SIGNS OF UNIVERSAL VECTOR-MESON COUPLING CONSTANTS $f_{\rho^0}, f_\omega, f_\phi$ WITH PHOTON*

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Universal vector-meson coupling constants $f_{\rho^0}, f_\omega, f_\phi$ appear in the lepton decay widths of the corresponding vector mesons in a quadratic form, therefore, in their numerical evaluation from experimental values on $\Gamma(V \to e^+e^-)$ one does not know their signs. It is demonstrated strong dependence of the signs of $f_{\rho^0}, f_\omega, f_\phi$ on the ω–ϕ mixing forms. However, by an application of the ω–ϕ mixing directly to the electromagnetic currents of ω and ϕ vector mesons and by a comparison of obtained results with the Kroll–Lee–Zumino electromagnetic current to be identified with a linear combination of the re-normalized ρ^0, ω and ϕ fields, signs of all coupling constants $f_{\rho^0}, f_\omega, f_\phi$ are specified.

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1. Introduction

The Review of Particle Physics (2016) makes known three trinities of neutral vector mesons

$$\begin{array}{lll} \rho(770)\,, & \omega(782)\,, & \phi(1020)\,, \\ \omega'(1420)\,, & \rho'(1450))\,, & \phi'(1680)\,, \\ \omega''(1650)\,, & \rho''(1700)\,, & \phi''(2170) \end{array} \tag{1}$$

to be revealed experimentally mainly in the total cross section $\sigma_{\rm tot}(e^+e^- \to$ hadrons), and their lepton decay width

$$\Gamma(V \to e^+e^-) = \frac{\alpha^2 m_V}{3}\left(\frac{f_V^2}{4\pi}\right)^{-1} \tag{2}$$

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is specified by the corresponding universal vector-meson coupling constant f_V, describing the photon–vector-meson transition in the form of $e\frac{m_V^2}{f_V}$.

Thus, if one knows the values of the corresponding universal vector-meson coupling constants numerically, one could predict the lepton width $\Gamma(V \to e^+e^-)$ of all above-mentioned vector mesons.

However, there is no theory able to predict numerical values of $f_{\rho^0}, f_\omega, f_\phi$ up to now. Therefore, we are left only with an "inverse problem", *i.e.* with evaluation of $f_{\rho^0}, f_\omega, f_\phi$ values from existing data on $\Gamma(V \to e^+e^-)$. Here, we would like to note that still experimental values on $\Gamma(V \to e^+e^-)$ of excited states of neutral vector mesons $\omega'(1420)$, $\rho'(1450)$, $\phi'(1680)$; $\omega''(1650)$, $\rho''(1700)$, $\phi''(2170)$, though very desirable, are missing (see [1]) .

Despite of this fact, our further considerations will be concerned of all ground state and excited neutral vector mesons.

Even if one knows $\Gamma(V \to e^+e^-)$ experimentally, it can provide only the absolute value, without any sign, of the corresponding f_V, as this is contained in the expression for lepton width $\Gamma(V \to e^+e^-)$ quadratically.

Nevertheless, there are physical quantities in which $f_{\rho^0}, f_\omega, f_\phi$ appear in linear form, so, their signs play a very important role.

2. Signs of universal vector-meson coupling constants $f_{\rho^0}, f_\omega, f_\phi$

Further, we demonstrate that signs of $f_{\rho^0}, f_\omega, f_\phi$ strongly depend on the applied ω–ϕ mixing forms [2]

$$\begin{aligned}
&1. \quad \omega = +\omega_8 \sin\theta + \omega_0 \cos\theta\,, && \phi = -\omega_8 \cos\theta + \omega_0 \sin\theta\,,\\
&2. \quad \omega = -\omega_8 \sin\theta + \omega_0 \cos\theta\,, && \phi = -\omega_8 \cos\theta - \omega_0 \sin\theta\,,\\
&3. \quad \omega = +\omega_8 \sin\theta - \omega_0 \cos\theta\,, && \phi = +\omega_8 \cos\theta + \omega_0 \sin\theta\,,\\
&4. \quad \omega = -\omega_8 \sin\theta - \omega_0 \cos\theta\,, && \phi = +\omega_8 \cos\theta - \omega_0 \sin\theta\,,\\
&5. \quad \omega = +\omega_8 \sin\theta + \omega_0 \cos\theta\,, && \phi = +\omega_8 \cos\theta - \omega_0 \sin\theta\,,\\
&6. \quad \omega = -\omega_8 \sin\theta + \omega_0 \cos\theta\,, && \phi = +\omega_8 \cos\theta + \omega_0 \sin\theta\,,\\
&7. \quad \omega = +\omega_8 \sin\theta - \omega_0 \cos\theta\,, && \phi = -\omega_8 \cos\theta - \omega_0 \sin\theta\,,\\
&8. \quad \omega = -\omega_8 \sin\theta - \omega_0 \cos\theta\,, && \phi = -\omega_8 \cos\theta + \omega_0 \sin\theta
\end{aligned} \tag{3}$$

