NUCLEI USING TOPOLOGICAL SOLITONS: SKYRMIONS AND RHO MESONS*

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The Skyrme model is a low-energy effective field theory of strong interactions where nuclei and baryons appear as topological solitons, more concretely as collective excitations of pionic degrees of freedom. In the last years, there has been a revival of Skyrme's ideas and new related models have been proposed to overcome two of the main drawbacks of the theory, namely, the too large binding energies and the lack of cluster structures. In this paper, we shortly review how to address both issues by extending the standard Skyrme model with the inclusion of the rho meson and how important the pion mass contribution is.

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1. Introduction

The study of strong interactions is a well-established subject due to Quantum Chromodynamics (QCD). Although an accurate insight has been achieved in the high-energy limit thanks to perturbative techniques, important phenomena also occur at low energies. This is the realm of fields such as nuclear physics. To deal with the difficulties arising from a direct derivation from QCD, two main approaches are usually taken: lattice QCD and effective field theories. This paper takes the second path to obtain a systematic description of light nuclei by means of topological solitons [1], which are particle-like solutions of a non-linear field theory where their stability arises from a topological twist or winding.

More concretely, we will focus on Skyrmions. Named after the British physicist Tony Skyrme who proposed them in the sixties of last century [2], they appear as collective excitations of a pionic field, with their topological charge identified with the baryon number of nuclei. Skyrme's ideas drew

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some attention [3, 4] after being supported by the discovery that in the large-N_{C} limit of QCD an effective field theory of mesons arises [5], and a revival with an intense activity in the field has taken place in the last decades [6]. However, there are also two important drawbacks, namely, unphysical high binding energies and the lack of clustering structures. In the following, we will see how both issues can be overcome by the inclusion of the lightest of vector mesons, the rho meson.

2. The model

The Lagrangian of the Skyrme model in dimensionless units is given by

$$\mathscr{L}_{\mathrm{Sk}} = \int \left[-\frac{c_1}{2}\mathrm{Tr}(R_\mu R^\mu) + \frac{c_2}{16}\mathrm{Tr}([R_\mu, R_\nu][R^\mu, R^\nu]) - m^2\frac{c_1^2}{c_2}\mathrm{Tr}(1-U) \right] \mathrm{d}^3x\,, \tag{1}$$

with $R_\mu = \partial_\mu U U^\dagger$ the right invariant current of the Skyrme field $U \in \mathrm{SU}(2)$ which can be parametrised as $U = \sigma + \boldsymbol{\pi}\cdot\boldsymbol{\tau}$, with $\boldsymbol{\pi} = (\pi_1, \pi_2, \pi_3)$ the triplet of pionic fields, σ an auxiliary field such that $\sigma^2 + \pi_1^2 + \pi_2^2 + \pi_3^2 = 1$, and τ_i the corresponding Pauli matrix.

The first term corresponds to the sigma model, while the second one, quartic in first derivatives and known as Skyrme term, is needed to circumvent Derrick's theorem so stable solutions can exist. The last contribution, a potential, gives the mass to the pions. Sometimes it is neglected and we can talk about the massless Skyrme model. In the usual Skyrme units, $c_1 = c_2 = 1$; however, for convenience with the extended model introduced below, we will consider $c_1 = 0.141$ and $c_2 = 0.198$. Note that the theory has only two parameters corresponding to the energy and length scales. By following the usual practice of relating them to the properties of the proton and its excited state, the delta baryon [4], the physical value of the pion mass appearing in the Lagrangian is $m = 0.526$.

The topological charge comes from the integral of the baryon density,

$$B = \int \frac{1}{24\pi^2}\epsilon_{ijk}\mathrm{Tr}(R_i R_j R_k)\,\mathrm{d}^3x\,, \tag{2}$$

and is identified with the baryon number (or mass number A).

