

NEW APPROACH IN KNOWLEDGE OF $a_\mu^{(\text{LO})\text{had}}$ VALUE TO THE MUON $g - 2$ ANOMALY*

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Recently, a Pavia–Padova–Parma–Frascati group of theoreticians has suggested a novel approach to determine the leading order of hadronic contribution $a_\mu^{(\text{LO})\text{had}}$ to the muon $g - 2$ anomaly, consisting in a measurement of the running QED fine structure constant in the space-like region by the Bhabha $\mu e \rightarrow \mu e$ scattering at CERN and an extraction of $\Delta\alpha_{\text{had}}^{(5)}(s)$ from the latter, to be crucial in determination of $a_\mu^{(\text{LO})\text{had}}$. In this contribution, it is demonstrated how, by one elaborated Unitary and Analytic model of electromagnetic structure of hadrons, one can predict behavior of $\Delta\alpha_{\text{had}}^{(5)}(s)$ before measurements carried out at CERN.

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1. Introduction

The anomalous magnetic moment of the muon μ^- , $a_\mu = \frac{g-2}{2}$, to be measured and simultaneously evaluated theoretically, provides an extremely clean test of the Standard Model (SM) of elementary particle physics. Therefore, it is constantly important to achieve in its theoretical evaluation the inequality $(a_\mu^{\text{exp}} - a_\mu^{\text{th}}) < \Delta(a_\mu^{\text{exp}} - a_\mu^{\text{th}})$ between differences of the experimental and theoretical values and their evaluated errors. Dominant sources of the total uncertainties in theoretical predictions of a_μ is the Leading Order hadronic contribution $a_\mu^{(\text{LO})\text{had}}$ represented by the vacuum-polarization diagram in Fig. 1, the contribution of which is given [1] by the sum of three dispersion integrals

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$$a_{\mu}^{(\text{LO})\text{had}} = \frac{\alpha^2(0)}{3\pi^2} \left(\int_{m_{\pi^0}^2}^{s_{\text{cut}}} \frac{ds}{s} \frac{\sigma_{\text{tot}}(e^+e^- \rightarrow \text{had})}{\sigma_{\text{tot}}(e^+e^- \rightarrow \mu^+\mu^-)} K(s) + \int_{s_{\text{cut}}}^{s_{\text{pQCD}}} \frac{ds}{s} R^{\text{data}}(s) K(s) + \int_{s_{\text{pQCD}}}^{\infty} \frac{ds}{s} R^{\text{pQCD}}(s) K(s) \right), \quad (1)$$

where s_{cut} is usually taken around the value of 2–4 GeV², in which highly fluctuating (due to hadronic resonances and threshold effects) $\sigma_{\text{tot}}(e^+e^- \rightarrow \text{had})$ in the first integral is changed to smoother one in the second integral, the $\sigma_{\text{tot}}(e^+e^- \rightarrow \mu^+\mu^-) = \frac{4\pi\alpha^2(s)}{3s}$, $K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)\frac{s}{m_{\mu}^2}}$, $R^{\text{pQCD}}(s) = 3 \sum_f Q_f^2 \sqrt{(1 - 4m_f^2/s)(1 + 2m_f^2/s)} (1 + \frac{\alpha_s(s)}{\pi} + c_1(\frac{\alpha_s(s)}{\pi})^2 + c_2(\frac{\alpha_s(s)}{\pi})^3 + \dots)$, where Q_f and m_f are the charge and mass of the quarks, respectively, $\alpha_s(s)$ the running strong coupling constant of QCD, $c_1 = 1.9857 - 0.1153N_f$, $c_2 = -6.6368 - 1.2002N_f - 0.0052N_f^2 - 1.2395(\sum Q_f)^2/(3 \sum Q_f^2)$ [2] with the number of active flavors N_f .

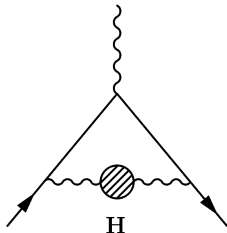


Fig. 1. The leading-order hadronic vacuum-polarization contribution to a_{μ} .

