SPACE-BASED GRAVITATIONAL WAVE OBSERVATIONS IN THE MID-BAND FREQUENCY REGION*

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The intermediate Gravitation Wave (GW) frequency detection band, *i.e.* the frequency region that is in between those accessible by space-based interferometers with million-kilometer armlengths and ground-based detectors, could be unveiled relatively soon. A space-based GW interferometer of arm-length equal to about 100,000 km would achieve an optimal sensitivity within an observational frequency band that is "blue-shifted" with respect to those of the LISA and TaiJi missions by about a factor 30. By opening the mid-band GW frequency region, such a mission would complement the scientific capabilities of both ground- and million-kilometer arm-length detectors and provide an enhanced scientific return over those obtainable by each detector operated as stand-alone.

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1. Introduction

The first direct observation of a GW signal announced by the LIGO project [1] on February 11, 2016 [2] represents one of the most important achievements in experimental physics today. By simultaneously measuring and recording strain data with two interferometers at Hanford (Washington) and Livingston (Louisiana), scientists were able to reach an extremely high level of detection confidence and infer unequivocally the GW source of the observed signal to be a coalescing binary system containing two black holes

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of masses $M_1 = 36^{+5}_{-4}\ M_\odot$ and $M_2 = 29^{+4}_{-4}\ M_\odot$ out to a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$ (the above uncertainties being at the 90 percent confidence level).

The direct observation of this GW event marks the beginning of GW astronomy [3], an historic moment comparable in magnitude to the early astronomical observations made in the year 1610 by Galileo Galilei [4]. Very much like Galileo then, we have just started to explore the capabilities of our new observational tool, which promises to unveil secrets of the Universe inaccessible by any other means.

Since the first detection announcement in 2016, several other GW signals have been observed by the LIGO–VIRGO Collaboration [1, 5]. Ground-based observations inherently require the use of multiple detectors widely separated on Earth and operating in coincidence. This is because a network of GW interferometers operating at the same time can discriminate a GW signal from random noise and provide enough information for reconstructing the source's sky-location, luminosity distance, mass(es), dynamic time scale and other observables [6, 7].

Space-based interferometers, on the other hand, have enough data redundancy to validate their measurements and uniquely reconstruct an observed signal with their six links along their three-arms [8, 9]. A mission such as LISA[1] or TaiJi [10], with their three operating arms, will be able to assess their measurements' noise levels and statistical properties over their entire observational frequency band. By relying on a time-delay interferometric (TDI) measurement [8] that is insensitive to GWs [11], space-based interferometers will assess their in-flight noise characteristics in the lower part of the band, *i.e.* at frequencies smaller than the inverse of the round-trip-light time. At higher frequencies instead, where they can synthesize three independent interferometric measurements, they will be able to perform a data consistency test by relying on the null-stream technique [7, 12, 13], *i.e.* a parametric non-linear combination of the TDI measurements that achieves a pronounced minimum at a unique point in the search parameter space when a signal is present. In addition, by taking advantage of the Doppler and amplitude modulations introduced by the motion of the array around the Sun on long-lived GW signals, space-based interferometers will measure the values of the parameters associated with the GW source of the observed signal [9].

Although a space-based array such as LISA and TaiJi can synthesize the equivalent of four interferometric (TDI) combinations (the Sagnac TDI combinations $(\alpha, \beta, \gamma, \zeta)$, for instance) [8], their best sensitivity levels are

[1] Over the years, several LISA mission designs were considered, each resulting in a different mission sensitivity. In this article, we will refer to LISA as being characterized by an arm-length of 2.5 Mkm, an acceleration noise of 3.0×10^{-15} m/s^2 (Hz)$^{-1/2}$ and a high frequency noise of 12 pm/$\sqrt{\text{Hz}}$.

achieved only over a relatively narrow region of the mHz frequency band. At frequencies lower than the inverse of the round-trip light time, the sensitivity of a space-based GW interferometer is determined by the level of residual acceleration noise associated with the nearly free-floating proof-masses of the onboard gravitational reference sensor and the size of the arm-length. In this region of the accessible frequency band, the magnitude of a GW signal in the interferometer's data scales in fact linearly with the arm-length. At frequencies higher than the inverse of the round-trip light time instead, the sensitivity is primarily determined by the photon-counting statistics at the photo-detectors [14]. The sensitivity in this part of the accessible frequency band grows linearly with the arm-length because shot-noise is inversely proportional to the square-root of the received optical power and the GW signal no-longer scales with the arm-length. From the above considerations, we may conclude that, for a given performance of the onboard science instrument and optical configuration, the best sensitivity level and the corresponding bandwidth over which it is reached are uniquely determined by the size of the array.

