gamma}$ represents its parent amplitude $A_{\gamma\gamma}$ reliably when their ratio

$$R_{\mathrm{as}}(s) = A^{\mathrm{as}}_{\gamma\gamma}(s)/A_{\gamma\gamma}(s) \tag{14}$$

is close to unity. Numerical calculations yield that $R_{\mathrm{as}}(s) \geq 0.9$, when $s > s_{\mathrm{min}}$, with

$$s_{\mathrm{min}} = 10^6 \sqrt{Q_1^2 Q_2^2}\,. \tag{15}$$

We remind that similar estimates for the applicability region of the BFKL Pomeron are absent in the literature.

4. Conclusion

In the present paper, we explained how to calculate the DL contribution to Pomeron and listed estimates for its intercept obtained for the cases of fixed and running QCD coupling, for the case of pure gluon contribution as well as for the mixture of gluon and quark contributions. We also demonstrated that although the Regge asymptotics are given by simple and convenient expressions, they should not be used beyond their applicability regions, which for the gamma–gamma scattering is given by Eq. (15). At $s < s_{\min}$, one should use the parent amplitudes $A^{M,D}_{\gamma\gamma}$ but not their asymptotics. We think that Eq. (15) can also be used to estimate an applicability region of the BFKL Pomeron, so the BFKL Pomeron can be used at energies $s > s_{\min}$ and in this region it should be used together with the DL Pomeron.

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