# CHIRAL PERTURBATION THEORY IN THE ENVIRONMENT WITH CHIRAL IMBALANCE*

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We discuss the description of chiral imbalance imprint in pion matter. It is based on the Chiral Perturbation Theory supplemented by medium induced chiral chemical potential in the covariant derivative. Such an implementation of chiral imbalance recently explored was in linear sigma model inspired by QCD. The relationship between two sigma models is examined. A possible experimental detection of chiral imbalance in charged pion decays inside the fireball is pointed out.

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## 1. Chiral imbalance

The generation of a phase with local parity breaking (LPB) in strong interactions in central heavy-ions collisions (HIC) has attracted much interest [1, 2] although the experimental evidences are still poor. After a HIC, the LPV in the so-called fireball is possible as a result of the appearance in a dense and hot nuclear environment a difference between the densities of the right- and left-handed chiral fermion fields (chiral imbalance). Such a chiral medium can be simulated by a chiral chemical potential $\mu_5$. Adding to the QCD Lagrangian the term $\Delta\mathcal{L}_q = \mu_5 q^\dagger\gamma_5 q \equiv \mu_5 Q_5$, we open the way of accounting for non-trivial topological fluctuations in the nuclear (quark) fireball, which are related to fluctuations of gluon fields. The behavior of various spectral characteristics for light scalar and pseudoscalar $(\sigma, \pi^a, a_0^a)$ mesons by means of the QCD motivated $\sigma$ model Lagrangian was recently derived

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for $SU_L(2) \times SU_R(2)$ flavor symmetry with an isosinglet chiral chemical potential [3]. It is shown, in particular, that exotic decays of scalar mesons arise due to mixing of $\pi$ and $a_0$ meson states in the presence of chiral imbalance. The pion electromagnetic form-factor obtains an unusual parity-odd supplement from the Wess–Zumino vertices which entails a photon polarization asymmetry of the pion polarizability in $\pi\gamma \to \pi\gamma$ process [4]. However, the structural constants of the model have to be taken as input parameters suitable to describe the light meson properties in a chiral medium. In this way, there is no real predictability as far as the hadron system response to chiral imbalance is concerned and reaching quantitative predictions requires a phenomenologically justified hadron dynamics. This is provided to some extent by Chiral Perturbation theory (ChPT) [5] with structural constants verified in low-energy interactions of pseudoscalar mesons.

To follow this way, one can reckon on the quark–hadron continuity [6] when passing through hadronization and, for the detection of LPB in the hadron fireball, implement the chiral Lagrangian model with a background 4-vector of axial chemical potential [4], symmetric under $SU_L(N_f)\times SU_R(N_f)$, for $u$-, $d$-quarks ($N_f = 2$).

Chiral imbalance can be associated with gradient density of isosinglet pseudoscalar condensate which can be formed as a result of large-scale, "long-lived" topological fluctuations of gluon fields in the fireball in central heavy-ion collisions (see [2] for details). To describe various effects of hadron matter in a fireball with LPB, we must introduce the chiral chemical potential [2].

## 2. Chiral Lagrangian with chiral chemical potential

For the detection of LPB in the hadron fireball, one considers the chiral Lagrangian for pions describing mass spectra and decays of pseudoscalar mesons in the fireball carrying a chiral imbalance. The latter can be implemented with the help of softly broken chiral symmetry in QCD transmitted to hadron media, properly constructed long derivative

$$D_\nu \Longrightarrow \bar{D}_\nu - \mathrm{i}\{\boldsymbol{I}_q \mu_5 \delta_{0\nu}, \star\} = \boldsymbol{I}_q \partial_\nu - \mathrm{i} A_\nu [Q_{\mathrm{em}}, \star] - 2\mathrm{i} \boldsymbol{I}_q \mu_5 \delta_{0\nu} \,, \qquad (1)$$

where $A_\mu, Q_{\mathrm{em}}$ are the electromagnetic field and charge respectively. The axial chemical potential is introduced as a constant time component of an isosinglet axial-vector field.

Further on, we skip the electromagnetic field. In the large-$N_c$ framework [7], the SU(3) chiral Lagrangian in the strong interaction sector contains the following dim $= 2$ operators [7]:

$$\mathcal{L}_2 = \frac{F_0^2}{4} \left\langle -j_\mu j^\mu + \chi^\dagger U + \chi U^\dagger \right\rangle \,, \qquad (2)$$

where $\langle \ldots \rangle$ denotes the trace in flavor space, $j_\mu \equiv U^\dagger \partial_\mu U$, the chiral field $U = \exp(\mathrm{i}\hat{\pi}/F_0)$, $F_0 \simeq 92$ MeV, $\chi(x) = 2B_0 s(x)$ and $M_\pi^2 = 2B_0 m_{u,d}$, the tree-level neutral pion mass. The constant $B_0$ is related to the chiral quark condensate $\langle \bar{q}q \rangle$ as $F_0^2 B_0 = -\langle \bar{q}q \rangle$. Taking now the covariant derivative in (1), it yields

$$\mathcal{L}_2(\mu_5) = \mathcal{L}_2(\mu_5 = 0) + \mu_5^2 N_\mathrm{f} F_0^2 \,. \tag{3}$$