The best sensitivity level of a space-based interferometer and the corresponding bandwidth over which it is reached become particularly important when considering signals sweeping upwards in frequency such as those emitted by coalescing binary systems containing black-holes. As pointed out by Sesana [15], it is theoretically expected that a very large ensemble of coalescing binary systems, with masses comparable to those of GW150914, will have characteristic wave's amplitudes that could be observable by both LISA and TaiJi over an accessible frequency region from about 1.5×10^{-2} Hz to about 7.6×10^{-2} Hz. The lower frequency limit corresponds to the assumption of observing a GW150914-like signal for a period of five years (approximately equal to its coalescing time). The upper limit instead corresponds to the value at which the signal's amplitude equals the interferometer's sensitivity, in this case that of LISA. Although one could in principle increase the size of the optical telescopes and rely on more powerful lasers so as to increase the upper frequency cut-off to enlarge the observational bandwidth, in practice, pointing accuracy and stability requirements together with the finiteness of the onboard available power would result in a negligible gain.

A natural way to broaden the mHz band, so as to fill the frequency gap between the region accessible by LISA and TaiJi and that by ground interferometers, is to fly an additional interferometer of smaller arm-length. An interferometer such as the geosynchronous Laser Interferometer Space Antenna (gLISA) [14, 16, 17] or the TianQin interferometer mission under study in China [18] could naturally accomplish this scientific objective. gLISA in particular, which was analyzed for about eight years by a collaboration of scientists and engineers at the Jet Propulsion Laboratory, Stanford University, University of California San Diego, the National Institute for

Space Research (INPE, Brazil), and Space Systems Loral, was shown to fit the cost limits of the NASA astrophysics probe class mission program. With an arm-length of about 7.3×10^4 km, it could achieve a shot-noise limited sensitivity in the higher region of its accessible frequency band that is about a factor of 35 better than that of LISA[2]; gLISA will display a minimum of its sensitivity in a frequency region that perfectly complements those of LISA and TaiJi, and advanced LIGO (aLIGO), resulting in an overall accessible GW frequency band equal to $(10^{-4}$–$10^3)$ Hz (see Figs. 1 and 2 below).

Regarding the gLISA onboard science payload components (primarily the laser, the optical telescope, and the inertial reference sensor), we will assume them to have a similar noise performance as those of LISA [16]. Other subsystems are regarded to have a noise performance that results in a high-frequency noise spectrum that is essentially determined by the photon-counting statistics. For further details, we refer the reader to Appendix A of Ref. [14].

Although the following analysis will focus on LISA and gLISA, similar results should hold with the pair of Chinese interferometers TaiJi and TianQin.

2. LISA–gLISA joint sensitivity

To derive the expression of the joint LISA–gLISA sensitivity, we first note that the noises in the TDI measurements made by the two arrays will be independent. This means that the joint signal-to-noise ratio squared, averaged over an ensemble of signals randomly distributed over the celestial sphere and random polarization states, $\langle \mathrm{SNR}^2_{\mathrm{L+gL}} \rangle$, can be written in the following form:

$$\begin{aligned} \left\langle \mathrm{SNR}^2_{\mathrm{L+gL}} \right\rangle &\equiv 4 \int_{f_1}^{f_2} \frac{\left|\widetilde{h}(f)\right|^2}{S_h^{\mathrm{L+gL}}(f)} \mathrm{d}f = \left\langle \mathrm{SNR}^2_{\mathrm{L}} \right\rangle + \left\langle \mathrm{SNR}^2_{\mathrm{gL}} \right\rangle \\ &= 4 \int_{f_1}^{f_2} \left\{ \frac{1}{S_h^{\mathrm{L}}(f)} + \frac{1}{S_h^{\mathrm{gL}}(f)} \right\} \left|\widetilde{h}(f)\right|^2 \, \mathrm{d}f \,, \end{aligned} \tag{1}$$

where the functions $S_h^i(f)$, $i = \mathrm{L},\ \mathrm{gL}$ correspond to the squared optimal sensitivities (as defined through the A, E, and T TDI combinations [8, 21])

