INSTABILITY OF A REISSNER–NORDSTRÖM–AdS BLACK HOLE FROM A DERIVATIVE COUPLING PERTURBATION*

B. Cuadros-Melgar

Escola de Engenharia de Lorena, Universidade de São Paulo
Estrada Municipal do Campinho S/N, 12602-810 Lorena, SP, Brazil

E. Abdalla

Instituto de Física, Universidade de São Paulo
Caixa Postal 66318, 05314-970 São Paulo, SP, Brazil

R.D.B. Fontana

Universidade Federal da Fronteira Sul, Campus Chapecó, 89802-112, SC, Brazil

Jeferson de Oliveira

Instituto de Física, Universidade Federal de Mato Grosso
78060-900 Cuiabá, MT, Brazil

Eleftherios Papantonopoulos

Department of Physics, National Technical University of Athens
Zografou Campus, 15773 Athens, Greece

A.B. Pavan

Instituto de Física e Química, Universidade Federal de Itajubá
37500-903 Itajubá, MG, Brazil

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We consider massive scalar perturbations coupled to the Einstein tensor, the so-called derivative coupling term, in the background of a Reissner–Nordström–AdS black hole. By studying the scalar potential, we identify

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the possible existence of instabilities due to the appearance of a negative well. This suspicion is confirmed through the calculation of the corresponding quasinormal modes. We show that there is a critical value of the derivative coupling that triggers this instability and that this critical value also depends on the black hole charge.

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1. Introduction

Scalar–tensor theories modifying Einstein gravity have been intensely studied in the last years. One of these theories, developed by Horndeski in 1974 [1], produces second order field equations and is built of a collection of Lagrangians which include first derivatives of a scalar field Φ. One of these terms corresponds to the kinetic coupling of a scalar field to Einstein tensor. The so-called derivative coupling (DC) term has proved to be useful in cosmological contexts [2] as it can mimic a friction term in the early inflationary evolution, provides a mechanism to suppress heavy particle overproduction after inflation [3], and can play the rôle of a cosmological constant introducing a new scale in the model. Moreover, several stability studies have been done using the DC term [4] yielding superradiance phenomena [5] and quasiresonant modes [6].

The aim of this work is to study the effect on the dynamics of the scalar field due to possible instabilities in AdS space owing to its natural boundary. We will find the critical value of the DC parameter that marks the transition between stability and instability. At the same time, we will analyze the interplay of DC and AdS scales.

2. Perturbation setup

We consider a perturbation given by a massive scalar field coupled to the Einstein tensor as

$$\mathcal{L}_{\text{pert}} = -\frac{\sqrt{-g}}{2}\left[\left(g^{\mu\nu} - \eta G^{\mu\nu}\right)\partial_\mu\Phi\partial_\nu\Phi + m^2\Phi^2\right] \tag{1}$$

in a spherically symmetric background given by

$$\mathrm{d}s^2 = -F(r)\ \mathrm{d}t^2 + F(r)^{-1}\ \mathrm{d}r^2 + r^2\ \mathrm{d}\Omega^2\,, \tag{2}$$

where $\mathrm{d}\Omega^2$ is the 2-sphere line element. The perturbation equation of motion obtained from Eq. (1) can be decoupled from the angular part by writing the scalar field as

$$\Phi(t, r, \theta, \varphi) = \sum_{l,\mathfrak{m}} \frac{Z(r,t)}{r(1+\eta A)^{1/2}} Y_{l,\mathfrak{m}}(\theta, \varphi)\,, \tag{3}$$

such that the remaining equation, once accommodated in a Schrödinger-like form, reads

$$-\frac{\partial^2 Z}{\partial t^2}+\frac{\partial^2 Z}{\partial r_*^2}-V_s(r)Z=0\,. \tag{4}$$

The effective potential V_s is given by

$$\begin{aligned} V_s(r) &= \frac{F}{(1+\eta A)}\left[\frac{l(l+1)}{r^2}(1-\eta B)+m^2+\frac{F'}{r}(1+\eta A)\right]+F^2V_\eta(r)\,, \\ V_\eta(r) &= \frac{\eta}{1+\eta A}\left(\frac{A''}{2}+\frac{A'F'}{2F}+\frac{A'}{r}\right)-\frac{1}{4}\left(\frac{\eta A'}{1+\eta A}\right)^2\,, \end{aligned} \tag{5}$$

and the functions A and B are, respectively,

$$A(r)=\left(-\frac{F'}{r}+\frac{1-F}{r^2}\right)\,,\qquad B(r)=A(r)-\frac{1}{2}\mathcal{R}\,. \tag{6}$$