Herein, we have used the identity for $U \in \mathrm{SU}(n)$, $\langle j_\mu \rangle = 0$. In the large-$N_\mathrm{c}$ approach, the dim $= 4$ operators [7] in the chiral Lagrangian are given by

$$\mathcal{L}_4 = \bar{L}_3 \langle j_\mu j^\mu j_\nu j^\nu \rangle + L_0 \langle j_\mu j_\nu j^\mu j^\nu \rangle - \frac{l_4}{4} \left\langle j_\mu j^\mu \chi^\dagger U + \chi U^\dagger \right\rangle , \tag{4}$$

where $\bar{L}_3, L_0$ are bare low-energy constants. For SU(3) and SU(2), $\langle j_\mu \rangle = 0$ and there is the identity

$$\langle j_\mu j_\nu j^\mu j^\nu \rangle = -2 \langle j_\mu j^\mu j_\nu j^\nu \rangle + \tfrac{1}{2} \langle j_\mu j^\mu \rangle \langle j_\nu j^\nu \rangle + \langle j_\mu j_\nu \rangle \langle j^\mu j^\nu \rangle , \tag{5}$$

whereas for SU(2), there is one more identity

$$2 \langle j_\mu j^\mu j_\nu j^\nu \rangle = \langle j_\mu j^\mu \rangle \langle j_\nu j^\nu \rangle \,. \tag{6}$$

Applying these identities, one finds the GL operators for the SU(3) chiral Lagrangian

$$\mathcal{L}_4 = L_1 \langle j_\mu j^\mu \rangle \langle j_\nu j^\nu \rangle + L_2 \langle j_\mu j_\nu \rangle \langle j^\mu j^\nu \rangle + L_3 \langle j_\mu j^\mu j_\nu j^\nu \rangle \tag{7}$$

with

$$L_1 = \tfrac{1}{2} L_0 \,, \qquad L_2 = L_0 \,, \qquad L_3 = \bar{L}_3 - 2L_0 \,. \tag{8}$$

For SU(2), one has a further reduction of the dim $= 4$ Lagrangian

$$\mathcal{L}_4 = \tfrac{1}{4} l_1 \langle j_\mu j^\mu \rangle \langle j_\nu j^\nu \rangle + \tfrac{1}{4} l_2 \langle j_\mu j_\nu \rangle \langle j^\mu j^\nu \rangle \tag{9}$$

with normalization, so that

$$l_1 = 2L_0 + 2L_3 \,, \qquad l_2 = 4L_0 \,, \qquad (l_1 + l_2) = 2L_3 + 6L_0 \,. \tag{10}$$

We stress that this chain of transformations is valid only if $\langle j_\mu \rangle = 0$.

The response of the chiral Lagrangian on chiral imbalance is derived with the help of the long derivative (1) applied to Lagrangian (4),

$$\begin{aligned} \Delta\mathcal{L}_4(\mu_5) &= -4\mu_5^2 \left\{ (l_1 + l_2) \langle j_\mu j^\mu \rangle - (l_1 + l_2) \left\langle j^0 j^0 \right\rangle \right\} \\ &= -4\mu_5^2 \left\{ 2(l_1 + l_2) \left\langle j^0 j^0 \right\rangle - (l_1 + l_2) \langle j_k j_k \rangle \right\} . \end{aligned} \tag{11}$$

We notice that this result is drastically different from what one could obtain from the final Lagrangian (9). This is because the identities (5) and (6) are violated if $\langle j_\mu \rangle \neq 0$. The mass term affected by chiral imbalance comes out from a dim = 4 GL vertex (4)

$$\Delta\mathcal{L}_4(\mu_5) = l_4 \mu_5^2 \left\langle \chi^\dagger U + \chi U^\dagger \right\rangle . \quad (12)$$

The modifications of the chiral Lagrangian in (11) and (12) change the coefficients in dispersion law differently in energy $p^0$ and three-momentum $|\vec{p}|$ as well as modify the mass shell for pions

$$\begin{aligned} K\left(p^2\right)(\mu_5) &= \left(F_0^2 + 32\mu_5^2(l_1 + l_2)\right) p_0^2 \\ &\quad - \left(F_0^2 + 16\mu_5^2(l_1 + l_2)\right) |\vec{p}|^2 - \left(F_0^2 + 4 l_4 \mu_5^2\right) m_\pi^2 \to 0 \,. \end{aligned} \quad (13)$$

The empirical values of the SU(2) GL SU(2) constants [5] normalized at the RG scale $\mu \simeq M_\pi \simeq 140$ MeV, $\log(m_\pi/\mu) \simeq 0$ are

$$\begin{aligned} l_1^{\rm r} &= (-0.4 \pm 0.6) \times 10^{-3} \,, \qquad & l_2^{\rm r} = (8.6 \pm 0.2) \times 10^{-3} \,, \\ l_1^{\rm r} + l_2^{\rm r} &= (8.2 \pm 0.8) \times 10^{-3} \,, \qquad & l_4^{\rm r} = (2,64 \pm 0.01) \times 10^{-2} \,. \end{aligned} \quad (14)$$

Hence, the distortion of mass shell can be detected in decays of charged pions when the effective pion mass approaches muon mass which may happen if $\mu_5 \sim F_\pi$.