[2] The derivation of the gLISA sensitivity and the related discussion on the magnitude of the noises that define it have been presented in the main body and Appendix of Ref. [14]. Although it was stated there that additional technology developments were needed to reduce the optical bench noise level below that of the photon-shot noise, the LISA Pathfinder experiment [20] has demonstrated this noise to be almost three orders of magnitude smaller than its anticipated value.

of the LISA and gLISA arrays, respectively. Note that the factor 4 in front of the integral in Eq. (1) reflects our adopted convention of using one-sided power spectral densities of the noises [22, 23].

From Eq. (1), it is straightforward to derive the following expression of the joint sensitivity squared, $S_h^{\mathrm{L+gL}}(f)$, in terms of the individual ones, $S_h^{\mathrm{L}}(f)$ and $S_h^{\mathrm{gL}}(f)$:

$$S_h^{\mathrm{L+gL}}(f) \equiv \frac{S_h^{\mathrm{L}}(f)\, S_h^{\mathrm{gL}}(f)}{S_h^{\mathrm{L}}(f) + S_h^{\mathrm{gL}}(f)}\,. \tag{2}$$

In Fig. 1, we plot the optimal sensitivities of LISA (solid black/blue) and gLISA (solid gray/red) averaged over sources randomly distributed over the sky and polarization states. For completeness, we have added the sensitivity of the third-generation aLIGO detector (first from the top/magenta) together with the amplitude of the GW signal GW150914 (second from the top/gray) as functions of the Fourier frequency f. Included are also *(i)* BBH with masses ranging from 10^3 to 10^7 solar masses emitting at a redshift $z = 3$, as well as *(ii)* the predicted ensemble of medium-mass BBH (gray/bright-green band) that sweep across the frequency band of space-based interferometers before coalescing in the band accessible by ground detectors. Such an ensemble was estimated by Sesana [15] shortly before the announcement made by aLIGO of the detection of a second signal emitted

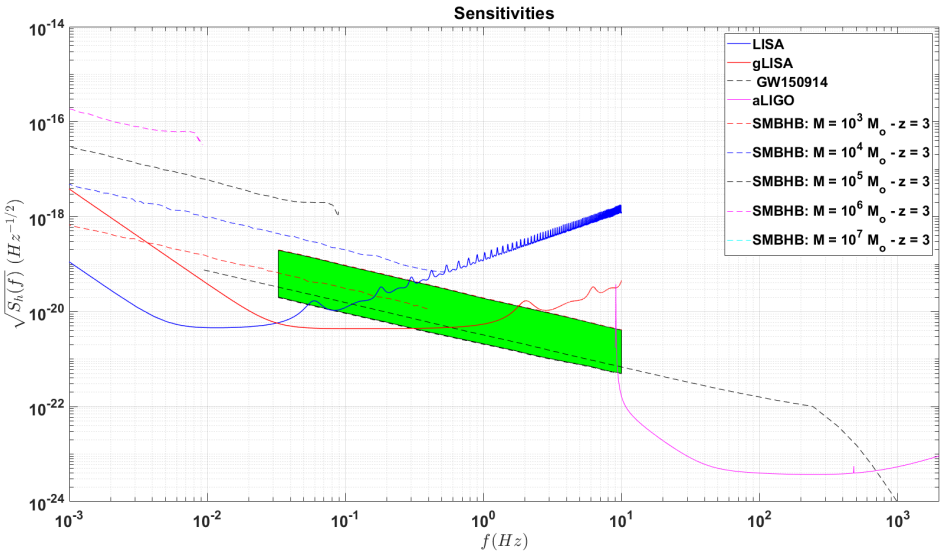

Fig. 1. (Color online) The optimal sensitivities of LISA (solid black/blue) and gLISA (solid gray/red) averaged over sources randomly distributed over the sky and polarization states. The third-generation aLIGO sensitivity curve and a set of signals' amplitudes across the band are also shown. See the main text for details.

by another black-hole binary system, GW151226, with masses roughly half of those of GW150914 [2, 25]. Sesana estimated that a large number of such systems [15] could be observed by LISA while they are still spiraling around each other for periods as long as the entire five years duration of the mission. Due to these compelling astrophysical reasons, to quantify the scientific advantages of flying gLISA jointly with LISA we will focus our attention on signals emitted by coalescing black-hole binaries with chirp-masses in the range (10–100) $M_{\odot}$.