from which only 1., 4., 5., 8. are physically acceptable.

The problem of $f_{\rho^0}, f_\omega, f_\phi$ signs will be made clear by using the rearranged hadronic electromagnetic (EM) current

$$J_\mu^{\rm h} = \tfrac{2}{3}\bar{u}\gamma_\mu u - \tfrac{1}{3}\bar{d}\gamma_\mu d - \tfrac{1}{3}\bar{s}\gamma_\mu s \tag{4}$$

into a sum of the ρ^0, ω, ϕ meson EM currents, then by an application of the ω–ϕ mixing directly to EM currents of ω and ϕ vector mesons and, finally, by a comparison of the obtained results with the Kroll–Lee–Zumino (KLZ) [3] hadronic EM current to be identified with a linear combination of the renormalized ρ^0, ω, ϕ fields, by means of which also $f_{\rho^0}, f_\omega, f_\phi$ coupling constants are defined.

Really, the hadronic EM current (4) can be formally arranged to the form of

$$J_\mu^{\rm h} = S_\rho \frac{1}{\sqrt{2}} J_\mu^{\rho^0} + S_\omega \frac{1}{3\sqrt{2}} J_\mu^\omega - S_\phi \frac{1}{3} J_\mu^\phi \,, \tag{5}$$

where

$$\begin{aligned} J_\mu^{\rho^0} &= \frac{1}{\sqrt{2}} \left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d \right) \,, \\ J_\mu^\omega &= \frac{1}{\sqrt{2}} \left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d \right) \,, \\ J_\mu^\phi &= \bar{s}\gamma_\mu s \end{aligned} \tag{6}$$

are ρ^0-meson, ω-meson and ϕ-meson EM currents, and S_ρ, S_ω, S_ϕ are the ρ-meson, ω-meson and ϕ-meson EM current signs, respectively.

Now, we accede to the ω–ϕ mixing. First, we write an explicit form of the ω_8 and ω_0 meson EM currents, which are showing to be useful in our further considerations

$$\begin{aligned} J_\mu^{\omega_8} &= \frac{1}{\sqrt{6}} \left(u\gamma_\mu \bar{u} + d\gamma_\mu \bar{d} - 2s\gamma_\mu \bar{s} \right) \,, \\ J_\mu^{\omega_0} &= \frac{1}{\sqrt{3}} \left(u\gamma_\mu \bar{u} + d\gamma_\mu \bar{d} + s\gamma_\mu \bar{s} \right) \,. \end{aligned} \tag{7}$$

Then, if the ω–ϕ mixing is applied directly to the ω, ϕ meson EM currents in (6), one finds