2. Running fine structure constant of QED $\alpha(s)$

The electromagnetic (EM) constant $\alpha(s)$ appearing in $\sigma_{\text{tot}}(e^+e^- \rightarrow \mu^+\mu^-)$ is the running fine structure constant of QED as a function of the energy, which can be expressed as

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha(s)} \quad (2)$$

with $\alpha(0) = 1/137.036$, whereby in $\Delta\alpha(s) = \Delta\alpha_l(s) + \Delta\alpha_{\text{had}}^{(5)}(s) + \Delta\alpha_{\text{top}}(s)$, one distinguishes contributions from leptons (e, μ, τ), from 5 light quarks u, d, c, s, b with the mass < 5 GeV and from “top” quark t with the mass ≈ 175 GeV.

The leptonic contributions are calculable in perturbation theory, where at leading order the free leptons yield

$$\Delta\alpha_l(s) = \frac{\alpha(0)}{3\pi} \sum_{l=e,\mu,\tau} \left[\ln \frac{s}{m_l^2} - \frac{5}{3} \right]. \quad (3)$$

Since the t -quark is heavy, one cannot use the light fermion approximation for it, and it behaves like

$$\Delta\alpha_{\text{top}}(s) \approx -\frac{\alpha(0)}{3\pi} \frac{4}{15} \frac{s}{m_t^2} \quad (4)$$

giving a negligible contribution to $\Delta\alpha(s)$. A serious problem is the 5 light quarks, u, d, s, c, b contribution $\Delta\alpha_{\text{had}}^{(5)}(s)$, which due to the light masses of these quarks cannot be calculated in the framework of the pQCD. Fortunately, one can evaluate it from $e^+e^- \rightarrow \text{had}$ data through the corresponding dispersion integral in time-like region

$$\Delta\alpha_{\text{had}}^{(5)}(s) = -\frac{\alpha(0)s}{3\pi} P \int_{m_{\pi_0}^2}^{\infty} ds' \frac{R(s')}{s'(s' - s - i\varepsilon)} \quad (5)$$

like in an evaluation of the Leading Order hadronic contribution (1), however, without the QED kernel function $K(s)$.

3. Novel approach in knowledge of $a_\mu^{(\text{LO})\text{had}}$ value

Let us rewrite the relation for $a_\mu^{(\text{LO})\text{had}}$ (1) in the abbreviated form

$$a_\mu^{(\text{LO})\text{had}} = \frac{\alpha^2(0)}{3\pi^2} \int_{m_{\pi_0}^2}^{\infty} \frac{ds'}{s'} R(s') \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)\frac{s'}{m_\mu^2}}. \quad (6)$$

Then, if one exchanges the x and s' integrations [3] and rearrange the function under the second integral by a suitable way, one gets

$$a_\mu^{(\text{LO})\text{had}} = \frac{\alpha^2(0)}{3\pi^2} \int_0^1 dx (1-x) \int_{m_{\pi_0}^2}^{\infty} \frac{ds'}{s'} R(s') \frac{x^2 m_\mu^2}{x^2 m_\mu^2 + s'(1-x)}, \quad (7)$$

from where the following relation

$$a_\mu^{(\text{LO})\text{had}} = \frac{\alpha(0)}{\pi} \int_0^1 dx (1-x) \frac{\alpha(0)}{3\pi} \int_{m_{\pi_0}^2}^{\infty} \frac{ds'}{s'} R(s') \frac{\frac{x^2 m_\mu^2}{x-1}}{\frac{x^2 m_\mu^2}{x-1} - s'} \quad (8)$$

follows or introducing new variable, the momentum transfer squared $t(x) = \frac{x^2 m_\mu^2}{x-1}$, to be negative in the integration interval $0 < x < 1$, one finally finds

$$a_\mu^{(\text{LO})\text{had}} = \frac{\alpha(0)}{\pi} \int_0^1 dx (1-x) \left[-\frac{\alpha(0)t(x)}{3\pi} \int_{m_{\pi 0}^2}^{\infty} ds' \frac{R(s')}{s'(s' - t(x))} \right]. \quad (9)$$

Here, the term in the angular brackets

$$\Delta\alpha_{\text{had}}^{(5)}(t(x)) = \left[-\frac{\alpha(0)t(x)}{3\pi} \int_{m_{\pi 0}^2}^{\infty} ds' \frac{R(s')}{s'(s' - t(x))} \right] \quad (10)$$

is actually the hadronic contribution $\Delta\alpha_{\text{had}}^{(5)}(t(x))$ (5) to the running fine structure constant of QED, however, now as a function of $t(x)$ to be defined in the space-like region.

