LIGHT STRANGE RESONANCES FROM ANALYTICITY AND AISPERSION RELATIONS*

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In this paper, we review our recent series of works where we determine the parameters of light strange resonances using data on $\pi K \to \pi K$ and $\pi\pi \to K\bar{K}$ and model-independent dispersive methods or techniques based on complex analysis. We also advance some preliminary results on a model-independent determination of the κ or $K_0^*(700)$ resonance parameters.

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1. Introduction

Most of the information on strange resonances below 2 GeV comes from πK scattering experiments. Unfortunately, this process is only observed indirectly as a sub-process in $\pi N \to \pi K N'$ and is plagued by large systematic uncertainties leading to conflicting data sets. In addition, the parameters of these resonances are affected by large model dependencies due to the use of simple models. The most extreme case is the lightest strange resonance, the so-called κ or $K_0^*(700)$ meson, whose very existence has been the matter of intense debate for over almost four decades. Actually, even today, the $\kappa/K_0^*(700)$ "Needs Confirmation" according to the Review of Particle Physics [1].

The only mathematically rigorous and process-independent feature of a resonance is its associated pole sitting in the second Riemann sheet of the complex plane of any amplitude in which the resonance appears. The pole

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position is related to the mass and width of the resonance by $\sqrt{s_{\rm pole}} \simeq M_{\rm R} - i\Gamma_{\rm R}/2$. When the resonance is narrow, this pole lies near the real or physical axis and, if the resonance is isolated from other resonances or analytic structures, it is seen in experiment in the form of a characteristic peak. In such cases, simple models that describe the data in the vicinity of the peak, like the familiar Breit–Wigner formula, can provide good approximations to the pole position. However, when resonances are wide, or overlap with other resonances, or lie near threshold cuts and other analytic or dynamical structures, simple models become unreliable to determine the pole position or even the very existence of a resonance.

All those problems can be overcome by taking rigorously into account the analytic structure of the singularities that appear in amplitudes. These give rise, through Cauchy's Integral Formula, to integral relations known as dispersion relations. In this paper, we review our recent use of such dispersion relations as constrains to obtain parameterizations of $\pi K \to \pi K$ and $\pi\pi \to K\bar{K}$ data that can be use as input later to obtain model-independent determinations of strange resonances.

2. Our series of works

Hence, over a series of works, we have followed our aim of determining the existence and parameters of strange resonances from the existing data avoiding model dependencies. Thus, in [2], we first obtained simple unconstrained fits to $\pi K \to \pi K$ data on S, P, D, F partial waves up to 1.8 GeV, paying particular attention to systematic uncertainties, and showed that they lead to inconsistencies with Forward Dispersion Relations (FDR). However, we were able to provide a set of Constrained Fits to Data (CFD) that satisfies a complete isospin set of FDR up to 1.6 GeV.

Unfortunately, FDRs do not provide a continuation to the complex plane for partial waves. Nevertheless, we made use of a powerful technique [3] based on the convergence on the complex plane of series of Padé approximants built from information on the real (physical) axis. The relevance of this method is that it does not rely on a specific parameterization choice for the resonance pole, thus avoiding such a model dependence. For instance, it does not assume that the residue is fixed by a Breit–Wigner-like formula once the pole position is known, as it was previously done in most studies of strange resonances below 2 GeV. Of course, the drawback is that we cannot calculate an infinite series of Padés, but some truncation is needed. Nonetheless, the pole determination becomes rather unstable with just a few Padés and that effect becomes a systematic uncertainty. The other important feature of this method is that it can be used both in the elastic, and more importantly, the inelastic region. Thus, the final result for the poles we obtained in [4] using this method for πK scattering in the inelastic region are shown in Fig. 1. Hollow symbols represent Breit–Wigner-like parameterizations and solid symbols stand for the T-matrix poles. For references, see the RPP [1]. Most of the spread in previous values is due to model dependence that can be avoided with the Padé sequence method.