Recently, it has been shown that the static energy functional of this massless version can emerge from a pure Yang–Mills theory in one higher dimension via dimensional deconstruction, coupled to an infinite tower of vector mesons [7]. As a consequence, the BPS energy bound (a lower topological energy bound) corresponding to self-dual instantons also takes place, $E \geq 2\pi^2|B|$, even when the tower is truncated. Restricting ourselves just to the lightest vector meson, the rho meson, the total energy presents three different contributions: $E = E_{\mathrm{Sk}} + E_\rho + E_{\mathrm{int}}$, where the first term is the static energy of the Lagrangian in Eq. (1), and E_ρ and E_{int} are the meson and interaction energy, respectively,

$$\begin{aligned} E_\rho &= \int -\mathrm{Tr}\Big\{\tfrac{1}{8}(\partial_i\rho_j - \partial_j\rho_i)^2 + \tfrac{1}{4}m_\rho^2\rho_i^2 + c_3(\partial_i\rho_j - \partial_j\rho_i)[\rho_i,\rho_j] \\ &\quad + c_4[\rho_i,\rho_j]^2\Big\}\mathrm{d}^3x\,, \qquad (3)\\ E_{\mathrm{int}} &= \int -\mathrm{Tr}\Big\{c_5([R_i,\rho_j]-[R_j,\rho_i])^2 - c_6[R_i,R_j](\partial_i\rho_j - \partial_j\rho_i) \\ &\quad - c_7[R_i,R_j][\rho_i,\rho_j] + \tfrac{1}{2}c_6[R_i,R_j]([R_i,\rho_j]-[R_j,\rho_i]) \\ &\quad - \tfrac{1}{8}([R_i,\rho_j]-[R_j,\rho_i])(\partial_i\rho_j - \partial_j\rho_i) \\ &\quad - \tfrac{1}{2}c_3([R_i,\rho_j]-[R_j,\rho_i])[\rho_i,\rho_j]\Big\}\mathrm{d}^3x\,, \qquad (4) \end{aligned}$$

with ρ_i the three $\mathfrak{su}(2)$-valued rho-meson fields. The values of the constants are $m_\rho = 1/\sqrt{2}$, $c_3 = 0.153$, $c_4 = 0.050$, $c_5 = 0.038$, $c_6 = 0.078$ and $c_7 = 0.049$, which come directly from the Yang–Mills theory by integrating out the additional dimension [7]. It is worth emphasising that besides the coupling constants, also the possible interactions among the Skyrme field and rho meson contributing to the energy functional are inherited from such a theory.

3. Results

The results of the theory with energy given by equations (1), (3) and (4) have been studied in detail in [8]. Due to the complexity of the functional to minimise, highly-demanding numerical methods have been carried out through parallel computations on a high-performance computing cluster. The simulation scheme used was a second-order time dynamics with a fourth-order Runge–Kutta method. Fourth-order accurate finite-difference approximations were considered for spatial derivatives in a cubic lattice of 128^3 points with a lattice spacing $\Delta x = 0.08$ and a time step $\Delta t = 0.02$. The values at the boundaries are $U = 1$ for the Skyrme field and $\rho_i = 0$ for the vector meson.

The computed Skyrmions in the extended theory with rho mesons (image (c)) are shown in Fig. 1 together with those from the standard model of pions (image (a)). The colour scheme is based on the parametrisation of the SU(2)-valued field U and encodes the dominant pionic field π_i and its sign according to the displayed colours. In subfigure (b), we have a comparison of the mass per nucleon within both Skyrme models and the corresponding experimental nuclear data in units of the single Skyrmion and the proton mass, respectively. It is clear here how the addition of the rho mesons significantly improves the too large binding energies of the standard Skyrme model lowering them down closer to the corresponding experimental values.