$$\begin{aligned}
&1. \quad J_\mu^\omega = +J_\mu^{\omega_8} \sin\theta + J_\mu^{\omega_0} \cos\theta \,, && J_\mu^\phi = -J_\mu^{\omega_8} \cos\theta + J_\mu^{\omega_0} \sin\theta \,, \\
&2. \quad J_\mu^\omega = -J_\mu^{\omega_8} \sin\theta + J_\mu^{\omega_0} \cos\theta \,, && J_\mu^\phi = -J_\mu^{\omega_8} \cos\theta - J_\mu^{\omega_0} \sin\theta \,, \\
&3. \quad J_\mu^\omega = +J_\mu^{\omega_8} \sin\theta - J_\mu^{\omega_0} \cos\theta \,, && J_\mu^\phi = +J_\mu^{\omega_8} \cos\theta + J_\mu^{\omega_0} \sin\theta \,, \\
&4. \quad J_\mu^\omega = -J_\mu^{\omega_8} \sin\theta - J_\mu^{\omega_0} \cos\theta \,, && J_\mu^\phi = +J_\mu^{\omega_8} \cos\theta - J_\mu^{\omega_0} \sin\theta \,, \\
&5. \quad J_\mu^\omega = +J_\mu^{\omega_8} \sin\theta + J_\mu^{\omega_0} \cos\theta \,, && J_\mu^\phi = +J_\mu^{\omega_8} \cos\theta - J_\mu^{\omega_0} \sin\theta \,, \\
&6. \quad J_\mu^\omega = -J_\mu^{\omega_8} \sin\theta + J_\mu^{\omega_0} \cos\theta \,, && J_\mu^\phi = +J_\mu^{\omega_8} \cos\theta + J_\mu^{\omega_0} \sin\theta \,, \\
&7. \quad J_\mu^\omega = +J_\mu^{\omega_8} \sin\theta - J_\mu^{\omega_0} \cos\theta \,, && J_\mu^\phi = -J_\mu^{\omega_8} \cos\theta - J_\mu^{\omega_0} \sin\theta \,, \\
&8. \quad J_\mu^\omega = -J_\mu^{\omega_8} \sin\theta - J_\mu^{\omega_0} \cos\theta \,, && J_\mu^\phi = -J_\mu^{\omega_8} \cos\theta + J_\mu^{\omega_0} \sin\theta \,.
\end{aligned} \tag{8}$$

Substituting $J_\mu^{\omega_8}, J_\mu^{\omega_0}$ forms from (7) into (8), by an explicit calculation of $J_\mu^{\omega}, J_\mu^{\phi}$ currents with the ideal mixing angle $\theta = 35.3^0$, when $\cos\theta = \sqrt{\frac{2}{3}}$ and $\sin\theta = \sqrt{\frac{1}{3}}$, one finds that only 1., 4., 5., and 8. ω–ϕ mixing forms reproduce the $J_\mu^{\omega}, J_\mu^{\phi}$ currents completely up to the sign.

The 2., 3., 6., and 7. ω–ϕ mixing forms make always to J_μ^{ω} admixture of the J_μ^{ϕ} current and to J_μ^{ϕ} admixture of the J_μ^{ω} current contributions, respectively, though the ideal mixing angle value has been used. Even more, these admixtures are dominant.

The latter is another evidence that the 2., 3., 6., and 7. ω–ϕ mixing forms are physically non-acceptable.

In order to demonstrate our previous assertions in more detail, let us present them here as follows. Substituting $J_\mu^{\omega_8}, J_\mu^{\omega_0}$ explicit forms from (7) *e.g.* into 4. of (8), one finds

$$\begin{aligned}
J_\mu^{\omega} &= -\frac{1}{3\sqrt{2}}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d - 2\bar{s}\gamma_\mu s\right) - \frac{2}{3\sqrt{2}}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d + \bar{s}\gamma_\mu s\right)\\
&= -\frac{1}{\sqrt{2}}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right)\\
&= -J_\mu^{\omega}\,, \qquad (9)\\
J_\mu^{\phi} &= \frac{1}{3}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d - 2\bar{s}\gamma_\mu s\right) - \frac{1}{3}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d + \bar{s}\gamma_\mu s\right)\\
&= -\left(\bar{s}\gamma_\mu s\right)\\
&= -J_\mu^{\phi}\,. \qquad (10)
\end{aligned}$$

Similar results are obtained from 1., 5., and 8. in (8).

Now, substituting $J_\mu^{\omega_8}, J_\mu^{\omega_0}$ explicit forms from (7) *e.g.* into 2. of (8), one finds

$$\begin{aligned}
J_\mu^{\omega} &= -\frac{1}{3\sqrt{2}}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d - 2\bar{s}\gamma_\mu s\right) + \frac{2}{3\sqrt{2}}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d + \bar{s}\gamma_\mu s\right)\\
&= +\frac{1}{3\sqrt{2}}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right) + \frac{4}{3\sqrt{2}}\left(\bar{s}\gamma_\mu s\right)\\
&= +\frac{1}{3}J_\mu^{\omega} + \frac{4}{3}\frac{1}{\sqrt{2}}J_\mu^{\phi}\,, \qquad (11)\\
J_\mu^{\phi} &= -\frac{1}{3}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d - 2\bar{s}\gamma_\mu s\right) - \frac{1}{3}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d + \bar{s}\gamma_\mu s\right)\\
&= +\frac{1}{3}\left(\bar{s}\gamma_\mu s\right) - \frac{2}{3}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right)\\
&= +\frac{1}{3}J_\mu^{\phi} - \frac{2}{3}\sqrt{2}J_\mu^{\omega}\,. \qquad (12)
\end{aligned}$$

Similar results are obtained from 3., 6., and 7. in (8).