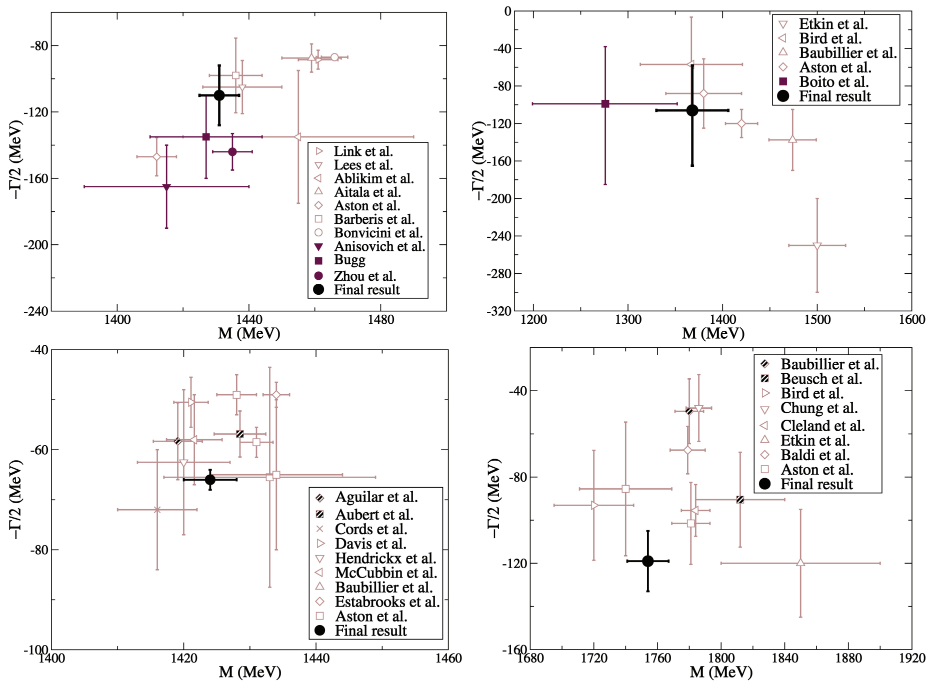

Fig. 1. From left to right and top to bottom, pole positions of the $K_0^*(1430)$, $K_1^*(1410)$, $K_2^*(1430)$, $K_3^*(1780)$ obtained from fits to data constrained with Forward Dispersion Relations and a sequence of Padé approximants for the analytic continuation to the complex plane. We plot the determinations listed in the RPP (see references therein) and the final result comes from [4]. Figures are from [4].

In the elastic region, even our simple CFD parameterization, which is constructed piecewise, yields a fairly reasonable pole when extrapolated naively to the complex plane. It is labelled Conformal CFD in Fig. 2. Of course, that relies on a particular parameterization and has some model dependence. In the figure, it is particularly evident that this resonance, being very wide, has an associated pole very deep in the complex plane. As a consequence, there is a big deviation when using inappropriately the Breit–Wigner parameterization or any of its variants (hollow symbols), which rely on the narrow resonance approximation. More sound T-matrix pole determinations use analytic or dispersive methods (solid symbols) that may also include chiral symmetry constraints (Adler zeros at least or some matching with Chiral Perturbation Theory). Actually, we also show the best dispersive determination obtained in [5]. This is a very rigorous analysis using partial-wave hyperbolic dispersion relations to continue to the complex plane a numerical solution (not a fit to data) of the Roy–Steiner equations. Despite this rigorous result the RPP still considered that the κ, still called $K_0^*(800)$ in 2016, "Needs Confirmation".

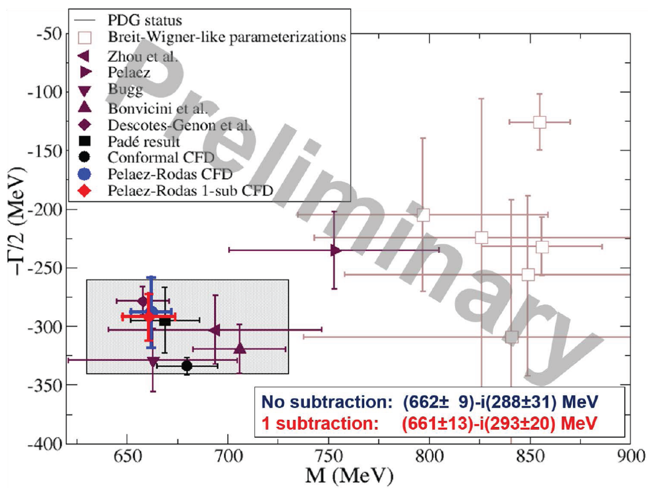

Fig. 2. (Colour on-line) Preliminary result for the $\kappa/K_0^*(700)$ pole from a Roy–Steiner analysis of a constrained fit to data. For comparison, we show the T-matrix poles listed in the RPP [1] (see references therein). The grey rectangle corresponds to the present uncertainty estimated in the RPP [1]. The "Pelaez–Rodas" poles and the values quoted in the inset rectangle are still preliminary.

Remarkably, we have also shown in [4] that the Padé technique described above yields a pole for the controversial $\kappa/K_0^*(700)$ when using as input our constrained fits to data in [2]. This one is labelled "Padé result" in Fig. 2. The fact that this result, with its very reduced model dependence, agrees so well with the dispersive prediction of [5] leads to the κ changing its name at the 2018 RPP revision from $K_0^*(800)$ to its present denomination $K_0^*(700)$. However, even with this additional piece of evidence, it still "Needs Confirmation" in the 2018 RPP [1].