In Fig. (2), we plot the joint LISA–gLISA sensitivity (dashed-thick black line) given by Eq. (2), which visually exemplifies the scientific advantages of flying simultaneously two space-based missions of different arm-lengths [24]. As expected, in the lower part of the mHz band, LISA defines the network sensitivity; in the overlapping region, the two sensitivities smoothly blend, resulting in a maximum sensitivity gain of $\sqrt{2}$. In the higher part of the accessible band, gLISA defines the joint LISA–gLISA sensitivity.

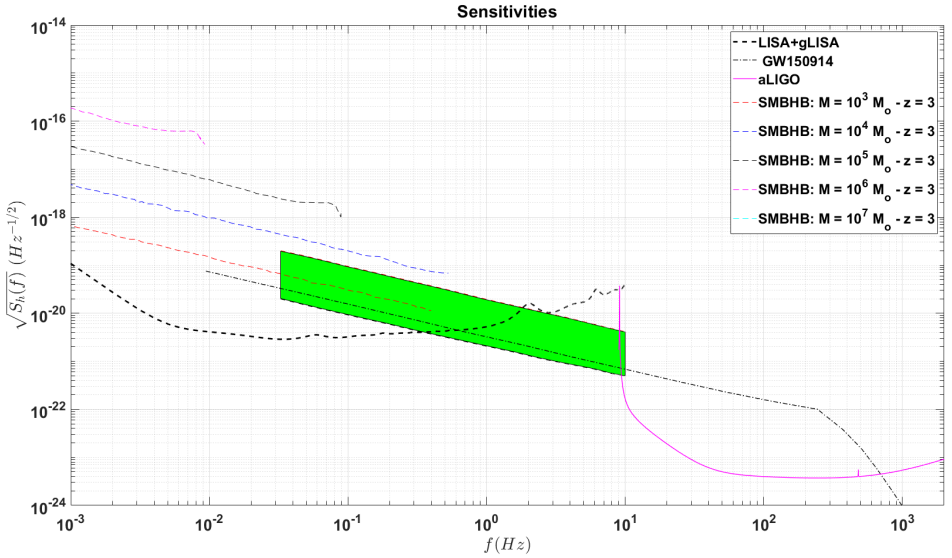

Fig. 2. The dashed-thick line (black) represents the optimal sensitivity achievable by combining the LISA and gLISA data when the two missions operate at the same time [24]. See the main text for details.

3. Results

In our analysis, we will use the following expressions (valid for circular orbits) of the Fourier transform of the amplitude of the GW signal emitted by such systems, $\widetilde{h}(f)$, and the time, $t_{\rm c}$, it takes them to coalesce [26]:

$$\widetilde{h}(f) = \sqrt{\frac{5}{24}}\,\pi^{-2/3}\left[\frac{(1+z)GM_{\rm c}}{c^3}\right]^{5/6}\frac{c}{D_{\rm L}}\,f^{-7/6}\,, \tag{3}$$

$$t_{\rm c} = \frac{5}{256}\left[\frac{(1+z)GM_{\rm c}}{c^3}\right]^{-5/3}(\pi f_{\rm gw})^{-8/3}\,. \tag{4}$$

In Eqs. (3) and (4), G is the gravitational constant, c is the speed of light, z is the cosmological redshift, $D_{\rm L}$ is the corresponding luminosity distance, $M_{\rm c} \equiv (M_1 M_2)^{3/5}/(M_1+M_2)^{1/5}$ is the chirp mass associated with the binary system whose components have masses M_1 and M_2, f is the Fourier frequency and $f_{\rm gw}$ the instantaneous frequency of the emitted GW.

By using the above expression for the signal amplitude and a time to coalescence of 5 years (which, in the case of the GW150914 signal, defines the lower-limit of integration in the integral of the SNR to be equal to 0.0151 Hz), we have estimated the SNRs achievable by the two interferometers when operating either as stand-alone or jointly. We find LISA can observe a GW150914-like signal with a SNR equal to 10.7, while gLISA with an SNR of 14.4 due to its better sensitivity over a larger part of its observable band (see Fig. 1). As expected, the joint LISA–gLISA network further improves upon the SNR of gLISA-alone by reaching a value of about 18.0. From these results, we can further infer that gLISA will achieve a sufficiently high SNR to warrant the detection of a GW150914-like signal by integrating for a shorter time. We find that, by integrating for a period that corresponds to 135 days prior to the moment of coalescence of a GW150914-like system, gLISA can achieve an SNR of 10.7.

