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TOWARDS HEAVY–LIGHT AXIALVECTOR TETRAQUARKS IN A DYSON–SCHWINGER/BETHE–SALPETER APPROACH*

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We discuss previous results for the light scalar mesons as four-quark states as well as recent results for the heavy–light axialvector states using the Dyson–Schwinger and Bethe–Salpeter equations. We introduce a new technique for solving the axialvector heavy–light four-quark system.

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1. Introduction

1.1. Exotic charmonium states

Before the discovery of the $X(3872)$ in 2003, the spectrum of charmonium states could be well-understood by potential model calculations. Since then, however, evidence has increased that there are other states in the same mass region whose interpretation needs to go beyond the naive $q\bar{q}$ assignment. The appearance of the charged Z states, for example, is discussed as a smoking gun signature for four-quark/anti-quark states. Other possible states of exotic nature are hybrids, glueballs, or even states with more than four valence (anti-)quarks, see [1, 2] for reviews.

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In this work, we focus on four-quark states. Their internal structure has been the topic of many studies with suggestions ranging from hadronic molecules (heavy–light meson clusters with long-range interactions), diquark–antidiquark states, or hadro-charmonia (separate clustering of heavy and light quarks). These have been investigated using effective field theory methods, lattice QCD, QCD sum rules, potential models and other approaches, see *e.g.* [1, 3–9] and references therein. In this respect, the functional approach using the Dyson–Schwinger (DSE) and Bethe–Salpeter (BSE) equations can offer additional insight. The Bethe–Salpeter amplitude of a four-quark state contains components from wave functions for heavy–light mesons (MOL), diquark–antidiquark configurations (DA), and hadro-charmonium combinations (MES), and the system is able to determine which of these components dominate. This leads to additional information on the nature of a state, as we will see below.

1.2. Light scalar mesons

The light scalar mesons ($J^{PC} = 0^{++}$) have been under discussion for many years [10]. In principle, one would expect their masses to be roughly in the same range as other p-wave mesons such as axialvectors and tensors, *i.e.* between 1–1.5 GeV. This is indeed where many scalar states appear. The lightest scalar nonet, however, has masses between 0.5–1 GeV and a very peculiar decay pattern: The $f_0(500)/\sigma$ is very broad and decays predominantly to $\pi\pi$. In addition, the κs which carry strangeness are broad states well above the $K\pi$ threshold. By contrast, the isosinglet/isotriplet $f_0/a_0(980)$ lie at the $K\bar{K}$ threshold and are rather narrow in comparison. Furthermore, the mass ordering within the multiplet is not compatible with an ordinary $q\bar{q}$ nonet, where the three a_0s have only light quarks in their wave function but their masses are degenerate with the $f_0(980)$ which carries a strange-quark pair. This is also in contradiction to their decay channels with hidden strangeness. A possible solution was introduced by Jaffe in 1977 [11]: He interpreted the light scalars as bound states of scalar diquarks, which led to a better understanding of the mass ordering within the multiplet, the large width of the OZI-superallowed decays of σ/κ into $\pi\pi/K\pi$, and the decay pattern of the a_0s that can be explained by the additional $s\bar{s}$ pair in their wave function as compared to a $q\bar{q}$ assignment. The non-$q\bar{q}$ nature of the light scalars is supported by calculations in many different frameworks such as unitarized ChPT [12, 13], quark models [14], the extended linear sigma model [15], and QCD sum rules [16].

2. The method

The DSE/BSE framework for bound states [17–19] is based on calculating the QCD propagators and vertices from their DSEs, which are subsequently used as ingredients in the calculation of bound-state properties from BSEs. This has successfully been applied to mesons and baryons [19], glueballs [20], and light scalar tetraquarks [21, 22]. We use the rainbow-ladder approximation [23], which effectively limits the tensor structure of the quark–gluon vertex to the one of a bare vertex, with a dressing function that depends on the gluon momentum only and whose product with the gluon dressing function is replaced by an effective coupling $\alpha_{\text{eff}}(k^2)$. From here, one can obtain a self-consistent solution for the quark propagator and also derive a consistent two-body scattering kernel for the BSE resulting in a dressed gluon exchange. This kernel also describes the two-body interactions in the tetraquark BSE shown in Fig. 1.

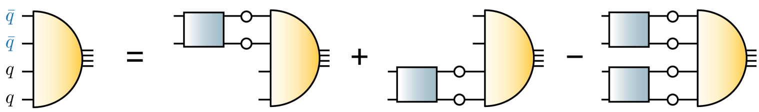

Fig. 1. (Color online) Schematic diagram (without permutations) of the four-quark BSE including two-body interactions only. The blue box and yellow half-ellipse denote the gluon exchange kernel and the tetraquark amplitude, respectively.

The tetraquark amplitude (yellow half-ellipse) is given by

$$\Gamma = \text{Dirac} \otimes \text{color} \otimes \text{flavor} = \sum_i f_i(\Omega)\tau_i \otimes \text{color} \otimes \text{flavor}\,. \tag{1}$$

It consists of two color structures and 256 Dirac tensors (32 with s waves only) in the scalar and 768 (48) in the axialvector case. The set Ω stands for all possible Lorentz invariant combinations of four vectors, which gives a total of nine for fixed total momentum P. It is convenient to use a set of Lorentz invariants that transform under the permutation group S_4 [24], which leads to a singlet (S_0), a doublet (D) and two triplets (T_0, T_1). In [22], it was found that the amplitude is dominated by products of two-body poles which can be parameterized by the kinematic variables contained in S_0 and D. This implies that the amplitudes can be parameterized to good approximation by a residue times a product of two-body poles

$$f_i(\Omega) \approx \frac{f_i(S_0)}{\left((p_1+p_2)^2 + m_{12}^2\right)\left((p_3+p_4)^2 + m_{34}^2\right)}\,, \tag{2}$$

where p_1, p_2 are the momenta of the contributing (anti-)quarks. It turns out that the dependence of the residue on the kinematic variables characterizing the triplets is weak.