Incidentally, we showed in [6] that using our CFD or Padé pole position as the only input for a dispersive representation of the Regge trajectory, the resulting slope of the $\kappa/K_0^*(700)$ trajectory does not come out linear with respect to the mass squared and has a magnitude much smaller that of ordinary mesons. This is an additional model-independent piece of evidence supporting the non-$q\bar{q}$-dominant nature of the $\kappa/K_0^*(700)$ and, therefore, of the light scalar-meson nonet. This is a consequence not only of being a wide resonance, but of its pole residue (*i.e.* its coupling to πK), being related to the mass and width differently than for ordinary resonances (generically well-described with simple Breit–Wigner-like formulas).

Thus, in order to provide the needed confirmation for the $\kappa/K_0^*(700)$, two of us are presently finishing an analysis of fits to data constrained with partial-wave hyperbolic and fixed-t dispersion relations up to $\simeq$1 GeV [8]. The use of dispersion relations takes into account correctly all analytic structures in πK partial waves, which are shown in Fig. 3. In that figure, it is shown that in terms of the Mandelstam variable s, which is the relevant one for analyticity arguments, the distance of the κ pole to the data in its nominal mass region is similar to the distance to threshold, to the Adler zero (a Chiral Symmetry requirement), or to the circular and left cuts. Thus, for a rigorous and precise $\kappa/K_0^*(700)$ pole determination, the contributions for those structures are relevant. They can be correctly calculated using dispersion relations with crossing built in, called the Roy–Steiner equations.

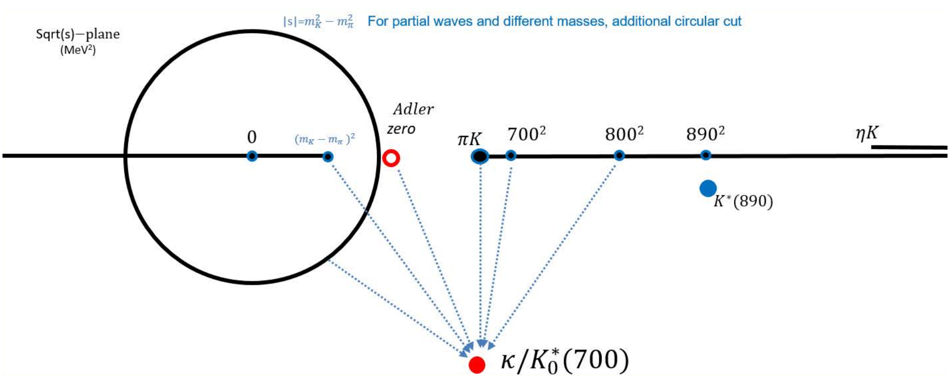

Fig. 3. (Colour on-line) Cut structure of the πK partial waves. Note that the pole associated to the $K^*(892)$ resonance in the lower half of the complex plane (in light grey/blue) is very close to the real axis at its nominal mass. In contrast, the pole of the $\kappa/K_0^*(700)$ (in black/red) is very deep in the complex plane and as close to its nominal mass region in the real axis as to the threshold, the Adler zero, or the circular and left cuts. A precise determination of the $\kappa/K_0^*(700)$ requires a careful calculation of such effects.

Let us emphasize that in our present analysis we are using data in the elastic region of the S-, P-, D-, F-waves and dispersion relations are used as constraints, in contrast to [5] who found solutions for S- and P-waves only but without input from data on those very same waves in the elastic region. In order to complete this analysis, we have first obtained constrained fits to data for $\pi\pi \to K\bar{K}$ scattering [7]. Actually, we showed that some of the data in the literature, as well as unconstrained fits to data, fail to satisfy hyperbolic dispersion relations. But once again, we provided Constrained Fits to $\pi\pi \to K\bar{K}$ Data (CFD), consistent with the dispersive representation, while describing data up to 1.47 GeV. Our fits extend up to 2 GeV, but we showed that 1.47 GeV is the maximum applicability region of hyperbolic dispersion relations.

In order to ensure that our work provides the rigorous confirmation needed to finally settle the $\kappa/K_0^*(700)$ discussion, we have included the following improvements in our calculation. In particular, our isospin 1/2 P-wave describes the existing data, our $\pi\pi \to K\bar{K}$ input has associated uncertainties and satisfies the Roy–Steiner representation, our $\pi K \to \pi K$ input satisfies FDRs up to 1.6 GeV, we have improved the Pomeron determination to be consistent with factorization of kaon–nucleon data. We have also imposed partial-wave hyperbolic dispersion relations on data fits. In addition, we have constrained the fits and calculated the pole with both one or no subtractions for the asymmetric amplitude (only the subtracted one was used before). All in all, we show our preliminary results for the $\kappa/K_0^*(700)$ pole in Fig. 2. It is remarkable to see that both determinations from one or no-subtractions yield remarkably consistent poles, also with our Padé result and with the previous Roy–Steiner prediction. Therefore, we think our work provides the confirmation needed by the RPP to finally settle the existence and parameters of the $\kappa/K_0^*(700)$ meson and complete the members of the light scalar nonet.

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