The level of SNR achievable by the LISA–gLISA network with an integration time of five years is about 80 percent higher than that of LISA alone. This implies, from the estimated parameter precisions derived by Sesana [15] and their dependence on the value of the SNR [27], that the LISA–gLISA network will estimate the parameters associated with the GW source of the observed signal with a precision that is 1.8 better than that obtainable by LISA alone. It should be said, however, that this is a lower bound on the improved precision by which the parameters can be estimated since it is based only on SNR considerations. As pointed out by McWilliams in an unpublished document [28], the Doppler frequency shift together with the larger diurnal amplitude modulation experienced by the GW signal in the gLISA TDI measurements will further improve the precision of the reconstructed parameters beyond that due to only the enhanced SNR. We will analyze and quantify this point in a follow-up article.

From the expressions of the SNR and of the Fourier amplitude of the GW signal, $\tilde{h}(f)$ (Eqs. (1) and (3)), and by fixing the SNR to a specific value for each operational configuration (stand-alone *vs.* network), it is possible to infer the corresponding average luminosity distance to a BHB in terms of its

chirp-mass parameter[3]. By assuming an SNR = 10, the corresponding three luminosity distances are monotonically increasing functions of the chirp-mass. In [24] it was shown that, for chirp-masses in the range [10–36) $M_\odot$, gLISA can see signals further away than LISA due to its better sensitivity at higher frequencies. Systems with chirp-mass larger that 36 $M_\odot$ instead can be seen by LISA at a larger luminosity distance than that achievable by gLISA alone. The LISA–gLISA network instead out-performs the stand-alone configurations by as much as 40 percent for BHBs with chirp-masses in the interval (30–40) $M_\odot$. This results in a number of observable events that is about 3 times larger than that detectable by each interferometer alone. Finally, a GW150914-like signal characterized by a chirp-mass of 28.1 $M_\odot$ can be seen by LISA at an average luminosity distance of about 400 Mpc, by gLISA out to 600 Mpc, and by the LISA–gLISA system out to about 800 Mpc.

4. Comments and conclusions

Our analysis has quantified the scientific advantages of flying a smaller-size GW interferometer jointly with a "bigger sister" detector. Due to its smaller arm-length, an array such as gLISA or TianQin will survey the region of the GW frequency band that is in between those accessible by LISA and TaiJi, and aLIGO. By covering the entire mHz and kHz GW frequency band, LISA, gLISA (or TaiJi together with a smaller array), and aLIGO will detect all known sources emitting in this band, and observe signals requiring multi-band detection for better understanding the physical nature of their sources.

Since the early studies of the gLISA mission [14, 16], other mission concepts for accessing the GW mid-band frequency region have appeared in the literature with some pursued experimentally. The most notable example of this group is the TianQin mission [18]. This project has been making significant technical advances during the past few years and it would not be surprising to see it flying before the end of the next decade. An additional geosynchronous GW mission concept called SAGE [29] has been getting some attention for relying on cubesats. Although its expected sensitivity in its long-wavelength regime is significantly degraded over that of gLISA and TianQin due to its bare-bone architecture (which does not include an onboard drag-free system) its good sensitivity in the high-frequency region might scientifically complement LISA and the TaiJi missions.

[3] To derive the redshift, z, we have assumed a fiducial ΛCDM flat cosmology with the matter density parameter $\Omega_{\rm M} = 0.3$, the cosmological constant density parameter $\Omega_\Lambda = 0.7$, and the value of the Hubble parameter at the present time $H_0 = 70\ {\rm km\,s^{-1}\ Mpc^{-1}}$. To obtain the luminosity distance, $D_{\rm L}$, in terms of the chirp-mass, $M_{\rm c}$, we have rewritten the SNR as a function of the redshift by expressing $D_{\rm L}$ in terms of z (see Eq. (13) in [14]). Then, for a given SNR, we have solved a transcendental equation to obtain z, and derived from it the luminosity distance.

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