As a result, the corresponding S_ρ, S_ω, S_ϕ signs in (5) are as follows:

$$\begin{array}{llll} 1. & S_\rho = +, & S_\omega = +, & S_\phi = +, \\ 4. & S_\rho = +, & S_\omega = -, & S_\phi = -, \\ 5. & S_\rho = +, & S_\omega = +, & S_\phi = -, \\ 8. & S_\rho = +, & S_\omega = -, & S_\phi = +, \end{array}$$

and the following four various forms of the hadronic EM current are found:

$$1. \quad J_\mu^{\mathrm{h}} = +\frac{1}{\sqrt{2}} J_\mu^{\rho^0} + \frac{1}{3\sqrt{2}} J_\mu^\omega - \frac{1}{3} J_\mu^\phi , \tag{13}$$

$$4. \quad J_\mu^{\mathrm{h}} = +\frac{1}{\sqrt{2}} J_\mu^{\rho^0} - \frac{1}{3\sqrt{2}} J_\mu^\omega + \frac{1}{3} J_\mu^\phi , \tag{14}$$

$$5. \quad J_\mu^{\mathrm{h}} = +\frac{1}{\sqrt{2}} J_\mu^{\rho^0} + \frac{1}{3\sqrt{2}} J_\mu^\omega + \frac{1}{3} J_\mu^\phi , \tag{15}$$

$$8. \quad J_\mu^{\mathrm{h}} = +\frac{1}{\sqrt{2}} J_\mu^{\rho^0} - \frac{1}{3\sqrt{2}} J_\mu^\omega - \frac{1}{3} J_\mu^\phi , \tag{16}$$

every of which depends on the applied ω–ϕ mixing form 1., 4., 5., 8. from (3) on the ω, ϕ meson EM currents in (6).

On the other hand, there is KLZ hadronic EM current to be a linear combination of the re-normalized ρ^0, ω, ϕ fields as follows:

$$\left(J_\mu^{\mathrm{h}}\right)_{\mathrm{KLZ}} = -\frac{m_{\rho^0}^2}{f_\rho} \rho_\mu^0 - \frac{m_\omega^2}{f_\omega} \omega_\mu - \frac{m_\phi^2}{f_\phi} \phi_\mu , \tag{17}$$

which is equal to the hadronic EM current J_μ^{h} expressed by means of the quark currents up to the real constant A, *i.e.*

$$\left(J_\mu^{\mathrm{h}}\right)_{\mathrm{KLZ}} = A\, J_\mu^{\mathrm{h}} . \tag{18}$$

If the latter equality is used to four various forms (13)–(16) of the hadronic EM current, dependent on the applied ω–ϕ mixing form 1., 4., 5., 8., separately, one finds relations for the reverse universal vector-meson coupling constants

$$\begin{array}{llll} 1. & -\dfrac{1}{f_\rho} = +A\,\dfrac{1}{\sqrt{2}}; & -\dfrac{1}{f_\omega} = +A\,\dfrac{1}{3\sqrt{2}}; & -\dfrac{1}{f_\phi} = -A\,\dfrac{1}{3}; \\ 4. & -\dfrac{1}{f_\rho} = +A\,\dfrac{1}{\sqrt{2}}; & -\dfrac{1}{f_\omega} = -A\,\dfrac{1}{3\sqrt{2}}; & -\dfrac{1}{f_\phi} = +A\,\dfrac{1}{3}; \\ 5. & -\dfrac{1}{f_\rho} = +A\,\dfrac{1}{\sqrt{2}}; & -\dfrac{1}{f_\omega} = +A\,\dfrac{1}{3\sqrt{2}}; & -\dfrac{1}{f_\phi} = +A\,\dfrac{1}{3}; \\ 8. & -\dfrac{1}{f_\rho} = +A\,\dfrac{1}{\sqrt{2}}; & -\dfrac{1}{f_\omega} = -A\,\dfrac{1}{3\sqrt{2}}; & -\dfrac{1}{f_\phi} = -A\,\dfrac{1}{3}; \end{array}$$

