DOUBLY HEAVY QQ TETRAQUARKS*

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With the discovery of a doubly charmed Ξ_{cc} baryon, a somewhat forgotten issue of tetraquarks containing two heavy and two light (anti)quarks, $\mathcal{T}_{QQ}$, triggered theorist's interest. We discuss quark model estimates of $\mathcal{T}_{QQ}$ masses and a model where the light sector is treated as a soliton. We show that this model has different large-N_c limit than other approaches.

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1. Introduction

Recent discovery of a doubly charmed $\Xi_{cc}^{++}(3621)$ baryon by the LHCb Collaboration at CERN [1] renewed interest in $\bar{Q}\bar{Q}q_1q_2$ tetraquarks or their antiparticles, essentially for two reasons. Firstly, the LHCb result shows that it is possible to create a cc pair in an attractive channel that can form a bound state with a light quark. Secondly, on theoretical side, the heavy cc pair can be described to a good approximation as a pointlike color $\bar{\mathbf{3}}$ source that can form a bound state with two light antiquarks. In the heavy-quark limit, the $\bar{\mathbf{3}}$ source acts as a heavy antiquark and, therefore, the underlying dynamics is identical to the heavy antibaryon case (neglecting spin effects). In this paper, we shall consider heavy $\bar{Q}\bar{Q}$ pairs acting as $\mathbf{3}$ color source.

Similar excitement arose approximately 17 years ago when SELEX experiment at FermiLab announced a discovery of $\Xi_{cc}^{+}(3519)$ [2], which is today (despite later report [3]) considered as unconfirmed [4]. Phenomenological attempt to estimate the $\bar{c}\bar{c}q_1q_2$ mass based on the SELEX result can be found *e.g.* in Ref. [5], where also large-N_c limit for such states is discussed.

In 1993, Manohar and Wise [6] showed within heavy-quark symmetry approach that doubly heavy tetraquarks are bound in the limit of $m_Q \to \infty$. These arguments were reanalyzed in 2006 and also very recently, at the time of this conference, by Cohen and collaborators in Refs. [7, 8].

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Asymptotic theorems, however, do not provide any hint at what scale they become operative. The aim of this paper is to recall some simple quark model estimates of the doubly heavy tetraquark mass and then apply a phenomenological model based on a soliton description of the light sector to tetraquarks. At the end, we discuss the difference between the soliton model and regular quark models in the large-N_c limit.

2. Quark model estimates of the tetraquark mass

To the best of our knowledge, the first phenomenological attempt to estimate doubly heavy QQ tetraquark mass was published by Lipkin in 1986 [9] (although the fourfold heavy tetraquarks were discussed even earlier in 1982 [10]). He used a variational method in the nonrelativistic quark model and one, rather natural assumption, that light quarks see heavy (anti)quark pair as a single object. Lipkin tried to use experimental data available at that time to derive (almost) model-independent estimate of the tetraquark mass. This is schematically shown in Fig. 1 and leads to the following mass formulae for tetraquark ($\mathcal{T}_{QQ}$), J/Ψ and Λ_Q:

$$\begin{aligned} M_{\mathcal{T}_{QQ}} &= 2M + 2m + T_{QQ} + V_{QQ} + 2V_{Qq} + 2T_q + V_{qq}\,, \\ M_{J/\Psi} &= 2m_Q + T_{QQ} + 2V_{QQ}\,, \\ M_{\Lambda_Q} &= m_Q + 2m_q + 2T_q + 2V_{Qq} + V_{qq}\,, \end{aligned} \tag{1}$$

where for simplicity we have suppressed *bars* over heavy-quark symbol Q. Notation confronted with Fig. 1 is self-explanatory. Note that quark–quark interaction in color $\bar{\mathbf{3}}$ is two times weaker than antiquark–quark in $\mathbf{1}$.

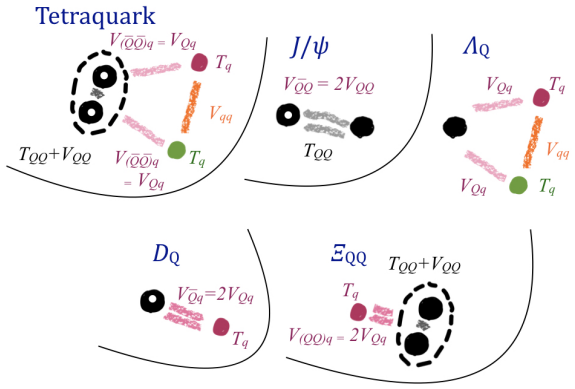


Fig. 1. Schematic illustration of multiquark states. First row shows states taken into account by Lipkin [9], second row shows two more states used additionally in this paper. One thick line joining quarks represents interaction in color $\mathbf{3}$ or $\bar{\mathbf{3}}$, whereas double line corresponds to color singlet.