We will work with a Reissner–Nordström (RN)–AdS black hole with metric coefficient given by

$$F=1-\frac{2M}{r}+\frac{Q^2}{r^2}+\frac{r^2}{L^2}\,. \tag{7}$$

The field transformation in Eq. (3) that we used to decouple the scalar equation has a discontinuity at

$$r_{\rm d}=\left(\frac{\eta Q^2}{\frac{3\eta}{L^2}-1}\right)^{1/4}\,. \tag{8}$$

Thus, we will choose the position of the event horizon such that $r_{\rm h}>r_{\rm d}$.

3. Results

3.1. Effective potentials

The potential in Eq. (5) is plotted in Fig. 1. It depends on 4 parameters, multipole number l, DC parameter η, perturbation mass m, and black hole charge Q. The most interesting features appear when we turn on the DC parameter. As η grows, the potential develops a negative well, initially hidden by the event horizon when η is small, which is shifted outside $r_{\rm h}$ for intermediate values of η. Thus, we see that η triggers the well emergence that can eventually lead to instabilities in the background metric. In fact, when the well becomes deep enough to provoke an instability, η reaches a critical value $\eta_{\rm c}$ that can be numerically calculated. In addition, the effect of the black hole charge and multipole number is to deepen the well. Finally, the effect of the scalar field mass as it grows is to make the well shallow. Therefore, the most interesting case where we can find instabilities corresponds to massless perturbations.

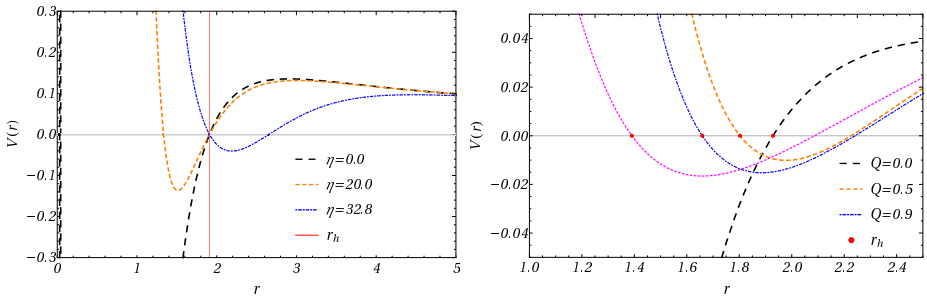


Fig. 1. Effective scalar potential with parameters $M = L/10 = 1$ and $m = 0$. Left panel: different values of η with $l = 1$ and $Q = 0.2$. Right panel: different values of charge Q, with $l = 0$ and $\eta = 30$.

3.2. Quasinormal modes

We show our results in Fig. 2. From the left panel, we see that the field evolution for multipole number $l = 0$ is always stable. For $l \neq 0$, there is a critical value of η for which the evolution becomes unstable. However, after certain value of η, stability returns and the field has a bounded evolution. The right panel shows the evolution for different charges. For Q less than a threshold value $Q_{\rm ext}$, the field decays with a ring-down signal and as Q approaches $Q_{\rm ext}$, the field has an exponential decay. After the threshold value, the field destabilizes the geometry and a phase transition is expected.

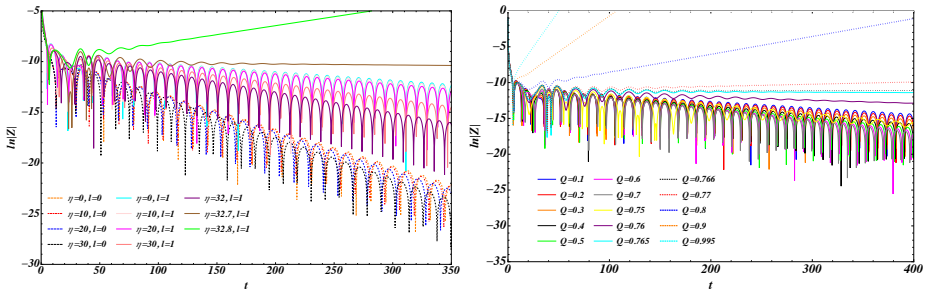


Fig. 2. Scalar field behavior for an AdS charged black hole with varying η for $M = L/10 = 5Q = 1$ (left) and different charges for $M = L/10 = \eta/20 = 1$ (right).