Figure 2 shows results for the masses of the light scalar mesons from the four-body equation [22], Fig. 1, together with those from a two-body approximation in terms of mesons and diquarks [21]. The mass ordering of the experimental states is nicely reproduced, as can be seen in the left plot. The right plot shows that the four-body calculation for the σ so far produces a resonance above $\pi\pi$ threshold for quark masses from the physical light quark masses up to the charm region, where it becomes bound again. The lightest scalar tetraquark is dominated by the internal $\pi\pi$ components in agreement with the experimentally observed $\sigma/f_0(500)$.

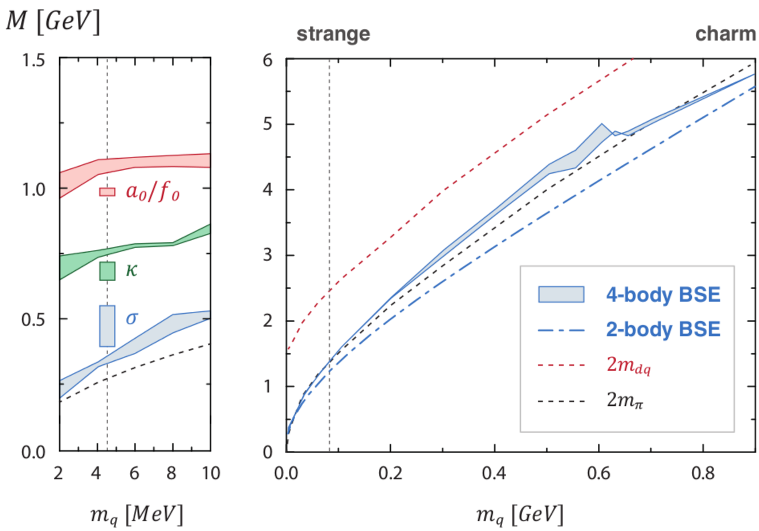

Fig. 2. Left: Real part of the light scalar meson masses as a function of the quark mass. Right: Mass evolution of the two- and four-body results for the lightest scalar tetraquark [21, 22] in comparison to the $\pi\pi$ and diquark–antidiquark thresholds.

3. Axialvector states in the charm energy region

The same framework can also be applied to heavy–light four-quark states in the charm energy region. Currently, we focus on the ground state in the axialvector channel, which might be identified with the $X(3872)$. We again solve the four-quark equation in Fig. 1. We expand the corresponding Bethe–Salpeter amplitude into heavy–light meson (MOL) and hadro-charmonium-like (MES) tensors inspired by the physical constituents that these state can have, as well as diquark–antidiquark (DA) components. If enough terms are considered, this procedure eventually leads to a complete basis. Encouraged by the outcome of the calculation of scalar mesons, we parameterize the dressing functions $f_i(\Omega)$ of the amplitude by the residue-pole structure discussed around Eq. (2), where the respective pole positions are calculated from their two-body BSEs. This leads us to

$$\Gamma = D\bar{D}^{*} + D^{*}\bar{D} + D^{+}D^{*-} + D^{*+}D^{-} + J/\Psi\omega + \sum_{q=u,d} (\mathrm{SA} + \mathrm{AS})_{q} \,. \quad (3)$$

Here, the MOL contribution is represented by the D mesons, the MES part by $J/\Psi\omega$ and the DA part by scalar and axialvector diquarks (S/A), where the sum goes over both light quark flavors.

In Fig. 3, the results for two different versions of Γ in Eq. (3) are presented. The red circles are obtained by retaining the MOL component only and the green triangles include both MOL and MES components. Keeping in mind that the points at the lightest two quark masses are plagued by technical difficulties, the extrapolation of the larger mass points leads to an axialvector four-quark state in the mass region of the $X(3872)$. Furthermore, the results clearly show that the $\omega J/\Psi$ (MES) component is negligible and the molecular component dominates.

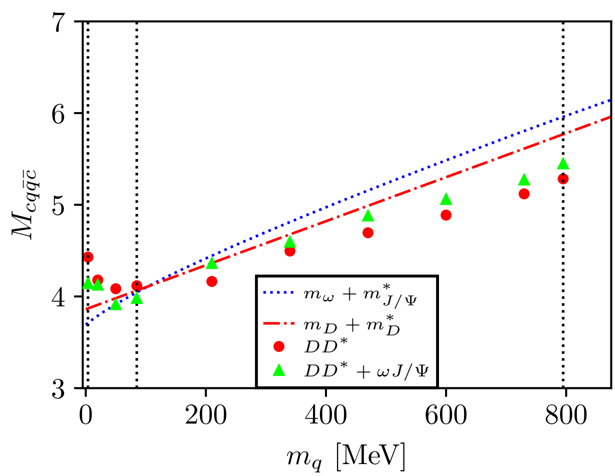

Fig. 3. (Color online) Heavy–light axialvector tetraquark mass as a function of the light quark mass.

4. Outlook

We presented first results for the axialvector ground state in the heavy–light quark sector using a new method to deal with the four-body equation. In order to systematically explore four-quark states in the charm energy region, we work on further generalisations to other quantum numbers.

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