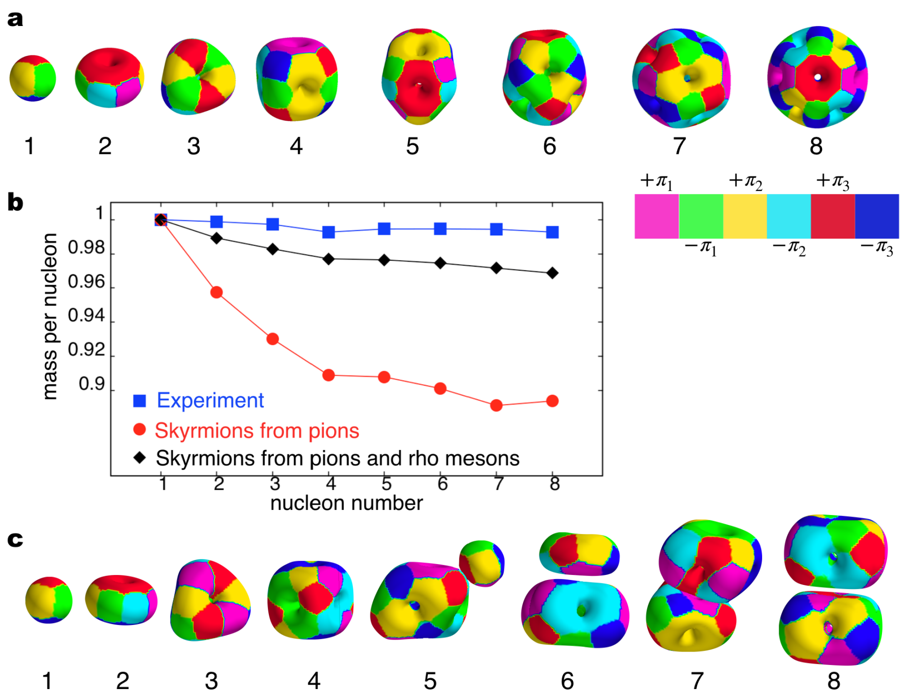

Fig. 1. (Colour on-line) Comparison of the standard Skyrme model and the extended version with rho mesons. (a) Baryon density isosurfaces for mass number up to 8 in the standard version; (b) Mass per nucleon in units of the single Skyrmion (proton) mass in the Skyrme model of pions (red circles) and in the extended version with pions and rho mesons (black diamonds), compared to experimental data (blue squares); (c) Baryon density isosurfaces for mass number from 1 to 8 in the Skyrme model with rho mesons.

On the other hand, the other main issue of the original theory is also met. As commented before, the solutions are usually too symmetric and lack a clustering structure (see Fig. 1 (a)). This is good for baryon number up to four, allowing to reproduce the spin and isospin states found in nature. However, for larger values of the mass number, light nuclei appear in clusters, where for instance, the alpha particle plays an important role. From our results in Fig. 1 (c), we see that this is achieved once the rho-meson contribution is taken into account but keeping, at the same time, the desired old symmetries for the first four Skyrmions. For mass number $A = 5$, we see that the obtained configuration corresponds to an alpha particle and a neutron modelling the ^{5}He. Indeed, it is easy to see that their constituents are nothing but the slightly deformed Skyrmions with mass number one and four.

Similarly, we find the Skyrmions with $A = 6$ and $A = 7$. These agree with nuclei ^{6}Li and ^{7}Li made of an alpha particle and a deuteron (^{2}H) or triton (^{3}H), respectively. The last solution shown is an example of the well-known α-particle clusters, where the combination of two alpha particles is reproduced by the Skyrmion with $A = 8$ corresponding to the ^{8}Be nucleus. In addition, another important success is the next α-cluster, the ^{12}C (see [8] for details). Its ground state together with the excited Hoyle state are also well-reproduced. In this case, an arrangement of 3 α-particles with triangular clustering is obtained for the ground state, whereas the Hoyle state is given by a linear chain. Let us mention that these configurations concerning carbon-12 are achieved in the standard massive Skyrme model too, but for a large value of the pion mass [9].

Finally, it is important to note that the inclusion of the pionic mass term is crucial for the results achieved. Indeed, it has been shown that the massless version of the model, despite decreasing the binding energies in a considerable way, cannot help with the clustering existing in nature [10].

4. Conclusions

The Skyrme model is an effective field theory of QCD where baryons appear as collective excitations of a pionic field. Despite its success in the past years as a bridge from the fundamental theory of strong interactions to nuclear physics, some important problems were still in place. In this work, we have shown how by adding a vector meson to the theory in a novel way (through a dimensional deconstruction of a pure Yang–Mills theory in one higher dimension), the two main issues with the model can be mitigated and small binding energies closer to their experimental values and clustering in light nuclei can be achieved. On the other hand, it is not expected to accurately reproduce experimental data just with the inclusion of the rho meson and, indeed, we have seen how important can be the contribution of heavier mesons. However, these are important results encouraging a further study to boost the model pursuing a precise description of nuclei.

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