3.3. Critical η

The critical value $\eta_{\rm c}$ where the field evolution destabilizes depends on the parameters of the geometry, especially on the multipole number l. Some of these values are shown in Table I. It is possible to analyze this critical behavior by expanding the potential near the horizon as $V_{r_{\rm h}} \sim F(r)\Omega(r)$,

TABLE I

Critical value of η for different charges of the geometry (in unities of $Q_{\rm ext}$) and multipole numbers. For the geometry parameters, $Q_{\rm ext} \sim 0.99518$. The corresponding values of $\eta_{\rm lim}$ are also shown for reference.

$10M = L = 10$							
l	$Q = 0.1$	$Q = 0.2$	$Q = 0.4$	$Q = 0.6$	$Q = 0.8$	$Q = 0.95$	$Q = 0.99517$
1	$33.15^{\pm 0.05}$	$32.75^{\pm 0.05}$	$30.65^{\pm 0.05}$	$26.35^{\pm 0.05}$	$18.25^{\pm 0.05}$	$7.95^{\pm 0.05}$	$1.75^{\pm 0.05}$
2	$33.05^{\pm 0.05}$	$32.15^{\pm 0.05}$	$28.65^{\pm 0.05}$	$22.35^{\pm 0.05}$	$13.45^{\pm 0.05}$	$5.25^{\pm 0.05}$	$1.25^{\pm 0.05}$
3	$32.95^{\pm 0.05}$	$31.85^{\pm 0.05}$	$27.45^{\pm 0.05}$	$20.35^{\pm 0.05}$	$11.55^{\pm 0.05}$	$4.35^{\pm 0.05}$	$1.15^{\pm 0.05}$
Lowest limit of stability for large l using Eq. (11)							
∞	32.54	30.29	22.91	14.44	7.10	2.63	0.92

where

$$\Omega(r_{\rm h}) = \frac{m^2 r_{\rm h}^2 + l(l+1)\left[1-\left(\frac{3}{L^2}+\frac{Q^2}{r_{\rm h}^4}\right)\eta\right]}{r_{\rm h}^2\left[1+\left(-\frac{3}{L^2}+\frac{Q^2}{r_{\rm h}^4}\right)\eta\right]} + \frac{F'(r_{\rm h})}{r_{\rm h}} - \frac{2Q^2\eta F'(r_{\rm h})}{r_{\rm h}^5\left[1+\left(-\frac{3}{L^2}+\frac{Q^2}{r_{\rm h}^4}\right)\eta\right]}\,. \tag{9}$$

Since $F(r)$ is a positive function, the change of sign in the potential comes from $\Omega(r_{\rm h})$ and occurs at

$$\eta_0 \sim \frac{-3L^2 r_{\rm h}^8 + L^4 r_{\rm h}^4\left[Q^2 - r_{\rm h}^2\left(1+l(l+1)+m^2 r_{\rm h}^2\right)\right]}{\left(L^2Q^2+3r_{\rm h}^4\right)\left\{-3r_{\rm h}^4 + L^2\left[Q^2 - r_{\rm h}^2(1+l(l+1))\right]\right\}}\,. \tag{10}$$

When $m = 0$, Eq. (10) becomes independent of l,

$$\eta_{\rm lim} \sim \frac{L^2 r_{\rm h}^4}{L^2Q^2+3r_{\rm h}^4}\,, \tag{11}$$

which also corresponds to the limit $l \to \infty$. Moreover, our calculations show that as l increases, $\eta_{\rm c}$ approaches $\eta_{\rm lim}$, as we can see in Table I.

4. Conclusions

We studied the influence of a derivative coupling perturbation in a RN–AdS background. By analyzing the effective potential of the equation of motion of the scalar field perturbation, we determined the influence of multipole number l, perturbation mass m, black hole charge Q, and DC parameter η

on the perturbation dynamics. Massless perturbations display a potential with a negative well that is shifted out of the event horizon by η and whose depth depends on the black hole charge and multipole number. The presence of this well opens the possibility of having instabilities, a fact that was confirmed by our numerical results of the field evolution. For $l = 0$, no instability was found, while for $l \neq 0$, instabilities appear at a critical value of DC parameter η_c, and keep for a certain range after which we recover stability. This η_c strongly depends on the extremal value of the charge Q_{ext}. As $Q \to Q_{ext}$, η_c decreases. We found that the transitions from stability to instability and back may signal the scalarization of RN–AdS black hole [7].

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