## 3. Linear sigma model for light pions and scalar mesons in the presence of chiral imbalance

Let us compare these constants with the ones estimated from the sigma model built in [4, 8]. The sigma model was built with realization of SU(2) chiral symmetry to describe pions and isosinglet and isotriplet scalar mesons. Its Lagrangian reads

$$\begin{aligned} \mathcal{L} &= N_c \left\{ \frac{1}{4} \left\langle D_\mu H \, (D^\mu H)^\dagger \right\rangle + \frac{B_0}{2} \left\langle m \left( H + H^\dagger \right) \right\rangle + \frac{M^2}{2} \left\langle H H^\dagger \right\rangle \right. \\ &\quad \left. - \frac{\lambda_1}{2} \left\langle \left( H H^\dagger \right)^2 \right\rangle - \frac{\lambda_2}{4} \left\langle \left( H H^\dagger \right) \right\rangle^2 + \frac{c}{2} \left( \det H + \det H^\dagger \right) \right\} , \end{aligned} \quad (15)$$

where $H = \xi \, \Sigma \, \xi$ is an operator for meson fields, $N_c$ is a number of colours, $m$ is an average mass of current $u, d$ quarks, $M$ is a "tachyonic" mass generating the spontaneous breaking of chiral symmetry, $B_0, c, \lambda_1, \lambda_2$ are real constants.

The matrix $\Sigma$ includes the singlet scalar meson $\sigma$, its vacuum average $v$ and the isotriplet of scalar mesons $a_0^0, a_0^-, a_0^+$, for the details see in [4, 8]. The covariant derivative of $H$ including the chiral chemical potential $\mu_5$ is defined in (1). The operator $\xi$ realizes a nonlinear representation (see (2)) of the chiral group $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}$, namely, $\xi^2 = U$. From spectral characteristics of scalar mesons in vacuum, one fixes the Lagrangian parameters, $\lambda_1 = 16.4850$, $\lambda_2 = -13.1313$, $c = -4.46874 \times 10^4\ \mathrm{MeV}^2$, $B_0 = 1.61594 \times 10^5\ \mathrm{MeV}^2$.

The change of pion coupling constant $F_0 \simeq v$ is determined by potential parameters as compared to the ChPT definition

$$\Delta F_\pi^2\left(\mu_5^2\right) = \frac{1}{\lambda_1 + \lambda_2} \approx 0.3 \quad vs. \quad 32(l_1 + l_2) \approx 0.26\,. \tag{16}$$

It is quite a satisfactory correspondence. It makes it plausible that the above low-energy GL operators with chiral imbalance are saturated by the isotriplet scalar meson exchange.

Analogously, in the rest frame, using the pion mass correction, $m_\pi^2(\mu_5)$ $F_\pi^2(\mu_5) \simeq 2m_q B F_\pi(\mu_5)$, it is easy to find the estimation for

$$l_4 \approx 2,64 \times 10^{-2} \quad vs. \quad \frac{1}{8(\lambda_1 + \lambda_2)} \approx 3.8 \times 10^{-2}\,, \tag{17}$$

wherefrom one can also guess the relation $2(l_1 + l_2) \sim l_4$ following from the linear sigma model. We notice that the above relation is one-loop RG invariant.

## 4. Conclusions and prospects

We have found a convincing correspondence between the ChPT prognosis and the linear sigma model predictions for the dispersion law of distorted mass shell for pions under influence of chiral imbalance. The immediate consequence of such a distortion is a suppression of charged pion decay into muon and neutrino with increasing of chiral chemical potential. In the limiting case, this decay channel may even become closed.

A manifestation of LPB in the presence of chiral imbalance in the sector of $\rho$ and $\omega$ vector mesons [3] can also happen and, in this case, the Chern–Simons interaction plays the main role. It turns out [2] that the spectrum of massive vector mesons splits into three components with different polarizations having different effective masses $m^2_{\mathrm{V},+} < m^2_{\mathrm{V,L}} < m^2_{\mathrm{V},-}$.

We also draw attention to the recent proposal to measure the photon polarization asymmetry in $\pi\gamma$ scattering [4, 9] as a way to detect LPB due to chiral imbalance. It happens in the ChPT including electromagnetic fields due to the Wess–Zumino–Witten operators.

The independent check of our estimates could be done by lattice computation (see [10] for inspiration).

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