or for their ratios

$$1.\quad \frac{1}{f_\rho}:\frac{1}{f_\omega}:\frac{1}{f_\phi}=-\frac{1}{\sqrt{2}}:-\frac{1}{3\sqrt{2}}:+\frac{1}{3}\,,$$
$$4.\quad \frac{1}{f_\rho}:\frac{1}{f_\omega}:\frac{1}{f_\phi}=-\frac{1}{\sqrt{2}}:+\frac{1}{3\sqrt{2}}:-\frac{1}{3}\,,$$
$$5.\quad \frac{1}{f_\rho}:\frac{1}{f_\omega}:\frac{1}{f_\phi}=-\frac{1}{\sqrt{2}}:-\frac{1}{3\sqrt{2}}:-\frac{1}{3}\,,$$
$$8.\quad \frac{1}{f_\rho}:\frac{1}{f_\omega}:\frac{1}{f_\phi}=-\frac{1}{\sqrt{2}}:+\frac{1}{3\sqrt{2}}:+\frac{1}{3}\,.$$

Now, multiplying the right-hand side of the previous relations by $\sqrt{6}$, one obtains

$$1.\quad \frac{1}{f_\rho}:\frac{1}{f_\omega}:\frac{1}{f_\phi}=-\sqrt{3}:-\frac{1}{\sqrt{3}}:+\sqrt{\frac{2}{3}}\,,$$
$$4.\quad \frac{1}{f_\rho}:\frac{1}{f_\omega}:\frac{1}{f_\phi}=-\sqrt{3}:+\frac{1}{\sqrt{3}}:-\sqrt{\frac{2}{3}}\,,$$
$$5.\quad \frac{1}{f_\rho}:\frac{1}{f_\omega}:\frac{1}{f_\phi}=-\sqrt{3}:-\frac{1}{\sqrt{3}}:-\sqrt{\frac{2}{3}}\,,$$
$$8.\quad \frac{1}{f_\rho}:\frac{1}{f_\omega}:\frac{1}{f_\phi}=-\sqrt{3}:+\frac{1}{\sqrt{3}}:+\sqrt{\frac{2}{3}}\,.$$

Finally, if an advantage of the ideal mixing angle θ, $\sqrt{\frac{1}{3}}=\sin\theta$, and $\sqrt{\frac{2}{3}}=\cos\theta$ is taken, then the form of relations demonstrating a dependence of the universal vector-meson coupling constants signs on the ω–ϕ mixing, currently presented in the literature, is found

$$1.\quad \frac{1}{f_\rho}:\frac{1}{f_\omega}:\frac{1}{f_\phi}=-\sqrt{3}:-\sin\theta:+\cos\theta\,,$$
$$4.\quad \frac{1}{f_\rho}:\frac{1}{f_\omega}:\frac{1}{f_\phi}=-\sqrt{3}:+\sin\theta:-\cos\theta\,,$$
$$5.\quad \frac{1}{f_\rho}:\frac{1}{f_\omega}:\frac{1}{f_\phi}=-\sqrt{3}:-\sin\theta:-\cos\theta\,,$$
$$8.\quad \frac{1}{f_\rho}:\frac{1}{f_\omega}:\frac{1}{f_\phi}=-\sqrt{3}:+\sin\theta:+\cos\theta\,.$$

3. Conclusions

By a rearrangement of the hadronic electromagnetic (EM) current (4) into a sum of the ρ^0, ω, ϕ meson EM currents (6), then by an application of the ω–ϕ mixing directly to EM currents of ω and ϕ vector mesons (8) and, finally, by a comparison of the obtained results with the Kroll–Lee–Zumino hadronic EM current (17), to be identified with a linear combination of the re-normalized ρ^0, ω, ϕ fields, we have elucidated the many years' standing problem of universal vector-meson coupling constants $f_{\rho^0}, f_\omega, f_\phi$ signs.

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