Formulae (1) lead to the following upper bound for the tetraquark mass:

$$M_{\mathcal{T}_{QQ}} \leq M_{\Lambda_Q} + \tfrac{1}{2} M_{J/\Psi} + \tfrac{1}{2}\langle T_{QQ}\rangle \,. \tag{2}$$

With the data available in 1986, one could not eliminate the unknown average $\langle T_{QQ}\rangle$. Nevertheless, an important lesson can be drawn from Eq. (2) when plugging in numerical values with a spin averaged mass $M_{\bar{c}c} = (3M_{J/\Psi} + M_{\eta_c})/4$ rather than $M_{J/\Psi}$

$$M_{\mathcal{T}_{cc}} - M^c_{\rm thr} \leq -55 \text{ MeV} + \tfrac{1}{2}\langle T_{cc}\rangle \tag{3}$$

with $M^c_{\rm thr} = M_D + M_{D^*}$. We see from Eq. (3) that the very existence of the bound heavy tetraquark depends on the subtle balance between -55 MeV and $\langle T_{cc}\rangle$. It is easy to convince oneself that adding the D meson does not eliminate $\langle T_{cc}\rangle$. Today, we can repeat the same estimate for the b case

$$M_{\mathcal{T}_{bb}} - M^b_{\rm thr} \leq -262 \text{ MeV} + \tfrac{1}{2}\langle T_{bb}\rangle \,. \tag{4}$$

Now, with the discovery of Ξ_{cc} [1], one can form a linear combination where the troublesome $\langle T_{QQ}\rangle$ term drops out [5]. To this end, we have

$$\begin{aligned} M_{D_Q} &= m_Q + m_q + T_q + 2V_{Qq} \,, \\ M_{\Xi_{QQ}} &= 2m_Q + m_q + T_{QQ} + V_{QQ} + T_q + 2V_{Qq} \,. \end{aligned} \tag{5}$$

With this new input, we have one new relation

$$M_{\mathcal{T}_{QQ}} \leq M_{\Xi_{QQ}} + M_{\Lambda_Q} - M_{D_Q} \,, \tag{6}$$

where again we use $M_{D_c} = (3M_{D^*} + M_D)/4$ (for a more accurate choice, see [5]) and obtain numerically

$$M_{\mathcal{T}_{cc}} \leq 3935 \text{ MeV} \,, \tag{7}$$

which is 60 MeV above the threshold (as *e.g.* in [11]). This implies that, if inequality (2) were saturated, $\langle T_{cc}\rangle \sim 230$ MeV. Since $m_b/m_c \sim 3$, one can reasonably assume that $\langle T_{bb}\rangle \sim \langle T_{cc}\rangle/3$ and we arrive at a conclusion

$$M_{\mathcal{T}_{bb}} - M^b_{\rm thr} \sim -224 \text{ MeV} \tag{8}$$

in a surprising agreement with much sophisticated quark model of Ref. [12]. In the literature, one finds predictions for (8) ranging from -60 [13] through -100 [11], -120 [14, 15] to -400 MeV [16] (see Table V in [11] for other predictions). Our simple analysis confirms that the binding of a putative doubly heavy QQ tetraquark increases with increasing m_Q. This is true also in the case when the structure of a heavy diquark can be resolved by the light quarks and repulsive **6** channel is included [17].

3. Soliton model for tetraquarks

While the nonrelativistic approach may be applicable to heavy quarks, its credibility for the light sector is certainly not at the same footing. In recent papers [18], we have proposed a mean-field description of heavy baryons as a light quark–soliton and a heavy quark, where the soliton is constructed from N_c-1 rather than from N_c quarks. The model passes phenomenological tests.

In the $m_Q \to \infty$ limit, the soliton should not be sensitive to the properties of an object it is interacting with. Moreover, since the ground state soliton is in this case in flavor antitriplet of spin zero [18], the spin of the heavy "nucleus" is irrelevant. Mass formula for the nonstrange heavy baryons in flavor antitriplet is very simple

$$M^{Q,\bar{\mathbf{3}}}_{\text{baryon}} = M^{Q}_{\bar{\mathbf{3}}} + \frac{2}{3}\delta_{\bar{\mathbf{3}}}\,, \tag{9}$$

where $\delta_{\bar{\mathbf{3}}}$ can be extracted from the light hyperon spectrum, and $M_{\bar{\mathbf{3}}}$ is an average mass of the $\bar{\mathbf{3}}$ flavor multiplet including heavy-quark mass m_Q, classical soliton mass M_{sol} and soliton rotational energy [18]. For strange heavy baryons, the coefficient in front of $\delta_{\bar{\mathbf{3}}}$ is equal to $-1/3$. In [18], we never needed m_Q and M_{sol} separately. For nonstrange tetraquarks, we therefore naturally have

$$M^{Q,\bar{\mathbf{3}}}_{\text{baryon}} = M^{Q}_{\bar{\mathbf{3}}} + \frac{2}{3}\delta_{\bar{\mathbf{3}}} + (m_{\bar{Q}\bar{Q}} - m_Q) \tag{10}$$

and it is clear that now we need not only m_Q but also $m_{\bar{Q}\bar{Q}}$. For a rough estimate, we can approximate $m_{\bar{Q}\bar{Q}} - m_Q \sim m_Q$, or we may assume a few percent binding following *e.g.* Ref. [19].