So, knowing the behavior of $\Delta\alpha_{\text{had}}^{(5)}(t(x))$ from the elastic scattering of μ^- on atomic electrons of Be and C in the space-like region [4], one can, by means of the following integral

$$a_\mu^{(\text{LO})\text{had}} = \frac{\alpha(0)}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{\text{had}}^{(5)}(t(x)), \quad (11)$$

calculate the Leading Order hadronic contribution $a_\mu^{(\text{LO})\text{had}}$ to the anomalous magnetic moment of the muon a_μ .

4. Evaluation of hadronic contribution $\Delta\alpha_{\text{had}}^{(5)}(t(x))$ to $\alpha(s)$

With this aim, we decompose (10) into sum of three integrals, like in (1)

$$\Delta\alpha_{\text{had}}^{(5)}(t(x)) = -\frac{\alpha(0)t(x)}{3\pi} \left(\int_{m_{\pi 0}^2}^{s_{\text{cut}}} \frac{ds'}{s'(s' - t(x))} \frac{\sigma_{\text{tot}}(e^+e^- \rightarrow \text{had})}{\sigma_{\text{tot}}(e^+e^- \rightarrow \mu^+\mu^-)} + \int_{s_{\text{pQCD}}}^{s_{\text{pQCD}}} \frac{ds'}{s'(s' - t(x))} R^{\text{data}}(s') + \int_{s_{\text{pQCD}}}^{\infty} \frac{ds'}{s'(s' - t(x))} R^{\text{pQCD}}(s') \right), \quad (12)$$

where

$$\begin{aligned}
 \sigma_{\text{tot}}(e^+e^- \rightarrow \text{had}) &= \sigma_{\text{tot}}(e^+e^- \rightarrow \pi^+\pi^-) + \sigma_{\text{tot}}(e^+e^- \rightarrow K^+K^-) \\
 &+ \sigma_{\text{tot}}(e^+e^- \rightarrow K^0\bar{K}^0) + \sigma_{\text{tot}}(e^+e^- \rightarrow \pi^0\gamma) + \sigma_{\text{tot}}(e^+e^- \rightarrow \eta\gamma) \\
 &+ \sigma_{\text{tot}}(e^+e^- \rightarrow \eta'\gamma) + \sigma_{\text{tot}}(e^+e^- \rightarrow \pi^+\pi^-\pi^0) + \sigma_{\text{tot}}(e^+e^- \rightarrow 2\pi^+2\pi^-) \\
 &+ \sigma_{\text{tot}}(e^+e^- \rightarrow \pi^+\pi^-2\pi^0) + \sigma_{\text{tot}}(e^+e^- \rightarrow \eta\pi^+\pi^-) + \sigma_{\text{tot}}(e^+e^- \rightarrow \pi^0K^+K^-) \\
 &+ \sigma_{\text{tot}}(e^+e^- \rightarrow \pi^0K^0\bar{K}^0) + \sigma_{\text{tot}}(e^+e^- \rightarrow \pi^+K^-K^0) + \sigma_{\text{tot}}(e^+e^- \rightarrow \pi^-K^+K^0) ,
 \end{aligned} \tag{13}$$

and in order to compare obtained results with other recent evaluations, we fix integration borders to be $s_{\text{cut}} = 3.752 \text{ GeV}^2$ and $s_{\text{pQCD}} = 125.417 \text{ GeV}^2$. The first six electron–positron annihilation total cross sections in sum (13) are dominant and they can be expressed through the corresponding EM form factors (FFs) squared. Then in evaluation of their contributions to $\Delta\alpha_{\text{had}}^{(5)}(t(x))$, the Unitary and Analytic model of pseudoscalar meson EM structure, having in this approach abundant experience from the past [5–7], can be applied. With this aim, the corresponding EM FFs are split into iso-scalar and iso-vector parts as follows:

$$\begin{aligned}
 F_{\pi^\pm}(s) &= F_\pi^{I=1}[W(s)] , \\
 F_{K^\pm}(s) &= F_K^{I=0}[V(s)] + F_K^{I=1}[W(s)] , \\
 F_{K^0}(s) &= F_K^{I=0}[V(s)] - F_K^{I=1}[W(s)] , \\
 F_{\pi^0\gamma}(s) &= F_{\pi^0\gamma}^{I=0}[V(s)] + F_{\pi^0\gamma}^{I=1}[W(s)] , \\
 F_{\eta\gamma}(s) &= F_{\eta\gamma}^{I=0}[V(s)] + F_{\eta\gamma}^{I=1}[W(s)] , \\
 F_{\eta'\gamma}(s) &= F_{\eta'\gamma}^{I=0}[V(s)] + F_{\eta'\gamma}^{I=1}[W(s)] ,
 \end{aligned} \tag{14}$$

where $F^{I=1}(s)$ FFs are saturated with ρ, ρ', ρ'' and $F^{I=0}(s)$ with $\omega, \phi, \omega', \phi', \omega'', \phi''$.