In order to estimate effective m_Q, we first observe that differences of mean multiplet values, both for flavor $\bar{\mathbf{3}}$ and $\mathbf{6}$, for bottom and charm, should be equal to $m_b - m_c = M^b_{\bar{\mathbf{3}}} - M^c_{\bar{\mathbf{3}}} = M^b_{\mathbf{6}} - M^c_{\mathbf{6}}$. Numerically, we have

$$M^b_{\bar{\mathbf{3}}} - M^c_{\bar{\mathbf{3}}} = 3327 \text{ MeV}\,, \qquad M^b_{\mathbf{6}} - M^c_{\mathbf{6}} = 3326 \text{ MeV} \tag{11}$$

with $M^Q_{\bar{\mathbf{3}},\mathbf{6}}$ taken from [18]. We see perfect agreement between both flavor multiplets. Another piece of information comes from the hyperfine splittings in $\mathbf{6}$ that are inversely proportional to the quark masses and have been estimated in [18] yielding

$$m_c/m_b = 0.29\text{–}0.31\,, \tag{12}$$

which is compatible with the ratio obtained from the PDG [4]. Now, from Eqs. (11) and (12), we can determine absolute effective masses

$$m_c = 1357 \div 1495 \text{ MeV}\,, \qquad m_b = 4685 \div 4821 \text{ MeV}\,. \tag{13}$$

The uncertainty in (13) is due to the uncertainty in ratio (12). Masses (13) are lower than masses extracted from meson spectra: $m_c = 1643$ and $m_b = 4979$ MeV, which are compatible with *e.g.* [19].

We see from Fig. 2 that for heavy-quark masses in the range of Eq. (13), both cc and bb tetraquarks are rather deeply bound. For larger m_Q, compatible with mesonic spectra, cc tetraquark is most likely unbound and bb is most likely bound.

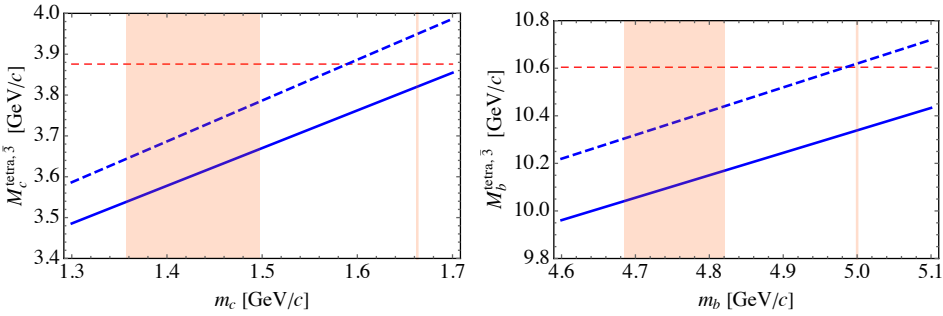


Fig. 2. (Color online) The lightest QQ tetraquark mass (charm — left and bottom — right) as a function of m_Q with (solid) and without (dashed) $\bar{Q}\bar{Q}$ binding contribution. Thin horizontal dashed (red) line corresponds to the $D\,D^*$ or $B\,B^*$ threshold. Shaded areas indicate the heavy-quark mass range (13). Solid vertical line shows the heavy-quark mass from Ref. [19].

4. Summary

We have recalled arguments that QQ tetraquarks are bound in the limit of $m_Q \to \infty$ and analyzed the mass spectrum of cc and bb tetraquarks with the help of the variational approach of Lipkin [9]. We than employed the quark–soliton model describing light degrees of freedom in the $N_c \to \infty$ limit used previously for heavy baryons with one heavy quark [18] to the problem of QQ tetraquarks. We have argued that the light soliton does not distinguish the nature of the color **3** heavy source, so that heavy quark can be replaced by a heavy anti-diquark leaving the soliton unaffected. Unfortunately, the anti-diquark properties have not been calculated within the soliton model. In fact, in the present model, the diquark should be considered an $N_c - 1$ heavy-quark system to neutralize the color of the soliton for $N_c > 3$. This color structure (discussed briefly in [5]) is completely different from the quark model picture where QQ tetraquarks consist of two antiquarks and two quarks for any N_c. The "diquark" in the soliton approach is, therefore, amenable to an effective description, as the light sector that is represented by a soliton, and deserves further studies from this perspective.

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