The Unitary and Analytic model takes into account all known EM FFs properties as their normalization, asymptotic behavior as predicted by the quark model, analytic properties of FFs, unitarity conditions of FFs, reality conditions of FFs, and the experimental fact of a creation of vector mesons in $e^+e^- \rightarrow \text{had}$ processes by means of the saturation of FFs by experimentally confirmed vector-meson resonances as mentioned above.

Then, every $F^{I=1}[W(s)]$ and $F^{I=0}[V(s)]$ represents one analytic function in the whole complex s -plane, besides two cuts on the positive real axis, to be defined on the four-sheeted Riemann surface and depends only on physically interpretable parameters to be evaluated numerically with errors by a comparison of the model with existing experimental data [8].

Every such a contribution, calculated by using the Unitary and Analytic model, or carrying the integration through experimental data on $\sigma_{\text{tot}}(e^+e^- \rightarrow \text{had})$ in sum (13) with final-state particles more than two, to $\Delta\alpha_{\text{had}}^{(5)}(t(x))$, including also contributions from the second and third integral in (12), is given by similar curves to the presented one for the contribution of the process $e^+e^- \rightarrow \pi^+\pi^-$ in Fig. 2(left).

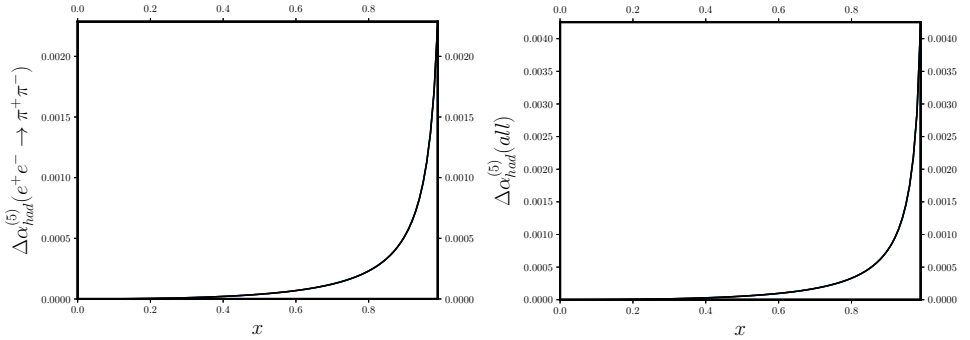


Fig. 2. Left figure: predicted curve representing contribution of $\sigma_{\text{tot}}(e^+e^- \rightarrow \pi^+\pi^-)$ to $\Delta\alpha_{\text{had}}^{(5)}(t(x))$; right figure: sum of all predicted curves by means of independent total cross sections in (13), including also curves obtained from the second and third integral to $\Delta\alpha_{\text{had}}^{(5)}(t(x))$.

The sum of all such curves obtained by (12) is given in Fig. 2(right). This curve is expected to be obtained in the measurement of the running QED fine structure constant in the space-like region by the Bhabha $\mu e \rightarrow \mu e$ scattering at CERN, and the value $a_\mu^{(\text{LO})\text{had}} = 713.80 \times 10^{-10}$ obtained through integral (11) confirms the curve in Fig. 2(right) to be correct.

5. Conclusions

We have presented a novel approach for a determination of the LO of hadronic contribution to muon $g - 2$ anomaly $a_\mu^{(\text{LO})\text{had}}$ consisting in a calculation of integral (11), if one knows $\Delta\alpha_{\text{had}}^{(5)}(s)$ in the space-like region.

The behavior of $\Delta\alpha_{\text{had}}^{(5)}(t(x))$ in the space-like region has been found in this contribution by a classical approach, and the calculation of the $a_\mu^{(\text{LO})\text{had}}$ by means of integral (11) confirms our evaluation of the $\Delta\alpha_{\text{had}}^{(5)}(t(x))$ to be correct.

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